SHORT COMMUNICATIONS

IRREGULAR COULOMB WAVE FUNCTIONS*

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In a recent paper (Learner and Robson 1958), henceforth referred to as I, a skeleton table of values of the regular Coulomb wave functions was given in the integral representation. Here, the values of g_L and g'_L (defined in I) are presented in Table 1 for the same range of parameters, $\eta \leqslant 10$, $L=5$, 6, 10, and 11, $\Delta\rho=1$, and $\Delta\eta=1$.

In the notation of I,§ the required values of G_L and G'_L are obtained by using the tabulated g_L and g'_L and noting that

$$G_L=A_L\rho^{L+1}g_L, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1a)$$

$$G'_L=A_L\rho^{L+1}\{g'_L+(L+1)g_L/\rho\}. \quad \ldots\ldots\ldots\ldots \quad (1b)$$

As for the regular functions, intermediate and other L values may be obtained by the recurrence formulae:

$$\rho^2Lg_{L+1}=2(2L+1)\{\eta\rho+L(L+1)\}g_L-4(L+1)(L^2+\eta^2)g_{L-1}, \quad \ldots (2a)$$

$$\rho Lg'_L=2(L^2+\eta^2)g_{L-1}-\{\eta\rho+L(2L+1)\}g_L, \quad \ldots\ldots\ldots\ldots \quad (2b)$$

and

$$2(L+1)g'_L=2\eta g_L-\rho g_{L+1}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2c)$$

The table was checked in three ways using (i) the above recurrence formulae, (ii) the Wronskian relation $F'_LG_L-G'_LF_L=1$, and (iii) the tables of Coulomb wave functions for $L=0$ (Tubis 1958). The accuracy was found to be at least 0·1 per cent.

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§ Equations **(3e)** and **(3f)** of I should read: $A_L=2^{-1}(L^2+\eta^2)^{-\frac{1}{2}}A_{L-1}$ and $A_0=\{C_0(\eta)\}^{-1}.e^{-\pi\eta}$ respectively.

TABLE 1

VALUES OF g AND g'

ρ	$L=5$		$L=6$		$L=10$		$L=11$	
	g_5	g'_5	g_6	g'_6	g_{10}	g'_{10}	g_{11}	g'_{11}
				$\eta=10$				
10	0·4074 5	−0·6882 5	0·1641 6	−0·3020 6	0·1531 9	−0·3826 9	0·1148 10	−0·3069 10
11	0·8174 4	−0·1248 5	0·2847 5	−0·4743 5	0·1418 8	−0·3212 8	0·9006 8	−0·2184 9
12	0·1910 4	−0·2642 4	0·5825 4	−0·8804 4	0·1630 7	−0·3366 7	0·8888 7	−0·1966 8
13	0·5115 3	−0·6409 3	0·1379 4	−0·1892 4	0·2259 6	−0·4270 6	0·1070 7	−0·2168 7
14	0·1554 3	−0·1758 3	0·3727 3	−0·4641 3	0·3699 5	−0·6410 5	0·1536 6	−0·2858 6
15	0·5314 2	−0·5396 2	0·1140 3	−0·1283 3	0·7028 4	−0·1118 5	0·2577 5	−0·4411 5
16	0·2038 2	−0·1837 2	0·3926 2	−0·3958 2	0·1530 4	−0·2233 4	0·4984 4	−0·7853 4
17	0·8760 1	−0·6905 1	0·1518 2	−0·1355 2	0·3786 3	−0·5057 3	0·1100 4	−0·1593 4
18	0·4206 1	−0·2863 1	0·6582 1	−0·5131 1	0·1057 3	−0·1286 3	0·2745 3	−0·3645 3
19	0·2233 1	−0·1318 1	0·3183 1	−0·2154 1	0·3320 2	−0·3650 2	0·7721 2	−0·9344 2
				$\eta=9$				
10	0·5486 4	−0·8736 4	0·2036 5	−0·3561 5	0·1468 8	−0·3560 8	0·1048 9	−0·2728 9
11	0·1214 4	−0·1733 4	0·3876 4	−0·6093 4	0·1464 7	−0·3208 7	0·8809 7	−0·2074 8
12	0·3138 3	−0·4016 3	0·8722 3	−0·1233 4	0·1813 6	−0·3607 6	0·9334 6	−0·1998 7
13	0·9350 2	−0·1068 3	0·2281 3	−0·2897 3	0·2715 5	−0·4919 5	0·1208 6	−0·2360 6
14	0·3186 2	−0·3222 2	0·6858 2	−0·7780 2	0·4809 4	−0·7946 4	0·1867 5	−0·3334 5
15	0·1236 2	−0·1091 2	0·2356 2	−0·2363 2	0·9926 3	−0·1495 4	0·3383 4	−0·5526 4
16	0·5442 1	−0·4133 1	0·9222 1	−0·8061 1	0·2360 3	−0·3229 3	0·7096 3	−0·1059 4
17	0·2692 1	−0·1756 1	0·4090 1	−0·3086 1	0·6422 2	−0·7935 2	0·1707 3	−0·2319 3
18	.....	.....	.....	.....	.....	.....	.....	.....
19	0·7553 0	−0·4629 0	0·1081 1	−0·6525 0	0·7036 1	−0·6884 1	0·1464 2	−0·1609 2
20	0·4684 0	−0·2856 0	0·5931 0	−0·3615 0	0·2806 1	−0·2423 1	0·5191 1	−0·5063 1
				$\eta=8$				
10	0·7879 3	−0·1168 4	0·2663 4	−0·4387 4	0·1442 7	−0·3384 7	0·9751 7	−0·2467 8
11	0·1949 3	−0·2559 3	0·5626 3	−0·8246 3	0·1542 6	−0·3285 6	0·8834 6	−0·2014 7
12	0·5674 2	−0·6559 2	0·1412 3	−0·1837 3	0·2089 5	−0·3986 5	0·1010 6	−0·2084 6
13	0·1926 2	−0·1938 2	0·4159 2	−0·4766 2	0·3402 4	−0·5873 4	0·1412 5	−0·2646 5
14	0·7575 1	−0·6531 1	0·1426 2	−0·1421 2	0·6582 3	−0·1028 4	0·2367 4	−0·4030 4
15	0·3417 1	−0·2507 1	0·5651 1	−0·4837 1	0·1493 3	−0·2101 3	0·4674 3	−0·7219 3
16	.....	.....	.....	.....	.....	.....	.....	.....
17	0·9363 0	−0·5598 0	0·1276 1	−0·8420 0	0·1203 2	−0·1340 2	0·2866 2	−0·3584 2
18	0·5091 0	−0·3242 0	0·6687 0	−0·4317 0	0·4236 1	−0·4140 1	0·8825 1	−0·9781 1
19	0·2503 0	−0·2048 0	0·3401 0	−0·2472 0	0·1691 1	−0·1460 1	0·3115 1	−0·3043 1
20	0·8467 −1	−0·1310 0	0·1464 0	−0·1492 0	0·7423 0	−0·5878 0	0·1240 1	−0·1080 1
				$\eta=7$				
10	0·1231 3	−0·1668 3	0·3724 3	−0·5699 3	0·1456 6	−0·3295 6	0·9289 6	−0·2278 7
11	0·3480 2	−0·4080 2	0·8881 2	−0·1189 3	0·1711 5	−0·3465 5	0·9107 5	−0·2003 6
12	0·1176 2	−0·1176 2	0·2547 2	−0·2956 2	0·2510 4	−0·4561 4	0·1129 5	−0·2236 5
13	0·4686 1	−0·3951 1	0·8694 1	−0·8624 1	0·4488 3	−0·7312 3	0·1721 4	−0·3073 4
14	.....	.....	.....	.....	.....	.....	.....	.....
15	0·1090 1	−0·7121 0	0·1587 1	−0·1168 1	0·2443 2	−0·3145 2	0·6891 2	−0·9920 2
16	0·5670 0	−0·3804 0	0·7814 0	−0·5425 0	0·7309 1	−0·8237 1	0·1772 2	−0·2263 2
17	0·2723 0	−0·2257 0	0·3836 0	−0·2870 0	0·2550 1	−0·2508 1	0·5345 1	−0·5997 1
18	0·9394 −1	−0·1376 0	0·1648 0	−0·1634 0	0·1010 1	−0·8856 0	0·1868 1	−0·1841 1
19	−0·1185 −1	−0·7713 −1	0·3999 −1	−0·9168 −1	0·4328 0	−0·3591 0	0·7348 0	−0·6531 0
20	−0·6533 −1	−0·3199 −1	−0·2654 −1	−0·4435 −1	0·1866 0	−0·1624 0	0·3093 0	−0·2641 0

TABLE 1 (*Continued*)

ρ	$L=5$				$L=6$				$L=10$				$L=11$			
	g_5		g'_5		g_6		g'_6		g_{10}		g'_{10}		g_{11}		g'_{11}	
							$\eta=6$									
10	0·2162	2	−0·2582	2	0·5693	2	−0·7905	2	0·1518	5	−0·3297	5	0·9076	5	−0·2149	6
11	0·7240	1	−0·7223	1	0·1578	2	−0·1863	2	0·1958	4	−0·3777	4	0·9692	4	−0·2047	5
12	.....		.....		.....		.....		.....		.....		.....		.....	
13	0·1338	1	−0·9780	0	0·2138	1	−0·1806	1	0·6334	2	−0·9582	2	0·2207	3	−0·3718	3
14	0·6542	0	−0·4692	0	0·9629	0	−0·7356	0	0·1534	2	−0·2031	2	0·4507	2	−0·6729	2
15	0·3047	0	−0·2566	0	0·4491	0	−0·3508	0	0·4462	1	−0·5122	1	0·1108	2	−0·1455	2
16	0·1077	0	−0·1473	0	0·1913	0	−0·1851	0	0·1530	1	−0·1530	1	0·3251	1	−0·3717	1
17	−0·3085	−2	−0·7871	−1	0·5340	−1	−0·9857	−1	0·5929	0	−0·5377	0	0·1114	1	−0·1120	1
18	−0·5657	−1	−0·3107	−1	−0·1699	−1	−0·4616	−1	0·2427	0	−0·2173	0	0·4275	0	−0·3945	0
19	−0·7027	−1	0·1203	−2	−0·4512	−1	−0·1275	−1	0·9481	−1	−0·9608	−1	0·1712	0	−0·1582	0
20	−0·5879	−1	0·1955	−1	−0·4700	−1	0·7004	−2	0·2861	−1	−0·4299	−1	0·6448	−1	−0·6848	−1
							$\eta=5$									
11	0·1788	1	−0·1514	1	0·3277	1	−0·3300	1	0·2358	3	−0·4288	3	0·1072	4	−0·2160	4
12	0·7979	0	−0·6266	0	0·1292	1	−0·1123	1	0·4298	2	−0·6828	2	0·1611	3	−0·2868	3
13	0·3550	0	−0·3067	0	0·5562	0	−0·4644	0	0·9787	1	−0·1344	2	0·3028	2	−0·4735	2
14	0·1283	0	−0·1627	0	0·2312	0	−0·2210	0	0·2738	1	−0·3229	1	0·7029	1	−0·9553	1
15	0·9057	−2	−0·8236	−1	0·7195	−1	−0·1100	0	0·9113	0	−0·9391	0	0·1985	1	−0·2333	1
16	−0·4596	−1	−0·3128	−1	−0·5239	−2	−0·4977	−1	0·3398	0	−0·3254	0	0·6598	0	−0·6846	0
17	−0·5981	−1	0·8261	−3	−0·3575	−1	−0·1439	−1	0·1295	0	−0·1289	0	0·2430	0	−0·2370	0
18	−0·4941	−1	0·1770	−1	−0·3926	−1	0·5241	−2	0·4330	−1	−0·5426	−1	0·9038	−1	−0·9272	−1
19	−0·2857	−1	0·2221	−1	−0·2907	−1	0·1353	−1	0·7413	−2	−0·2170	−1	0·2906	−1	−0·3801	−1
20	−0·7839	−2	0·1811	−1	−0·1478	−1	0·1396	−1	−0·5753	−2	−0·6529	−2	0·4349	−2	−0·1456	−1
							$\eta=4$									
10	0·1068	1	−0·9573	0	0·2003	1	−0·2089	1	0·1870	3	−0·3654	3	0·9535	3	−0·2074	4
11	0·4401	0	−0·3987	0	0·7550	0	−0·7022	0	0·3037	2	−0·5119	2	0·1245	3	−0·2368	3
12	0·1604	0	−0·1897	0	0·2967	0	−0·2875	0	0·6377	1	−0·9178	1	0·2108	2	−0·3484	2
13	0·2578	−1	−0·8975	−1	0·9873	−1	−0·1302	0	0·1689	1	−0·2070	1	0·4543	1	−0·6455	1
14	−0·3325	−1	−0·3330	−1	0·9557	−2	−0·5640	−1	0·5375	0	−0·5792	0	0·1217	1	−0·1486	1
15	−0·4866	−1	−0·7718	−3	−0·2535	−1	−0·1739	−1	0·1892	0	−0·1950	0	0·3869	0	−0·4199	0
16	−0·4038	−1	0·1499	−1	−0·3145	−1	0·2763	−2	0·6500	−1	−0·7415	−1	0·1345	0	−0·1410	0
17	−0·2276	−1	0·1855	−1	−0·2382	−1	0·1087	−1	0·1692	−1	−0·2876	−1	0·4519	−1	−0·5278	−1
18	−0·5835	−2	0·1444	−1	−0·1225	−1	0·1136	−1	−0·980	−3	−0·9601	−2	0·1130	−1	−0·1990	−1
							$\eta=3$									
10	0·2147	0	−0·2432	0	0·4207	0	−0·4352	0	0·2268	2	−0·4111	2	0·1041	3	−0·2142	3
11	0·5002	−1	−0·1047	0	0·1415	0	−0·1698	0	0·4279	1	−0·6539	1	0·1541	2	−0·2724	2
12	−0·1781	−1	−0·3819	−1	0·2930	−1	−0·6865	−1	0·1051	1	−0·1357	1	0·3013	1	−0·4527	1
13	−0·3691	−1	−0·3887	−2	−0·1346	−1	−0·2241	−1	0·3141	0	−0·3590	0	0·7525	0	−0·9653	0
14	−0·3186	−1	0·1148	−1	−0·2350	−1	−0·5611	−3	0·1023	0	−0·1153	0	0·2250	0	−0·2584	0
15	−0·1796	−1	0·1472	−1	−0·1896	−1	0·7955	−2	0·3055	−1	−0·4107	−1	0·7246	−1	−0·8273	−1
16	−0·4683	−2	0·1113	−1	−0·1011	−1	0·8879	−2	0·5066	−2	−0·1415	−1	0·2138	−1	−0·2907	−1
							$\eta=2$									
10	0·1584	−2	−0·4825	−1	0·5856	−1	−0·9351	−1	0·2996	1	−0·4939	1	0·1207	2	−0·2314	2
11	−0·2445	−1	−0·8989	−2	0·925	−3	−0·3102	−1	0·6643	0	−0·9163	0	0·2075	1	−0·3342	1
12	−0·2396	−1	0·7240	−2	−0·1523	−1	−0·5033	−2	0·1826	0	−0·2240	0	0·4717	0	−0·6438	0
13	−0·1413	−1	0·1093	−1	−0·1444	−1	0·4771	−2	0·5399	−1	−0·6727	−1	0·1305	0	−0·1601	0
14	−0·4190	−2	0·8323	−2	−0·8336	−2	0·6537	−2	0·1356	−1	−0·2197	−1	0·3841	−1	−0·4807	−1
							$\eta=1$									
10	−0·1634	−1	0·2151	−2	−0·5876	−2	−0·1155	−1	0·4326	0	−0·6474	0	0·1511	1	−0·2653	1
11	−0·1085	−1	0·7241	−2	−0·9899	−2	0·1163	−2	0·1070	0	−0·1420	0	0·3036	0	−0·4465	0
12	−0·3928	−2	0·5997	−2	−0·6660	−2	0·4291	−2	0·2827	−1	−0·3895	−1	0·7630	−1	−0·1006	0

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