# THE STRESS FIELDS AROUND SOME DISLOCATION ARRAYS*

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The stress fields around some edge dislocation arrays have been calculated using the approximation of Leibfried (1951) that the discrete dislocations are replaced by a continuous dislocation density. Haasen and Leibfried (1954) have calculated one such stress distribution (corresponding to (ii) below) via the stress function which they evaluated as a superposition integral over the dislocation distribution of the stress function of an infinitesimal dislocation. The results given here were evaluated directly as the superposition of the stress fields of the infinitesimal dislocations making up the distribution. Like most of the integrals which occur in this approximation, they can be evaluated most readily by contour integration (Haasen and Leibfried 1954). A multiple coordinate system is used and is such that the results can be written compactly and approximations and limiting cases can be easily obtained.

(i) Blocks at $x=\pm a$ with $n$ positive dislocations between $x=b$ and $x=a$ and $n$ negative dislocations between $x=-b$ and $x=-a$ which are held apart by an applied shear stress $\sigma$ (Head and Louat 1955 (vi)).

The components of stress at a point $P(x,y)$ are

$$\sigma_{xx}=\sigma\left(\frac{\rho_1\rho_2}{r_1r_2}\right)^{\frac{1}{2}}\left[\frac{Ry}{\rho_1\rho_2}\cos\{\eta-\tfrac{1}{2}(\varphi_1+\varphi_2+\theta_1+\theta_2)\}-\frac{Ry}{r_1r_2}\cos\{\eta+\tfrac{1}{2}(\varphi_1+\varphi_2)-\frac{3}{2}(\theta_1+\theta_2)\}+2\sin\tfrac{1}{2}(\varphi_1+\varphi_2-\theta_1-\theta_2)\right], \quad \ldots\ldots\ldots\ldots (1)$$

$$\sigma_{yy}=\sigma\left(\frac{\rho_1\rho_2}{r_1r_2}\right)^{\frac{1}{2}}\left[-\frac{Ry}{\rho_1\rho_2}\cos\{\eta-\tfrac{1}{2}(\varphi_1+\varphi_2+\theta_1+\theta_2)\}+\frac{Ry}{r_1r_2}\cos\{\eta+\tfrac{1}{2}(\varphi_1+\varphi_2)-\frac{3}{2}(\theta_1+\theta_2)\}\right], \quad \ldots (2)$$

$$\sigma_{xy}=\sigma\left(\frac{\rho_1\rho_2}{r_1r_2}\right)^{\frac{1}{2}}\left[-\frac{Ry}{\rho_1\rho_2}\sin\{\eta-\tfrac{1}{2}(\varphi_1+\varphi_2+\theta_1+\theta_2)\}+\frac{Ry}{r_1r_2}\sin\{\eta+\tfrac{1}{2}(\varphi_1+\varphi_2)-\frac{3}{2}(\theta_1+\theta_2)\}+\cos\tfrac{1}{2}(\varphi_1+\varphi_2-\theta_1-\theta_2)\right], \quad \ldots\ldots\ldots\ldots (3)$$

where $(R,\eta)$, $(r_1,\theta_1)$, $(r_2,\theta_2)$, $(\rho_1,\varphi_1)$, $(\rho_2,\varphi_2)$ are polar coordinates, as shown in Figure 1. The expression (3) for the shear stress is the total shear stress including the applied shear stress $\sigma$.

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(ii) Haasen and Leibfried (1954) have considered the special case of (i) for $b=0$. This can be derived from (i) by putting $R=\rho_1=\rho_2$, $\eta=\varphi_1=\varphi_2$, and then (**1**)–(**3**) are equivalent to their result.

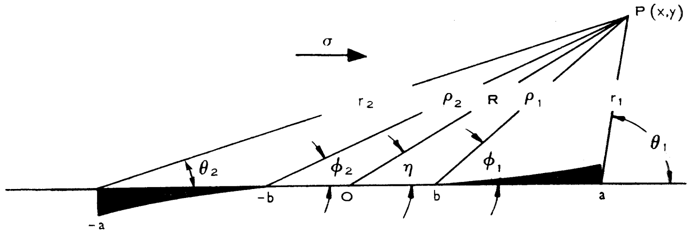


Fig. 1.—Coordinate system for equations (**1**)–(**3**).

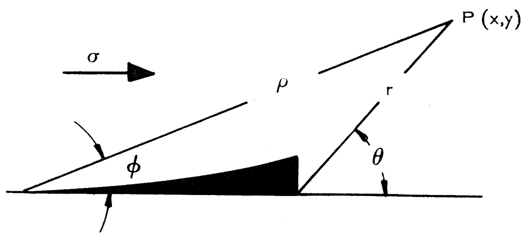


Fig. 2.—Coordinate system for equations (**4**)–(**6**).

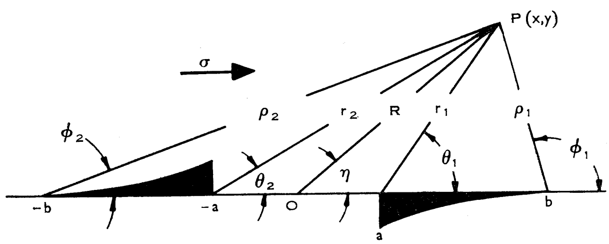


Fig. 3.—Coordinate system for Section (iv).

(iii) If the negative dislocations in (i) are removed to infinity by putting $r_2=\rho_2=2R=\infty$, $\varphi_2=\eta=\theta_2=0$, we are left with Figure 2, a group of positive dislocations forced against a block by a shear stress $\sigma$ (Head and Louat 1955 (iv)), and (**1**)–(**3**) become

$$\sigma_{xx}=\tfrac{1}{2}\sigma(\rho/r)^{\frac{1}{2}}\left[\frac{y}{\rho}\cos\tfrac{1}{2}(\theta+\varphi)-\frac{y}{r}\cos\tfrac{1}{2}(\varphi-3\theta)+4\sin\tfrac{1}{2}(\varphi-\theta)\right], \quad \ldots (4)$$

$$\sigma_{yy}=\tfrac{1}{2}\sigma(\rho/r)^{\frac{1}{2}}\left[-\frac{y}{\rho}\cos\tfrac{1}{2}(\theta+\varphi)+\frac{y}{r}\cos\tfrac{1}{2}(\varphi-3\theta)\right], \quad \ldots\ldots\ldots\ldots (5)$$

$$\sigma_{xy}=\tfrac{1}{2}\sigma(\rho/r)^{\frac{1}{2}}\left[\frac{y}{\rho}\sin\tfrac{1}{2}(\theta+\varphi)+\frac{y}{r}\sin\tfrac{1}{2}(\varphi-3\theta)+2\cos\tfrac{1}{2}(\varphi-\theta)\right]. \quad \ldots (6)$$

The stresses near the head of the pile-up will be given approximately by putting

$$\rho\simeq L,\ \varphi\simeq 0,\ r\ll L,\ y=r\sin\theta$$

(where $L$ is the length of the pile-up). Equations (**4**)–(**6**) then become the approximate expressions derived by Stroh (1954) from consideration of a pile-up of discrete dislocations.

(iv) Figure 3 is example (vii) from Head and Louat (1955) with $n$ positive dislocations beyond $x=-a$ and $n$ negative dislocations beyond $x=a$, driven together by a shear stress $\sigma$ but prevented from coalescing by blocks at $x=\pm a$. The stress field in this case is also given by (**1**)–(**3**) if the symbols are reinterpreted as in Figure 3.

*References*

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