THE ETA CARINAE NEBULA AND CENTAURUS A NEAR 1400 Mc/s

II. PHYSICAL DISCUSSION OF THE ETA CARINAE NEBULA

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Summary

The 1400 Mc/s observations of NGC 3372 described in Paper I (Hindman and Wade 1959) are discussed. Comparison of the flux densities at $85 \cdot 5$ Mc/s (Mills, Little, and Sheridan 1956) and at 1400 Mc/s leads to a value of $10{,}000 \pm 1000$ °K for the electron temperature of the nebula. Unpublished optical measurements of the distribution of surface brightness across the object, made by Gum, indicate that there is a dense core about 24 min of arc in diameter, surrounded by a much less dense region with a diameter of 120 min of arc. Adopting the optically determined distance of 1400 parsecs (Hoffleit 1953), we find r.m.s. densities of 71 ions cm^{-3} in the core and 11 ions cm^{-3} in the outer region. The total mass of the object is not more than 25,000 solar masses. It is shown that several O-stars probably are needed to maintain the ionization of the nebula.

I. INTRODUCTION

The present paper is a sequel to an earlier one (Hindman and Wade 1959) which described observations of the Eta Carinae Nebula (NGC 3372) at a frequency near 1400 Mc/s. The main results in the previous paper may be summarized as follows :

(i) the flux density of NGC 3372 near 1400 Mc/s is $5 \cdot 82 \times 10^{-24}$ W m^{-2} (c/s)$^{-1}$, with an estimated uncertainty less than ± 20 per cent,

(ii) the apparent surface brightness distribution appears to be fairly symmetrical,

(iii) the source shows a high degree of concentration towards its centre.

The objective of the present paper is to infer the temperature, density, and mass of the object from the available radio data. We shall consider also its excitation.

The physical discussion to follow rests upon several assumptions. These are.

(i) *The radio emission of the nebula is purely thermal.*

(ii) *The ionized region has a uniform electron temperature.* While this assumption is undoubtedly somewhat idealized, arguments presented by Spitzer (1954) show that the range of electron temperatures within a given nebula is not likely to be very great.

(iii) *The mass distribution within the nebula is spherically symmetrical.* This needs to be only approximately true for the discussion given below. The assumption is supported by the observed absence of any marked asymmetry in the distribution of the radio emission of the source.

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(iv) *The nebular gas consists entirely of singly ionized atoms.* This assumption is probably quite sound, since the only elements likely to be present in significant amounts are hydrogen and helium. Swihart (1952) has shown that, under such conditions as we expect to prevail in NGC 3372, helium is almost entirely in the singly ionized state; moreover, the He II and H II zones are almost exactly coincident. Therefore the numbers of positive and negative ions should be equal and Kramers' law may be used in the form appropriate for fully ionized hydrogen.

The discussion requires a knowledge of the solid angle subtended by the nebula and of the distribution of relative optical depth over its projected surface. Ideally, we should like to determine these from radio-frequency observations. Because of the insufficient angular resolution of the available instruments, however, the existing radio data consist mainly of measurements of flux density. Therefore it is necessary, in our case, to estimate the angular size and optical depth distribution from optical observations.

II. Geometrical Preliminaries

(*a*) *The Apparent Surface Distribution of Optical Depth*

Consider a spherically symmetrical nebula situated at some distance from the observer. The optical depth at a point on the projected surface of the sphere will depend only on the angular distance of the point from the centre. The ratio of the optical depth at fractional radius x to that at the apparent centre may be expressed by some function $g(x)$. Letting $x=1$ at the outer boundary of the object, we have $g(0)=1$ and $g(1)=0$. The values of $g(x)$ for intermediate radii depend upon the distribution of opacity within the sphere, the relationship being expressed by a form of Abel's integral equation

$$g(x)=\frac{2r_0\varkappa_0}{\tau_0}\int_x^1\frac{yh(y)\mathrm{d}y}{\sqrt{(y^2-x^2)}}, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{1})$$

where r_0 is the radius of the nebula, $\varkappa_0$ is the absorption coefficient at its centre, and τ_0 is the optical depth through its apparent centre. $h(y)$ is the ratio of the absorption coefficient at fractional radius y to $\varkappa_0$. Equation (**1**) has a unique solution (Bracewell 1956):

$$h(y)=-\frac{\tau_0}{\pi r_0\varkappa_0}\int_y^1\frac{g'(x)\mathrm{d}x}{\sqrt{(x^2-y^2)}}. \quad \ldots\ldots\ldots\ldots\ldots \quad (\mathbf{2})$$

Equations (**1**) and (**2**) provide a means of converting between corresponding distributions of opacity and optical depth.

C. S. Gum (unpublished data) has measured the distribution of surface brightness, in Hα light, along a line passing across the centre of NGC 3372 at position angle 335°, as shown in Plate 1. Since the object is optically thin in Hα, the radio surface brightness distribution at high frequencies should be very similar to that measured by Gum. The diameter along which his measurement was made appears to be relatively unaffected by the rather erratic obscuration overlying most of the nebula. The optical measurement is consistent with an optical depth distribution of the form

$$g(x)=\begin{cases}\dfrac{(1-c)\sqrt{(\alpha^2-x^2)}+c\sqrt{(1-x^2)}}{\alpha+c(1-\alpha)}, & 0\leqslant x\leqslant\alpha,\\[2ex] \dfrac{c\sqrt{(1-x^2)}}{\alpha+c(1-\alpha)}, & \alpha<x\leqslant 1,\end{cases} \qquad \ldots\ldots (3)$$

where $\alpha=0\cdot2$ and $c=0\cdot025$. The rather complicated expression of these equations is due to the desirability of having the opacity model (equation (4))

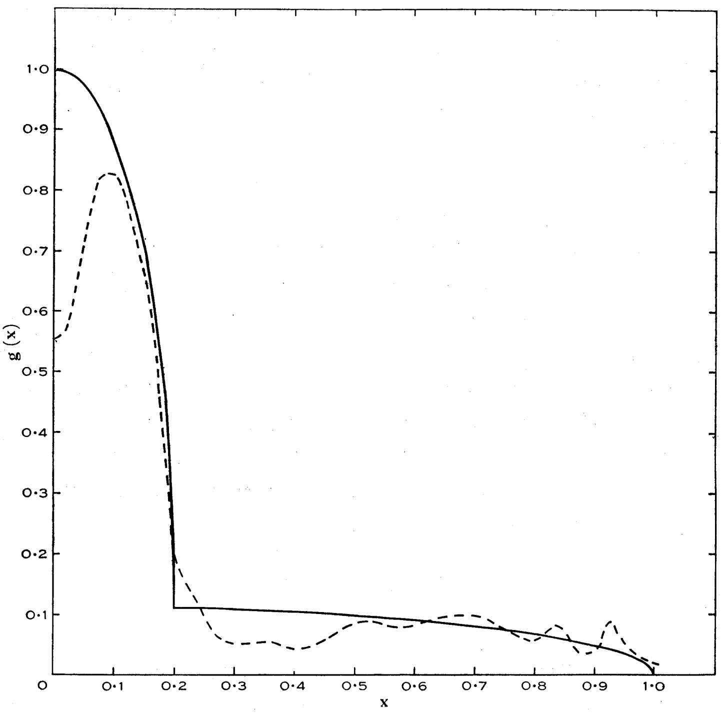


Fig. 1.—Radial optical depth distribution across NGC 3372. The quantity plotted is relative optical depth g as a function of fractional angular radius x; $x=1$ at 60′ from the centre.
······ Distribution in p.a. 335°, based on optical measurements by C. S. Gum.
——— Adopted distribution.

stated in the simplest possible form. Figure 1 shows the agreement between the above distribution and that measured by Gum, for an adopted angular diameter of 120 min of arc. The two distributions differ appreciably only where $x<0\cdot1$. The difference in that region is to be expected, because that is where Gum's line of measurement cuts across one of the strong lanes of absorbing matter seen against the nebula at optical wavelengths. The adopted optical depth distribution is undoubtedly somewhat idealized, but errors in it have only a slight effect on the values to be derived for the electron temperature and total mass, and the discussion of the excitation of the object is entirely independent of the geometry. The calculation of density and emission measure are more sensitive to errors in $g(x)$, and hence less weight should be attached to the values found for these quantities. Figure 2 shows the form of $h(y)$ corresponding to equations (3),

$$h(y)=\begin{cases}1, & 0\leqslant y\leqslant\alpha,\\ c, & \alpha<y\leqslant 1.\end{cases} \qquad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

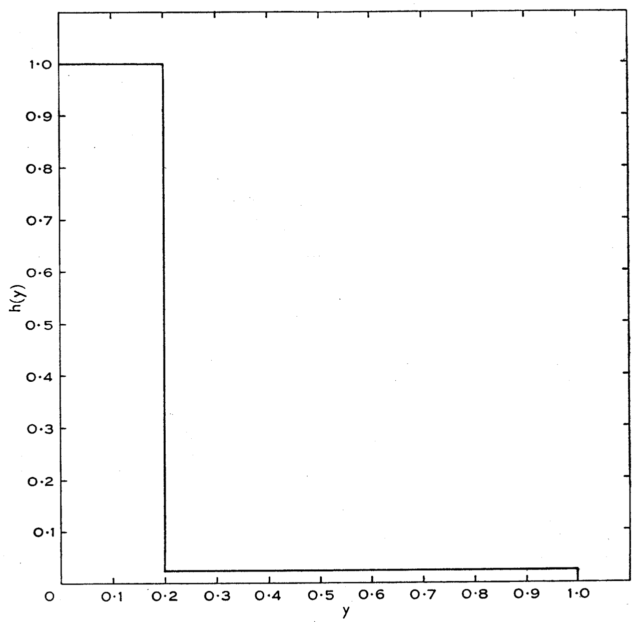


Fig. 2.—Radial opacity distribution corresponding to the adopted optical depth distribution. The relative opacity h is plotted as a function of fractional radius y. $\alpha=0\cdot 2$, $c=0\cdot 025$.

Other model-dependent parameters needed in the ensuing discussion may be derived from equations (3) and (4). These are:

$$\bar{g}=2\int_0^1 xg(x)\mathrm{d}x=0\cdot 0994,$$

$$u=\left[2\bar{g}\int_0^1 h(y)\mathrm{d}y\right]^{\frac{1}{2}}=0\cdot 2091.$$

The numerical values have been calculated for $\alpha=0\cdot 2$ and $c=0\cdot 025$.

(b) *The Apparent Flux Density*

The apparent flux density (defined by Mills, Little, and Sheridan 1956) of a nebula at a particular frequency, in units of W m^{-2} (c/s)$^{-1}$, is

$$F_{app}=2\cdot 04\times 10^{-35}\varphi^2 f_{mc}^2(T_e-T_b)\psi(\tau_0), \qquad \ldots\ldots\ldots (5)$$

where φ is its angular diameter in minutes of arc, f_{mc} is the frequency in Mc/s, T_e is the electron temperature of the nebular gas, and T_b is the brightness temperature (assumed to be uniform) of the sky background at the position of the nebula. $\psi(\tau_0)$ is the value of $1-e^{-\tau(x)}$ averaged over the projected surface of the object. If $T_b \ll T_e$, the apparent and true flux densities are equal.

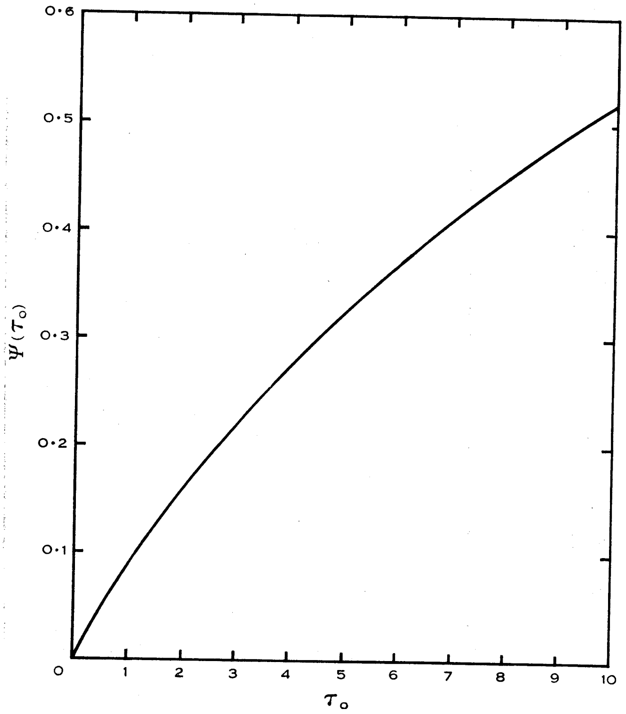


Fig. 3.—The optical depth parameter $\psi(\tau_0)$ for $\alpha=0\cdot2$, $c=0\cdot025$.

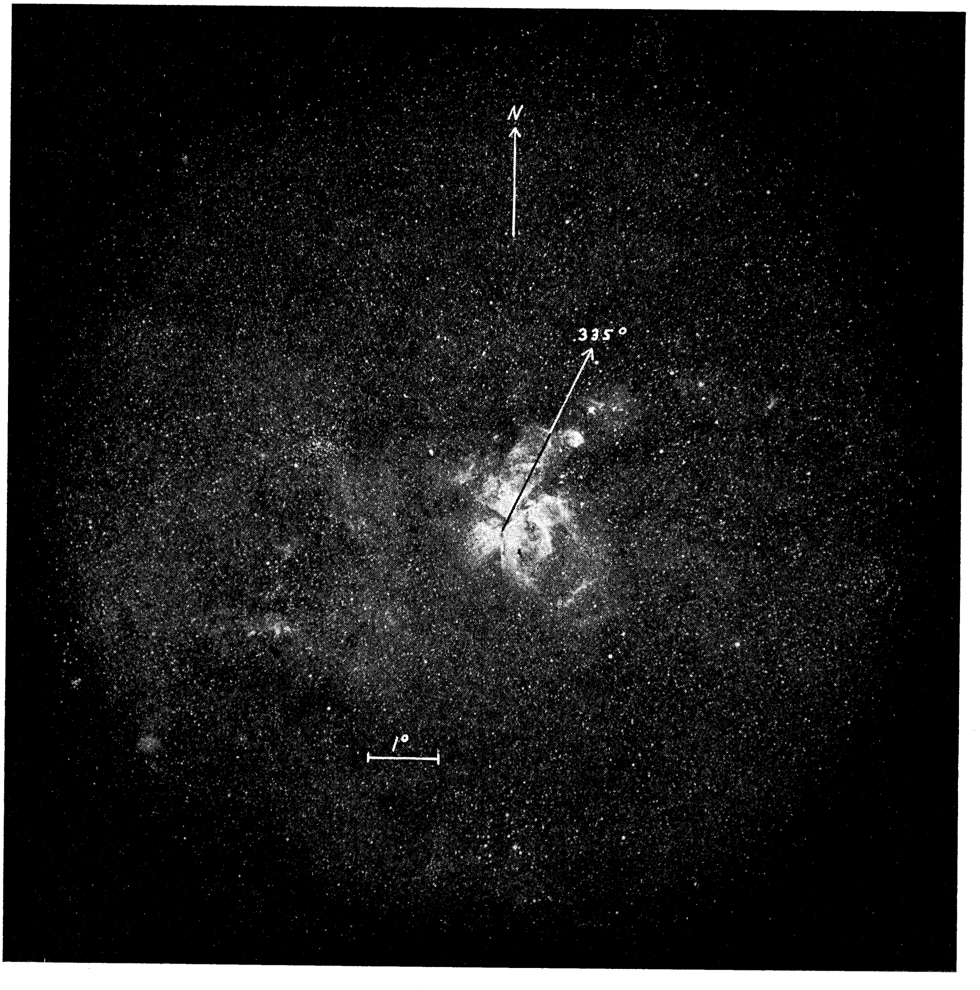
Figure 3 shows $\psi(\tau_0)$ in the range $0 \leqslant \tau_0 \leqslant 10$ for the adopted optical depth distribution $g(x)$. If $\tau_0 \ll 1$, we have simply

$$\psi(\tau_0)=\bar{g}\tau_0. \qquad (6)$$

III. The Electron Temperature

The flux density of a thermal radio source depends only weakly on frequency as long as the object is optically thin. The optical depth increases rapidly with decreasing frequency, however, causing the emitted flux to drop fairly sharply at the lower frequencies. The spectrum in the region of decreasing flux density is quite sensitive to the electron temperature of the source; this circumstance permits one to find the electron temperature of a nebula by comparing the flux densities measured at two well-separated frequencies, provided that the optical depth is appreciable at one at least of the frequencies. The observations of

THE ETA CARINAE NEBULA AND CENTAURUS A. II



Photograph of NGC 3372 in Hα light, taken with the 8-in. $f/1$ Meinel-Pearson Schmidt camera at the Mount Stromlo Observatory. The line along which C. S. Gum measured the surface brightness of the object is indicated. North is at the top, with east to the left.

NGC 3372 at 85·5 Mc/s by Mills, Little, and Sheridan (1956) fall in the range of intermediate optical depths; hence they are suitable for comparison with the 1400 Mc/s data.

The method, which was described by the author in a previous paper (Wade 1958), may be outlined as follows.

(i) The ratio of the optical depths at the projected centre of a nebula at the two frequencies must be equal to the ratio of the absorption coefficients, which is:

$$\frac{\tau_{0,1}}{\tau_{0,2}}=\frac{\zeta_1}{\zeta_2}\left(\frac{f_2}{f_1}\right)^2, \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

where

$$\zeta=9\cdot 70\times 10^{-3}\ln 3kT_e/2hf.$$

The subscripts 1 and 2 denote the two frequencies; in the present case we may take these to be 85·5 and 1400 Mc/s respectively. h and k are Planck's and Boltzmann's constants and f is the frequency in cycles per second. For temperatures in the vicinity of 10,000 °K, we have

$$\tau_{0,1}/\tau_{0,2}=326.$$

(ii) Using equations (5) and (6) and Figure 3, one may find the ratio of the central optical depths from the radio-frequency observations, assuming a value for the electron temperatures. If the assumed temperature is correct, the computed optical depth ratio will be equal to that required by equation (7). The simplest means of arriving at the correct temperature is to assume several values and compute the optical depth ratio corresponding to each. The temperature can then be found graphically from a plot of optical depth ratioversus electron temperature.

Mills, Little, and Sheridan (1956) found that the apparent flux density of NGC 3372 at 85·5 Mc/s is $3\cdot 42\times 10^{-24}$ W m^{-2} (c/s)$^{-1}$.* Hill, Slee, and Mills (1958) give 4700 °K for the brightness temperature of the sky near NGC 3372 at 85·5 Mc/s. This value must consist of two parts: (*a*) the true background temperature, i.e. the emission arising beyond NGC 3372, and (*b*) the emission occurring between the source and the Sun, which we shall call the *foreground component*. It is difficult to estimate reliably the relative strengths of the two components. Extrapolation of unpublished observations at 19·7 Mc/s by C. A. Shain (personal communication) leads to an estimate of 1400 °K for the foreground component. This is consistent with data obtained at 85·5 Mc/s (B. Y. Mills, personal communication); hence we adopt the above value, leaving a true background temperature of 3300 °K at 85·5 Mc/s. At 1400 Mc/s, the background temperature is less than 5 °K and may be neglected. Taking φ=120 min of arc, we can draw Figure 4; the intersection of the two curves

* Because of an error in calibration, the flux density published by Mills and his co-authors was too low by a factor slightly greater than two (Mills, personal communication). The correction has been incorporated in the figure given above.

gives an electron temperature of 10,000 °K. The principal sources of possible error in this result are.

(i) *The experimental uncertainties in the flux densities*, which lead to an uncertainty of ±700 °K.

(ii) *The uncertainty in the magnitude of the foreground component at 85·5 Mc/s.* The error due to this cause is probably not greater than ±300 °K.

The net uncertainty is therefore less than ±1000 °K.

The electron temperature deduced above is in good agreement with the 8000–10,000 °K expected on theoretical grounds (Spitzer 1954). It also accords well with the recent optical determinations by Pronyk (1957), who found that the temperature ranges from 7500 to 11,000 °K in NGC 1976 and from 7600 to 9000 °K in IC 405; in both cases the temperature tends to decrease with increasing distance from the exciting stars.

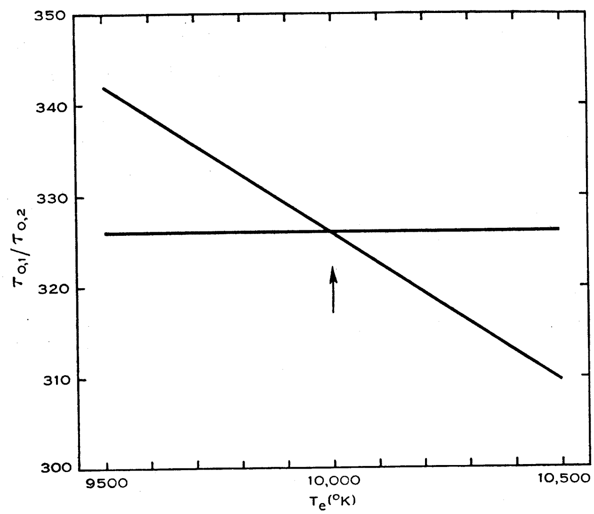


Fig. 4.—Determination of the electron temperature of NGC 3372.

IV. Density and Total Mass

If a nebula is optically thin, the r.m.s. density at its centre, in ions per cubic centimetre, is given by (see Appendix I)

$$n_0 = 1 \cdot 05 \times 10^{16} \frac{T_e^{\frac{1}{4}} F^{\frac{1}{2}}}{(\zeta R)^{\frac{1}{2}} \varphi^{3/2} u}, \qquad \dots\dots\dots\dots\dots \quad (8)$$

provided $T_b \ll T_e$, which is usually true at the frequencies where $\tau_0 \ll 1$. F is the true flux density of the nebula, and R is its distance in parsecs. Hoffleit (1953) has estimated the distance to NGC 3372 as 1400 parsecs, using available optical data. With the 1400 Mc/s flux density, we find that the r.m.s. density of the core region is 71 ions cm^{-3}. This agrees fairly well with Bok's (1932) early result of 60 ions cm^{-3}, derived from optical data. The density of the outer region may be found by multiplying the core value by $c^{\frac{1}{2}}$; it is 11 ions cm^{-3}.

An upper limit to the total mass of the nebula may be set by assuming that the r.m.s. densities are equal to the actual average ion densities, and integrating over the volume of the object. Before doing this, however, one must know the relative abundances of the hydrogen and helium components of the gas. No direct determination of the abundance ratio in NGC 3372 is available; but Mathis (1957) has evaluated it for the Orion Nebula (NGC 1976), finding that one helium ion is present for each eight of hydrogen, in good agreement with the ratio of about one in ten found in stellar atmospheres and nebulae by various other workers (see Allen 1955, p. 28). Adopting the eight to one ratio, we find that the total mass of NGC 3372 is less than or equal to 25,000 solar masses.

V. The Central Emission Measure

The surface brightness of a nebula may conveniently be described by its *emission measure*, which is defined (Strömgren 1948) as

$$\varepsilon=\int_S n_e n_i \mathrm{d}s,$$

where n_e and n_i are the electron and ion densities, and S is the path length of the line of sight through the ionized region in parsecs. Since we are assuming in the present discussion that the region consists of singly ionized atoms, we may take $n_e=n_i$. The emission measure at the apparent centre of an optically thin spherical nebula is related to the flux density by

$$\varepsilon_0=1\cdot 59\times 10^{28}T_e^{\frac{1}{2}}F/\varphi^2\zeta\bar{g} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

(see Appendix I). Using this relation and the 1400 Mc/s flux density, we find that the central emission measure of NGC 3372 is 54,000. This may be compared with the value 80,000 ± 30,000 which Gum estimated from his unpublished optical measurements.

We have ignored the small, very intense patch of nebulosity associated with the star Eta Carinae. Since this nebulosity is less than 10 sec of arc in diameter, it can have no measurable effect on the observed flux densities. Furthermore, its connexion with NGC 3372 is doubtful (Bok 1932; Gaviola 1950). Therefore we are justified in neglecting it in the present discussion.

VI. The Excitation of NGC 3372

The effectiveness of a star as a source of nebular excitation may be inferred from the size and density of the ionized region associated with it. Considering a spherical nebula with radius r_0 and uniform ion density n, Strömgren (1939) showed that $U(\equiv n^{\frac{2}{3}}r_0)$ depends only on the spectral type and the absolute luminosity of the exciting star. The excitation parameter U may be found directly from the flux density if the object is optically thin (Wade 1958)

$$U=6\cdot 32\times 10^6 R^{\frac{2}{3}}T_e^{\frac{1}{6}}\zeta^{-\frac{1}{3}}F^{\frac{1}{3}}. \quad \ldots\ldots\ldots\ldots\ldots \quad (10)$$

The value of U derived from this equation is independent of the geometry of the nebula. Applying it to NGC 3372, we find $U=134$. According to Pottasch (1956), the value of U for a typical O5 star is 100. Since the effective U of a group of stars is the cube root of the sum of the cubes of their individual U's, the value found for NGC 3372 implies that it is excited by the equivalent, roughly, of three O5 stars. No satisfactory identification has been made of the stars responsible for the ionization of NGC 3372, but our result for U makes it seem likely that a group of several hot stars is imbedded in it.

VII. Conclusions

The foregoing discussion has led to the interpretation of NGC 3372 shown in Figure 5. The available radio data are consistent with a spherical nebula having an electron temperature of 10,000 ± 1000 °K, comprising a relatively small dense core and a broad tenuous envelope. Adopting the optically determined distance of 1400 parsecs, we found that the r.m.s. ion densities of the core and the envelope are respectively 71 cm^{-3} and 11 cm^{-3}. The computed

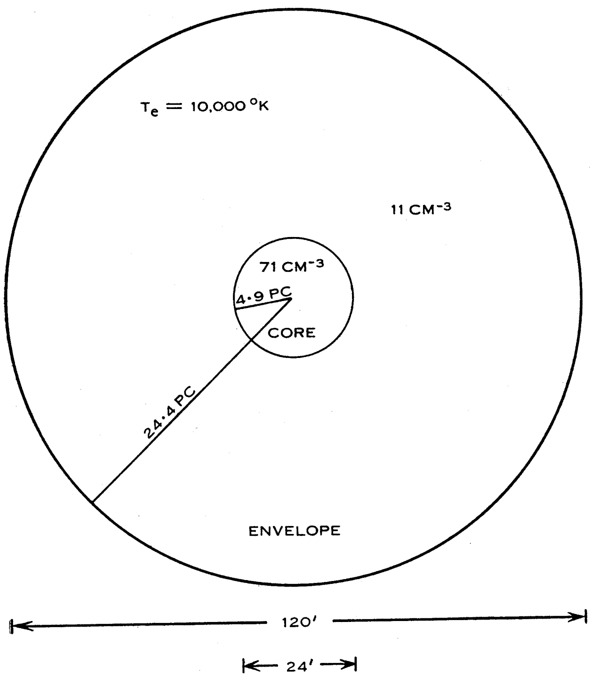


Fig. 5.—Derived model for NGC 3372.

central emission measure of 54,000 is in rough agreement with Gum's unpublished optical determination. The nebula almost certainly is excited by more than one O-star, since the ultraviolet flux required to maintain the ionization of such a large mass (about 25,000 solar masses) is about three times the amount which could be supplied by a single O5 star.

A high resolution determination of the surface brightness distribution across the object at a high frequency (i.e. at a frequency such that the nebula is everywhere optically thin) is desirable to check the form adopted in the preceding discussion and in order to ascertain the validity of the assumption of radial symmetry. Observations of the flux density at several other frequencies will serve as a general check on the derived model.

Figure 6 shows the integrated radio-frequency spectrum implied by our model. It was computed from equation (5), assuming that

$$T_b = 3300[f_{mc}/85 \cdot 5]^{-2 \cdot 70}.$$

At about 66 Mc/s, the background temperature should be equal to the electron temperature, making the object invisible at that frequency. At lower frequencies it should be seen in absorption. At the high frequencies, where the object is optically thin, about 75 per cent. of the flux arises in the envelope.

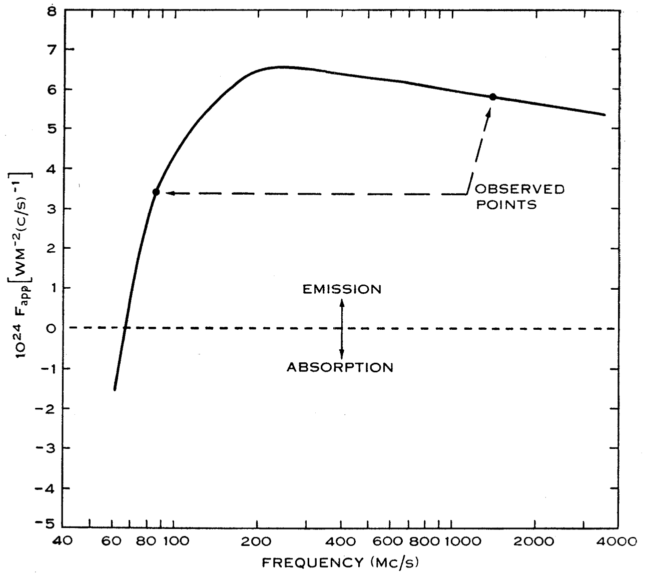


Fig. 6.—Radio spectrum of NGC 3372, computed from the derived model. The observed points are indicated.

The results obtained in the present paper agree well with previous conclusions about emission nebulae gained from theoretical and optical studies. This fact provides some confirmation of the basic physical assumption underlying the analysis; namely, that the nebula emits radio waves solely by the thermal mechanism of free-free transitions. The radio approach to the problem of determining the general physical properties of nebulae is, on the whole, simpler and more direct than contemporary optical methods. This is true of both the observational techniques and the physical theory required to discuss the observations.

VIII. Acknowledgments

The author is indebted to Dr. C. S. Gum for making available his unpublished optical measurements of the surface brightness of NGC 3372, and to Dr. B. Y. Mills and Mr. C. A. Shain for helpful discussions. The work was made possible by a Junior Research Fellowship granted by the Commonwealth Scientific and Industrial Research Organization.

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Appendix I

(*a*) *Derivation of Equation* (8)

If the nebula is optically thin, we may combine equations (5) and (6) to obtain

$$F=2\cdot 04\times 10^{-35}\varphi^2 f_{mc}^2 T_e \bar{g}\tau_0, \qquad \text{(A1)}$$

provided $T_b \ll T_e$, which ordinarily is true at the high frequencies where $\tau_0 \ll 1$.

According to Kramers' law (e.g. Piddington 1951), the absorption coefficient at the centre of the nebula is

$$\varkappa_0=10^{-12}\zeta n_0^2/f_{mc}^2 T_e^{3/2}, \qquad \text{(A2)}$$

if the gas is of low density and consists only of singly ionized atoms. The optical depth through the projected centre of the nebula is

$$\tau_0=2r\varkappa_0\int_0^1 h(y)\mathrm{d}y. \qquad \text{(A3)}$$

Now, if R is the distance to the nebula in parsecs and φ is its angular diameter in minutes of arc,

$$r_0=4\cdot 48\times 10^{14}R\varphi. \qquad \text{(A4)}$$

Substituting (A2) and (A4) in (A3), we get

$$\tau_0=448R\zeta\varphi n_0^2/f_{mc}^2 T_e^{3/2}. \qquad \text{(A5)}$$

Then, substituting (**A5**) in (**A1**) and solving for n_0, and defining

$$u=\left[2\bar{g}\int_0^1 h(y)\mathrm{d}y\right]^{\frac{1}{2}},$$

we obtain

$$n_0=1\cdot 05\times 10^{16}\frac{T_e^{\frac{1}{4}}F^{\frac{1}{2}}}{(R\zeta)^{\frac{1}{2}}\varphi^{3/2}u},$$

which is equation (**8**).

(*b*) *Derivation of Equation* (**9**)

According to equation (**16**) of a previous paper by the author (Wade 1958), the central emission measure of an optically thin nebula is

$$\varepsilon_0=\frac{f_{mc}^2 T_e^{3/2}\tau_0}{3\cdot 08\times 10^6\zeta}$$

Solving (**A1**) for τ_0 and substituting in the above expression, we obtain

$$\varepsilon_0=1\cdot 59\times 10^{28}T_e^{\frac{1}{2}}F/\varphi^2\zeta\bar{g},$$

which is equation (**9**).