THE OPTIMUM LINE WIDTH FOR THE TRANSITION USED IN A REFLECTION CAVITY MASER AMPLIFIER

By G. J. Troup*

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Summary

The line width of the amplifying transition in a reflection cavity maser is shown to have an optimum value, which will give maximum amplification bandwidth at a fixed gain. Difficulties associated with achieving the optimum line width in practice for the paramagnetic maser are briefly discussed.

I. Introduction

The bandwidth of a resonant cavity maser amplifier at a fixed gain depends upon the line width of the amplifying transition and is always considerably less than this line width (Gordon, Zeiger, and Townes 1955 ; Gordon and White 1958 ; Stich 1958). It will be shown below that, for a given reflection cavity maser system, there is an optimum transition line width which will give maximum amplification bandwidth at constant gain. The value of the optimum line width is a function of various amplifier parameters (frequency, filling factor for the active medium, etc.), of the dipole moment for the transition, and of the concentration of excess molecules in the emissive condition. (Molecule will be used throughout as a generic term.)

In order to take full advantage of the low noise input temperature of a maser amplifier, it must be operated at a gain of the order of 20 dB if followed by a conventional microwave receiver (see, for example, Wittke 1957). At such gains, reported cavity maser bandwidths are narrow ; for example, Morris, Kyhl, and Strandberg (1959) report a bandwidth of 4 Mc/s at $\sim$9000 Mc/s with a gain of 20 dB, actually a vast improvement on previously reported masers (for example, McWhorter and Meyer 1958 ; Autler and McAvoy 1958). Bandwidth increase is desirable, and the knowledge of the optimum transition line width is therefore useful.

Moreover, any limitation of the concentration of excess upper-state molecules in a maser also limits the obtainable bandwidth, and operation of the system at optimum line width may then be the only method of bandwidth increase. This is important in the case of the three-level paramagnetic maser (Bloembergen 1956), for the device cannot be placed in an emissive condition when the concentration of paramagnetic ions exceeds a certain value. One is led to the idea that the maximum bandwidth obtainable with a Bloembergen cavity maser will

* Weapons Research Establishment, Salisbury, S. Aust.

occur when the concentration of active centres is close to the limiting value and operation at optimum line width is used. A prediction of the ultimate bandwidth obtainable from a Bloembergen system then becomes possible, but this will not be undertaken here.

II. The Reflection Cavity Maser

Only the reflection cavity maser is considered, since it is superior in all respects (noise factor, bandwidth at a fixed gain, etc.) to the transmission cavity amplifier (Gordon and White 1958). Maser performance depends upon the " molecular Q " Q_m, which is defined by

$$-Q_m=\frac{2\pi f\times\text{Average energy stored in the cavity}}{\text{Average power emitted in the cavity}}. \quad \ldots . \quad (1)$$

Here, f is the resonant frequency of the cavity, taken as equal to the frequency of amplification and to the central frequency of the molecular response.

It can be shown (e.g. Strandberg 1957) that Q_m is of the form

$$|Q_m|=(K\delta/N^*), \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

where δ is the line width of the amplifying transition, N^* is the number density of the excess molecules in the emissive state, and K is a constant for the particular molecules and amplifier system. K includes the square of the dipole moment for the amplifying transition and the filling factor for the active medium in the cavity. (This last is the ratio of the field energy coupled to the molecules to the total field energy stored in the cavity.)

Subject to certain assumptions, the gain G of the reflection cavity maser can be written

$$G\simeq(|Q_m|+Q_L)^2(|Q_m|-Q_L)^{-2}, \quad \ldots\ldots\ldots\ldots \quad (3)$$

where Q_L is the loaded Q of the cavity. The assumptions are :

(i) that the gain has a reasonably high value (>10, say) ;

(ii) that the cavity has a high unloaded Q, so that the loaded Q is determined mainly by the external coupling to the cavity (McWhorter and Meyer 1958).

The bandwidth B of the reflection cavity maser, after some manipulation of the results of Gordon and White (1958) and of Stich (1958) (which are identical), can be written

$$B\simeq\delta[(|Q_m|/Q_L)-1][1+(\delta\,|Q_m|/f)]^{-1}, \quad \ldots\ldots\ldots\ldots \quad (4)$$

where f is the centre frequency of the molecular response and of the cavity resonance, as previously ; a Lorentz line shape is assumed for the transition. The amplification bandwidth B is seen to be always less than the line width δ, both because of the factor $[(|Q_m|/Q_L)-1]$, which may be regarded as the gain-dependent term (see equation (3)), and because of the form of the remainder of the expression.

III. Optimum Line Width for the Reflection Cavity Maser

From equation (2), any alteration in the line width δ will alter the molecular Q, and hence (equation (3)) the gain. However, the gain can be kept constant by suitably adjusting the loaded Q of the cavity. Then only the terms $\delta[1+(\delta \mid Q_m \mid /f)]^{-1}$ need be considered to determine whether the transition line width has an optimum value at constant gain. Using equation (2), the expression governing the bandwidth is rewritten as

$$\delta[1+(\delta \mid Q_m \mid /f)]^{-1}=b=\delta[1+(K\delta^2/N^*f)]^{-1}. \quad \ldots\ldots \quad (5)$$

N^*, the concentration of excess emissive molecules, is taken as constant for the amplifier system. Differentiation of this expression with respect to δ then shows that b has a maximum value

$$b_{\mathrm{max.}}=\tfrac{1}{2}(K/N^*f)^{-\frac{1}{2}}, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

when

$$\delta=(K/N^*f)^{-\frac{1}{2}}=2b_{\mathrm{max.}}.$$

Hence the amplification bandwidth of a given reflection cavity maser system can be given a maximum value by the adjustment of the line width of the amplifying transition to an optimum value.

Two conditions must be fulfilled before the optimum line width can be realized. Firstly, alteration of the line width must not alter N^*, the concentration of excess emissive molecules, i.e. the efficiency of excitation must remain constant. Secondly, any method used to alter the line width must not affect the magnitude of the dipole moment for the amplifying transition (contained in the factor K). This last requirement is not unrealistic. In the case of the three-level paramagnetic maser, the energy levels are mixtures of states, dependent on the strength and direction of a steady magnetic field: the use of magnetic field inhomogeneity to increase the line width might therefore affect the dipole moment of the amplifying transition.

For completeness, an important practical case is considered next. When paramagnetic ions are diluted in a host lattice, the transition line width can be directly proportional to the concentration of the ions, N (Bogle and Symmons 1959). Since N^*, the concentration of excess ions in the emissive state, will also be directly proportional to the total ionic concentration N, the molecular Q can be a constant, independent of the line width (equation (2)). If variation of the line width is then carried out solely by variation of the ionic concentration, equation (5) shows that the best width is the broadest obtainable; for

$$\left.\begin{array}{r} b\rightarrow(f/\mid Q_m\mid) \text{ as } \delta\rightarrow\infty, \\ \text{i.e. as } N\rightarrow\infty, \end{array}\right\} \quad \ldots\ldots\ldots\ldots\ldots \quad (7)$$

since δ/N is a constant.

Equations (6) and (7) show that the cavity maser amplification bandwidth is influenced by the concentration of excess emissive molecules. Therefore, limiting this concentration also limits the bandwidth at constant gain. The fact that operation of the Bloembergen maser cannot be achieved when the concentration of paramagnetic ions reaches a certain critical value (Autler and McAvoy 1958 ; Giordmaine *et al.* 1958) indicates that adjustment of the transition line width may become an important method of bandwidth increase (see also Bogle and Symmons 1959).

IV. Practical Considerations for Paramagnetic Masers

The actual realization of the optimum line width for a given maser system may not be easy. Increasing the effective line width for paramagnetic transitions might be done relatively simply, by the use of magnetic-field inhomogeneity. Decreasing the line width is more difficult, since this will involve changing the maser crystal in some way—for example, by using special isotopes to reduce hyperfine broadening. It is desirable to work with a substance in which the line width is less than the optimum width for a given maser system, but which will still give a low molecular Q when the line width is increased. Much research into the causes of line widths of paramagnetic ions in crystals still needs to be done ; in many cases the line widths are considerably in excess of calculated spin-spin widths (Bogle and Symmonds 1959).

V. Conclusion

It has been shown that there is an optimum line width for the transition used in a cavity maser, such that the amplification bandwidth is a maximum at a fixed gain. The realization of this optimum line width may be difficult in practice, since the line width must be variable without changing any other amplifier parameters.

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