SCATTERING OF HIGH ENERGY POLARIZED NUCLEONS BY COMPLEX NUCLEI

By B. A. Robson*

[*Manuscript received May* 20, 1959]

Summary

The elastic scattering of 220 MeV polarized nucleons by carbon and calcium is studied using a new expression for the polarization. Except at small angles of scattering, substantial agreement is obtained with the results of calculations using a simplified form of the WKB method. Optical model parameters are found which are compared with those obtained by other workers for this energy region.

I. Introduction

The elastic scattering of polarized protons by complex nuclei has been investigated (Hafner 1958) for incident energies of 220 MeV with sufficient accuracy to permit a severe test of the optical model. At this energy, the polarization shows large fluctuations with scattering angle. For most scattering angles, the polarization as usually defined (equation (**3**)) is large and positive. However, the polarization is negative for those angles for which the differential cross section shows a minimum. Both the latter two effects are diffraction phenomena associated with the scattering.

The theoretical interpretation of the experimental data has generally been carried out by the addition of a spin-orbit term to the complex central potential of the optical model. Fermi (1954) obtained qualitative agreement for the polarization of 340 MeV protons by carbon using the Born approximation. However, the Born approximation is not even qualitatively correct at large angles. Other workers have used the WKB approximation in a partial wave analysis. This method is discussed in Section III (*b*). It has been found to give excellent fits to the experimental data at all except small angles for a Woods-Saxon (1954) complex central potential with a spin-orbit term of the Thomas type. Unfortunately, the partial wave method tends to conceal the physics of the problem, which can be more easily understood in an approach such as the Born approximation. It is for this reason that an attempt has been made to obtain a closed formula for the polarization which does not have the defects of the Born approximation.

II. Approximations for Polarization

The nuclear potential is taken to be

$$V(r)=-(V+\mathrm{i}W)g(r)+\gamma r^{-1}(\mathrm{d}g/\mathrm{d}r)\mathbf{L}\,.\,\boldsymbol{\sigma}, \qquad \ldots\ldots\ldots \quad (\mathbf{1})$$

where $g(r)$ is the shape of the central potential, V and W are positive real functions so that the central force is attractive and absorptive, and γ is a positive real

* Physics Department, University of Melbourne.

quantity corresponding to the sign of the spin-orbit force in the shell model. $\mathbf{L}$ and $\boldsymbol{\sigma}$ represent the orbital and spin angular momenta of the nucleon.

The differential cross section and the polarization for the elastic scattering of unpolarized nucleons by a spinless target are given by

$$d\sigma(\theta)/d\Omega = |A(\theta)+B(\theta)\boldsymbol{\sigma}\,.\,\mathbf{n}|^2 = A^*A + B^*B, \quad \ldots\ldots\ldots (2)$$

and

$$\mathbf{P}(\theta) = P(\theta)\mathbf{n} = (A^*B + B^*A)\mathbf{n}/(A^*A+B^*B), \quad \ldots\ldots (3)$$

where $A(\theta)$ and $B(\theta)$ are the amplitudes for spin-independent and spin-dependent scattering respectively and $\mathbf{n}$ is a unit vector given by

$$\mathbf{n} = \mathbf{k}_i \times \mathbf{k}_f / k^2 \sin\theta, \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

where $\mathbf{k}_i$ and $\mathbf{k}_f$ are $\hbar^{-1}$ times the initial and final momenta of the nucleon.

In the Born approximation, it may be shown that for neutrons

$$A(\theta) = 2m(V+iW)\hbar^{-2}\int_0^\infty j_0(Kr)r^2g(r)dr, \quad \ldots\ldots\ldots\ldots (5)$$

$$B(\theta) = i2m\gamma k^2\hbar^{-2}\sin\theta\int_0^\infty j_0(Kr)r^2g(r)dr, \quad \ldots\ldots\ldots\ldots (6)$$

where $K=2k\sin\frac{1}{2}\theta$, m is the reduced mass of the nucleon, and j_0 is the zero order spherical Bessel function. Hence

$$P(\theta) = \frac{2\gamma k^2 W\sin\theta}{V^2+W^2+\gamma^2k^4\sin^2\theta}. \quad \ldots\ldots\ldots\ldots\ldots (7)$$

This result for the polarization has the following defects :

(*a*) the polarization is proportional to the absorption potential, so that $P(\theta)=0$ for $W=0$;

(*b*) the polarization is independent of $g(r)$ and is therefore the same for all nuclei for a given incident energy (assuming γ and W constant) ;

(*c*) the polarization vanishes only when $\theta=0$ or π for finite W.

These defects are a consequence of $A(\theta)$ and $B(\theta)$ having the same angular dependence apart from the $\sin\theta$ factor, so that the integral over r is the same for both amplitudes.

The Montroll and Greenberg (MG) approximation has been shown (Mohr and Robson 1956) to give better results for the differential cross section than the Born approximation, although both approximations are rather unsatisfactory for complex potentials, when absolute magnitudes are concerned. If the approximate Montroll and Greenberg (1952) wave function is used instead of a plane wave, one obtains for neutrons :

$$A(\theta) = 2m(V+iW)\hbar^{-2}p\int_0^\infty \{j_0(K_-r)+qj_0(K_+r)\}r^2g(r)dr, \quad \ldots\ldots (8)$$

$$B(\theta) = i2m\gamma kk_1\hbar^{-2}p\sin\theta\int_0^\infty \{j_0(K_-r)+qj_0(K_+r)\}r^2g(r)dr, \quad \ldots\ldots (9)$$

where $K_{\pm}=\{k^2+k_1^2\pm2kk_1\cos\theta\}^{\frac{1}{2}}$ (N.B.—Im $\{k_1\}$ is neglected here),

$$\begin{aligned} p&=2k_1\exp\{\mathrm{i}a(k_1-k)\}/(k_1+k)(1-q^2),\\ q&=(k_1-k)\exp\{2\mathrm{i}ak_1\}/(k_1+k),\\ k_1^2&=k^2-U,\\ U&=-2m(V+\mathrm{i}W)\hbar^{-2}, \end{aligned}$$

and a is the radius of the equivalent sphere of the diffuse boundary spherical scatterer $g(r)$. The assumption involved is that the MG approximate wave function does not differ too greatly from the accurate wave function in the diffuse boundary region of $g(r)$. The polarization is given by

$$P(\theta)=\frac{2\gamma k^2W\sin\theta}{V^2+W^2+\gamma^2k^2k_1k_1^*\sin^2\theta}. \qquad (\mathbf{10})$$

Equation (**10**) differs only slightly from equation (**7**) and has the same defects. Both the Born and MG approximations for the polarizations fail for the same reason; $A(\theta)$ and $B(\theta)$ have the same angular dependence under the integral over r. The fluctuations in the polarization arise from a change in wavelength and amplitude of the wave function over the diffuse nuclear boundary.

A simple approximate wave function which has changes in wavelength and amplitude is

$$\left.\begin{aligned} \psi(r)&=p\exp(\mathrm{i}k_1\mathbf{n}_0.\mathbf{r})+pq\exp(-\mathrm{i}k_1\mathbf{n}_0.\mathbf{r}), && r\leqslant a,\\ &=\exp(\mathrm{i}k\mathbf{n}_0.\mathbf{r}), && r>a. \end{aligned}\right\} \qquad (\mathbf{11})$$

where $\mathbf{n}_0$ is a unit vector along the incident direction, and a is some appropriate value of r. The above wave function is discontinuous at $r=a$ (although it is nearly continuous at high energies), its wavelength and amplitude changing abruptly at this radius. On the other hand, the wavelength and amplitude of the exact wave function will change gradually over the whole diffuse nuclear boundary. However, it will be seen that the use of a "two-step" wave function is sufficient to account for the observed fluctuations of the polarization, which are not predicted by either the Born or MG approximations, which use "single-step" wave functions.

In the following calculations $g(r)$ is taken, for convenience, to be of trapezoidal form

$$\left.\begin{aligned} g(r)&=1, && \text{for } 0<r\leqslant a,\\ &=(b-r)/(b-a), && a<r\leqslant b,\\ &=0, && b<r<\infty. \end{aligned}\right\} \qquad (\mathbf{12})$$

For simplicity, the "two-step" wave function was taken to be

$$\left.\begin{aligned} \psi(r)&=p\exp(\mathrm{i}k_1\mathbf{n}_0.\mathbf{r})+pq\exp(-\mathrm{i}k_1\mathbf{n}_0.\mathbf{r}), && 0<r\leqslant a,\\ &=\exp(\mathrm{i}k\mathbf{n}_0.\mathbf{r}), && a<r\leqslant b. \end{aligned}\right\} \qquad (\mathbf{13})$$

so that the change in the wavelength and amplitude takes place at $r=a$ rather than at $r=r_0=\frac{1}{2}(a+b)$, the radius of the equivalent sphere. It was found that substantially the same results were obtained if the discontinuity of the wave function was taken at either $r=a$ or $r=r_0$. One finds for neutrons

$$A(\theta)=2m(V+\mathrm{i}W)\hbar^{-2}\{Z(a)+I(a)\}, \quad \ldots\ldots\ldots\ldots \quad (\mathbf{14})$$

$$B(\theta)=\mathrm{i}2m\gamma k^2\hbar^{-2}\sin\theta\{Z'(a)+I(a)\}, \quad \ldots\ldots\ldots\ldots \quad (\mathbf{15})$$

where $I(a)=\int_a^b \mathrm{j}_0(Kr)r^2g(r)\mathrm{d}r$,

$$Z(a)=a^2p\{[\mathrm{j}_i(K_-a)/K_-]+[q\mathrm{j}_1(K_+a)/K_+]\},$$
$$Z'(a)=a^2\mathrm{j}_1(Ka)/K.$$

$Z(a)$ and $Z'(a)$ are equal only if $k_1=k$, so that $p=1$, $q=0$, and $K_-=K$. $Z(a)$ arises from the region $0<r<a$ of the $A(\theta)$ integration, while $Z'(a)$ arises from the region $a<r<b$ of the $B(\theta)$ integration and the difference between them is due entirely to the different wave functions in these regions. The polarization is

$$P(\theta)=\frac{2\gamma k^2L(a)\{W\lambda(a)+V\mu(a)\}\sin\theta}{(V^2+W^2)(\lambda^2+\mu^2)+\gamma^2k^4L^2\sin^2\theta}, \quad \ldots\ldots\ldots \quad (\mathbf{16})$$

where $L(a)=Z'+I$, $\lambda(a)=\mathrm{Re}\{Z\}+I$, $\mu(a)=\mathrm{Im}\{Z\}$.

The polarization, given by equation (**16**), does not exhibit the same defects as those given by the Born approximation (equation (**7**)) or the MG approximation (equation (**10**)). Firstly, $P(\theta)\neq 0$ if $W=0$, since the polarization depends also upon V. Secondly, the polarization vanishes whenever $L=0$ or $\{W\lambda+V\mu\}=0$. These two conditions are satisfied for slightly different scattering angles and give rise to the observed fluctuations of the polarization. Moreover, the differential cross section given by the denominator of equation (**16**) (apart from a factor $4m^2\hbar^{-4}$), does not become zero at the diffraction minima, as in the Born and MG approximations, since the two terms do not vanish simultaneously. (It should be noted that the denominators of equations (**7**) and (**10**) are not the differential cross sections for the Born and MG approximations.) Thirdly, the polarization depends upon $g(r)$ and is therefore different for different nuclei.

III. Comparison of Theory with Experiment

(*a*) *Carbon*

Figure 1 (*a*) shows the experimental results of Chesnut, Hafner, and Roberts (1956) and Hafner (1958) for 220 MeV protons elastically scattered by carbon, together with the theoretical curve of Hafner, who used the WKB phase shift method. The full curve is the prediction of equation (**16**) for neutrons of the same energy. The potential used was nearly that of Hafner, $V=10$ MeV, $W=25$ MeV, $\gamma=4\cdot 7$ MeV f^2 (f$=$1 fermi$=10^{-13}$ cm), the mean nuclear radius $r_0=\frac{1}{2}(a+b)=2\cdot 4$ f and the diffuseness parameter $\Delta=\frac{1}{2}(b-a)=0\cdot 2$ f.

Since the calculation is for neutrons, a discrepancy can be expected at small angles. Also, Fernbach, Heckrotte, and Lepore (1955) have shown that the effect of the Coulomb interference is to decrease the first maximum and to increase the angular width of the diffraction dip of the polarization. Thus, the full curve may be considered as a satisfactory prediction for the polarization of 220 MeV neutrons by carbon.

In the present calculations, it was found that the zeros of the polarization are largely determined by the radius r_0. Changing r_0 from $2\cdot4$ f to $2\cdot8$ f moved the zeros to smaller angles by about 4°, so the choice of r_0 is rather critical. A value of $3\cdot0$ f, corresponding to $1\cdot3\ A^{\frac{1}{3}}$ for r_0, gives the dip of the polarization at too small an angle. The largest radius, which is consistent with the polarization data, is about $1\cdot2\ A^{\frac{1}{3}}$. This agrees very well with the information about nuclear radii obtained from electron scattering experiments. It was found, in disagreement with Hafner, that reasonable changes in Δ (e.g. from $0\cdot2$ f to $0\cdot4$ f) did not change the polarization appreciably.

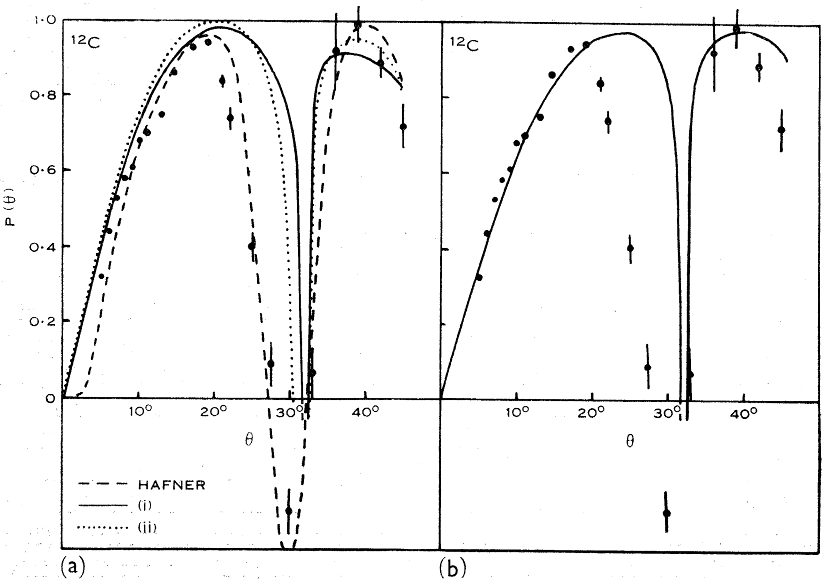

Fig. 1.—Polarization in elastic scattering of 220 MeV nucleons from ^{12}C. The points are the experimental results of Hafner for protons. (*a*) shows the theoretical predictions of Hafner (with $V=10$ MeV, $W=25$ MeV, $\gamma=5\cdot0$ MeV f^2, $r_0=2\cdot4$ f, and $\Delta=0\cdot1$ f) and of equation (**16**) (with (i) $V=10$ MeV, $W=25$ MeV, $\gamma=4\cdot7$ MeV f^2, $r_0=2\cdot4$ f, $\Delta=0\cdot2$ f and (ii) $V=10$ MeV, $W=40$ MeV, $\gamma=6$ MeV f^2, $r_0=2\cdot4$ f, $\Delta=0\cdot2$ f). (*b*) shows the theoretical prediction of equation (**20**) (with $V=10$ MeV, $W=20$ MeV, $\gamma=3\cdot0$ MeV f^2, $\varepsilon=-0\cdot5$, $r_0=2\cdot4$ f, $\Delta=0\cdot2$ f).

The effect of changes in V, W, and γ may be studied by considering the two angles $\theta=20$ and 40°, where $P(\theta)$ is a maximum. The theoretical curve cannot be a poor fit, if both the maxima and the zeros of the polarization data are satisfied. Figure 2 shows the predictions of $P(20°)$ and $P(40°)$ as a function of W and γ for $V=10$ and 20 MeV respectively. The polarization for these two angles is between $0\cdot9$ and $1\cdot0$. Table 1 shows the values of γ with the corresponding values of W for which fits at both angles may be obtained. Figure 1 (*a*) shows that a satisfactory fit is also obtained for $V=10$ MeV, $W=40$ MeV, and $\gamma=6$ MeV f^2.

(*b*) *Calcium*

Figure 3 shows the method applied to the heavier nucleus, calcium. It is seen that a satisfactory result is obtained for $V=10$ MeV, $W=25$ MeV, $\gamma=4\cdot7$ MeV f^2, $r_0=3\cdot9$ f, and $\Delta=0\cdot4$ f. For comparison, Hafner's parameters

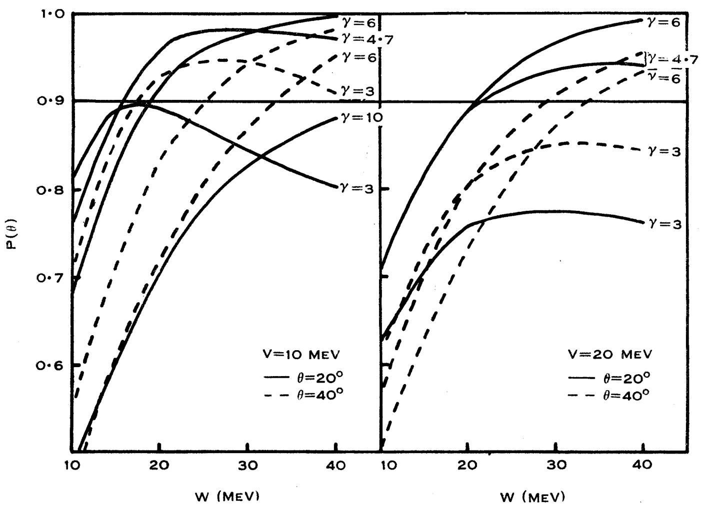

Fig. 2.—Polarization in elastic scattering of 220 MeV nucleons from ^{12}C at $\theta=20$ and 40°. The experimental polarization lies between $0\cdot9$ and $1\cdot0$ for both angles. The predictions of $P(20°)$ and $P(40°)$ given by equation (**16**) are shown for several values of γ and W for $V=10$ MeV and $V=20$ MeV.

are $V=10$ MeV, $W=25$ MeV, $\gamma=4\cdot7$ MeV f^2, $r_0=3\cdot6$ f, and $\Delta=0\cdot1$ f. Such a value of the diffuseness parameter, which is considerably smaller than that ($\sim0\cdot4$ f) given by electron scattering experiments, causes the differential cross

TABLE 1

VALUES OF THE NUCLEAR POTENTIAL PARAMETERS

V (MeV)	γ (MeV f^2)	W (MeV)
10	~3	~18
	$4\cdot7$	$\gtrsim25$
	6	$\gtrsim34$
20	~4	~27
	$4\cdot7$	$\gtrsim29$
	6	$\gtrsim34$

section to be too large at angles greater than the first diffraction minimum (Hafner 1958). This discrepancy would appear to arise from the failure of the WKB approximation at small angles.

The effect of increasing or decreasing γ is to move the maxima to smaller or larger angles respectively and make the peaks asymmetrical (Fig. 3 (*b*)).

The zeros of the polarization are unaffected by variations of γ and again they require the radius to be given by $1\cdot1\,A^{\frac{1}{3}}\leqslant r\leqslant1\cdot2\,A^{\frac{1}{3}}$. As for carbon, one can expect that an equally satisfactory fit may be obtained for a larger value of γ with a greater value of W.

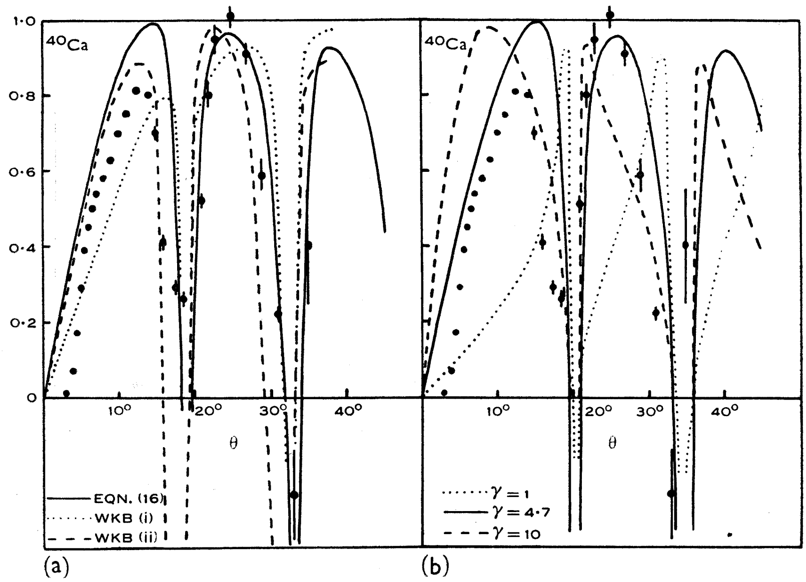

Fig. 3.—Polarization in elastic scattering of 220 MeV nucleons from ^{40}Ca. The points are the experimental results of Hafner for protons. (a) shows the theoretical predictions of equation (**16**) (with $V=10$ MeV, $W=25$ MeV, $\gamma=4\cdot7$ MeV f^2, $r_0=3\cdot9$ f, $\Delta=0\cdot4$ f) and of simplified WKB calculations (with $V=10$ MeV, $W=25$ MeV, $r_0=3\cdot8$ f, $\Delta=0\cdot4$ f and (i) $\gamma=4\cdot7$ MeV f^2, (ii) $\gamma=10$ MeV f^2. (b) shows the theoretical predictions of equation (**16**) (with $V=10$ MeV, $W=25$ MeV, $r_0=3\cdot7$ f, $\Delta=0\cdot4$ f, and (i) $\gamma=1$ MeV f^2, (ii) $\gamma=4\cdot7$ MeV f^2, (iii) $\gamma=10$ MeV f^2.

Figure 3 (a) also shows a comparison with the calculations obtained with a simplified form of the WKB approximation. In this approximation, the phase shifts δ_l are given by

$$\delta_l=\tfrac{1}{2}\int^{\infty}\frac{v\mathrm{d}r}{[k^2-(l+\frac{1}{2})^2/r^2]^{\frac{1}{2}}}, \quad \ldots\ldots\ldots\ldots\ldots \quad (\mathbf{17})$$

where $v=2m\hbar^{-2}V(r)$. Writing $\delta_l=x_l+\mathrm{i}y_l$, then, for the trapezoidal potential of equations (**1**) and (**12**), one finds

$$x_l=\frac{\alpha V\nu^2}{(B-A)}\left[\frac{\rho}{\nu}\sinh\left(\operatorname{arcosh}\frac{\rho}{\nu}\right)-\operatorname{arcosh}\frac{\rho}{\nu}\right]_{\rho=A}^{\rho=B}$$

$$+2\alpha\gamma k^2\mathrm{d}g/\mathrm{d}\rho\,\mathbf{L}.\boldsymbol{\sigma}\left[\operatorname{arcosh}\frac{\rho}{\nu}\right]_{\rho=A}^{\rho=B}, \quad \ldots\ldots \quad (\mathbf{18})$$

$$y_l=\frac{\alpha W\nu^2}{(B-A)}\left[\frac{\rho}{\nu}\sinh\left(\operatorname{arcosh}\frac{\rho}{\nu}\right)-\operatorname{arcosh}\frac{\rho}{\nu}\right]_{\rho=A}^{\rho=B}, \quad \ldots\ldots \quad (\mathbf{19})$$

where $\alpha=\frac{1}{2}m\hbar^{-2}k^{-2}$, $\nu=l+\frac{1}{2}$, $\rho=kr$, $A=ka$, and $B=kb$.

It is seen that the two approximations agree qualitatively. Quantitatively, the main difference between the two methods exists at small angles, where the WKB approximation is least satisfactory on account of its phase shift cut-off. At larger angles, both approximations show a similar movement of the polarization maxima as γ is varied but in the WKB approximation the polarization zeros also move and the shape of the polarization curve shows less asymmetry.

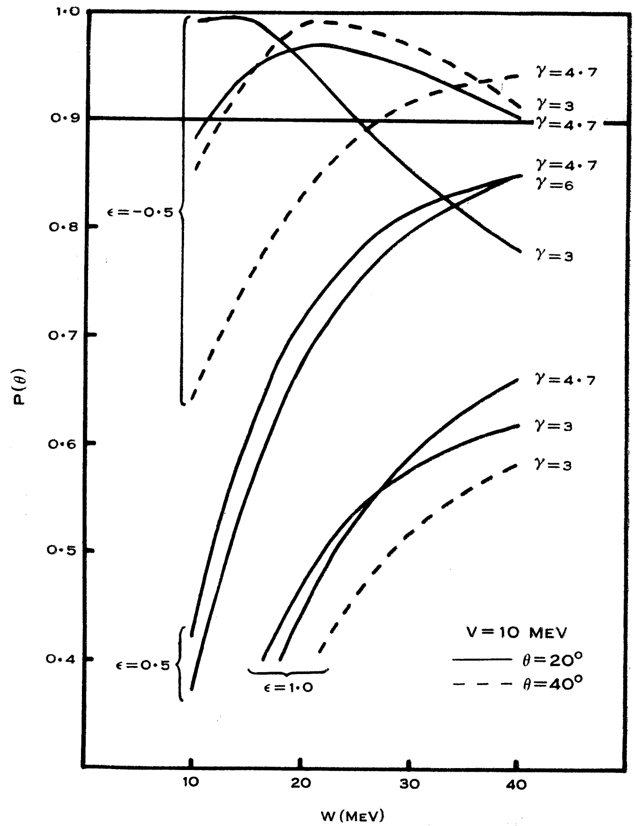

Fig. 4.—Polarization in elastic scattering of 220 MeV nucleons from ^{12}C at $\theta=20$ and $40°$. The experimental polarization lies between $0\cdot9$ and $1\cdot0$ for both angles. The theoretical predictions of $P(20°)$ and $P(40°)$ given by equation (**20**) are shown for several values of γ and W for $\varepsilon=1\cdot0$, $0\cdot5$, and $-0\cdot5$ and $V=10$ MeV.

IV. Complex Spin-orbit Potential

So far the discussion has been restricted to real spin-orbit interactions. However, Heckrotte (1956) required the spin-orbit potential to have an imaginary term in order to fit the small-angle elastic scattering polarization of 300 MeV protons by carbon. He found that, if the ratio of imaginary to real spin-orbit potential ε was $\sim -0\cdot5$ to $-1\cdot0$, satisfactory agreement could be obtained with experiment. It is thus of interest to investigate the effect of making $\varepsilon \neq 0$. Replacing γ by $\gamma_c = \gamma(1+i\varepsilon)$, the polarization is

$$P(\theta) = \frac{2\gamma k^2 L(a)\{W\lambda + \mu V + \varepsilon(W\mu - V\lambda)\}\sin\theta}{(V^2+W^2)(\lambda^2+\mu^2)+\gamma^2 k^4(1+\varepsilon^2)L^2\sin^2\theta}. \quad \ldots . \quad (20)$$

Figure 4 shows the predictions for $P(20°)$ and $P(40°)$ as a function of W, γ, and ε for V=10 MeV. Taking ε=0·5 or 1·0 is seen to give too small values at both angles. Further changes in γ would not cause a significant increase. On the other hand, an appreciable amount of negative ε may be added and agreement with experiment can still be maintained. Figure 1 (*b*) gives the result of using ε=–0·5, V=10 MeV, W=20 MeV, and γ=3·0 MeV f². Table 2 shows for comparison the values of γ_c determined by other workers.

TABLE 2

VALUES OF THE SPIN-ORBIT COUPLING CONSTANT

Reference	Energy (MeV)	Re$\{\gamma_c\}$ (MeV f²)	Im$\{\gamma_c\}$ (MeV f²)
Present paper	220	3 to 6	0 to –3
Sternheimer (1958)	150	11·2	0 to –11·2
Riesenfeld and Watson (1956)	190	(A) 2·4	–2·7
		(B) 3·3	–2·8
Hafner (1958)	220	5·0	0
Ohnuma (1958)	220	(GT) 3·3	–0·6
		(SM) 7·2	–1·1
Jastrow and Harris (1959)	287	4	+1
Bjorklund, Blandford, and Fernbach (1957)	300	2·2	–2·6
Batty (1958)	310	6·4	0 to –1·6
Bethe (1958)	310	3·6	–1·1

The two sets of results (A and B) refer to the two sets of nucleon-nucleon phase-shifts of Feshbach and Lomon (1956) used by Riesenfeld and Watson to calculate the optical model potential. Similarly, Ohnuma used the phase-shift data of Gammel and Thaler (1957) (GT) and of Signell and Marshak (1958) (SM).

V. ACKNOWLEDGMENTS

The author wishes to thank Associate Professor C. B. O. Mohr for his close interest in this work, and Dr. P. Swan for helpful discussions.

VI. REFERENCES

BATTY, C. J. (1958).—*Proc. Phys. Soc.* **73**: 185.
BETHE, H. A. (1958).—*Ann. Phys. N.Y.* **3**: 190.
BJORKLUND, F., BLANDFORD, I., and FERNBACH, S. (1957).—*Phys. Rev.* **108**: 795.
CHESNUT, W. G., HAFNER, E. M., and ROBERTS, A. (1956).—*Phys. Rev.* **104**: 449.
FERMI, E. (1954).—*Nuovo Cim.* **11**: 407.
FERNBACH, S., HECKROTTE, W., and LEPORE, H. (1955).—*Phys. Rev.* **97**: 1059.
FESHBACH, H., and LOMON, E. (1956).—*Phys. Rev.* **102**: 891.

GAMMEL, J. L., and THALER, R. M. (1957).—*Phys. Rev.* **107** : 291.
HAFNER, E. M. (1958).—*Phys. Rev.* **111** : 297.
HECKROTTE, W. (1956).—*Phys. Rev.* **101** : 1406.
JASTROW, R., and HARRIS, I. (1959).—*Nuc. Phys.* **9** : 437.
MOHR, C. B. O., and ROBSON, B. A. (1956).—*Proc. Phys. Soc.* A **69** : 365.
MONTROLL, E. W., and GREENBERG, J. M. (1952).—*Phys. Rev.* **86** : 889.
OHNUMA, S. (1958).—*Phys. Rev.* **111** : 1173.
RIESENFELD, W. B., and WATSON, K. M. (1956).—*Phys. Rev.* **102** : 1157.
SIGNELL, P. S., and MARSHAK, R. E. (1958).—*Phys. Rev.* **109** : 1229.
STERNHEIMER, R. M. (1958).—*Phys. Rev.* **112** : 1785.
WOODS, R. D., and SAXON, D. S. (1954).—*Phys. Rev.* **95** : 577.