A STUDY OF WHISTLING ATMOSPHERICS

II. DIFFUSENESS

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Summary

The diffuseness or spread, in time, of whistlers observed at four Australian stations is discussed. It is found to be approximately twice as large for long as for short whistlers, to be independent of season and time of day, and to be linearly related to geomagnetic latitude and whistler dispersion. The diffuseness may be accounted for by assuming that the energy travels over a range of paths in the exosphere, and estimates of this range are made (i) assuming a smooth variation of dispersion with latitude and (ii) assuming a duct-type of guiding.

I. Introduction

When one listens to whistlers replayed from magnetic tape records some sound like pure descending tones while others are much less pure or more " swishy " in character. In this paper the former are referred to as sharp whistlers and the latter as diffuse whistlers. The " diffuseness " of a whistler is herein described by the time duration of the component frequencies (rather than by the frequency band). Thus whistlers in which any particular frequency lasts more than $0 \cdot 1$ s sound, and are termed, " diffuse " while the remainder are classified as " sharp ". This paper discusses the diffuseness of those whistlers (discussed in Part I, Occurrence (Crouchley 1961)) which were suitable for analysis by spot-frequency tuned circuits.

II. Experimental Details

The signals, replayed from the magnetic tape (Part I), were fed to 18 parallel-tuned L-C circuits whose mid frequencies lay in the range $1 \cdot 10$–$20 \cdot 0$ kc/s. The voltage outputs of these filter circuits were displayed on 18 one-inch cathode-ray tubes, equally spaced along a straight line, which were photographed with an optical reduction of 150 to 1 on film moving at 45 cm/min.

As most whistlers closely followed the frequency-time relationship $t=Df^{-\frac{1}{2}}$ (Eckersley 1935, Storey 1953 ; " D " the dispersion) over the frequency range of the analyser, it was convenient to arrange the frequencies of the individual channels in such a fashion that they responded in succession to the whistler and that the response of each channel was separated from that of the next by an equal time interval, which depended on the value of D. This may be achieved by choosing the centre frequencies of the channels so that they are equally spaced in $f^{-\frac{1}{2}}$ instead of in f. In practice, this leads to a crowding of channels at the lower frequencies and accordingly in this instrument the frequency range

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was covered by two sets of filters, the first containing 8 channels and covering from 20·0 to 7·10 kc/s and the second containing 11 channels and covering from 7·10 to 1·10 kc/s. A whistler which covered the whole frequency range would thus ideally appear on the film as a succession of "dots" (lower frequencies later in time) lying on two straight lines of different slope which intersected on

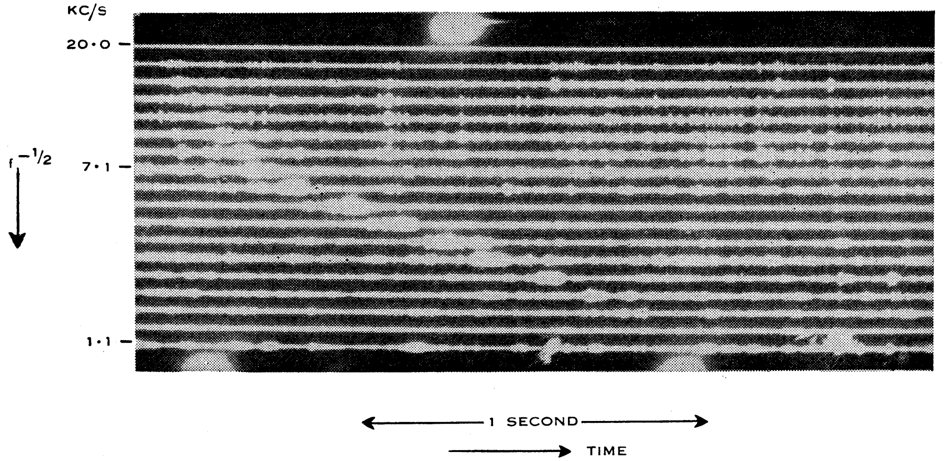

Fig. 1 (*a*).—Analyser record of sharp whistler (Brisbane, single hop).

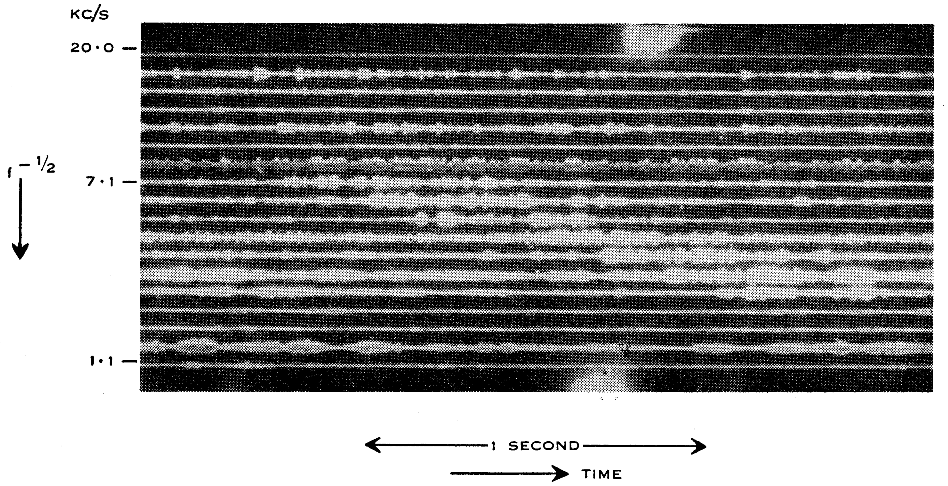

Fig. 1 (*b*).—Analyser record of diffuse whistler (Hobart, double hop).

the trace produced by the 7·10 kc/s channel. The slopes of the two intersecting straight lines are determined by D (as well as by the channel spacing) and hence a graticule marked with pairs of lines calculated for different values of D may be used to estimate this quantity from the film.

In practice a set of 18 parallel lines was traced on the film with zero-signal input to the analyser and the response of any channel was indicated by the line corresponding to that channel being broadened. Thus an atmospheric gave rise to set of marks lying on a line at right angles to the traces and a whistler caused a set of marks lying on the pair of lines corresponding to its dispersion. A reproduction of a typical sharp whistler (Brisbane) is shown in Figure 1 (*a*) and a typical diffuse whistler (Hobart) is shown in Figure 1 (*b*).

The response of an L-C filter to a gliding tone (Storey and Grierson 1958) consists of an oscillation bounded by a relatively slowly varying envelope whose shape depends only on a single parameter k which involves the band width of the filter and the rate of change of frequency of the gliding tone. The values of k were calculated for the filters used and a whistler of dispersion of 60 $s^{\frac{1}{2}}$ and hence the times between the two " 6-dB " points for channels 4, 8, and 12 were estimated as 6, 9, and 12 ms respectively. These times, which are small compared with the durations of the observed responses on these channels and which increase only as $D^{\frac{1}{2}}$, were ignored. In practice, the bandwidths of the filters were adjusted to give approximately the best signal-to-noise ratio for whistlers of average dispersion for the station being analysed.

The following details were read from the films for each whistler:

(i) the time of occurrence of 7·10 kc/s (channel 8) to the nearest 0·05 s,

(ii) the dispersion D (accuracy about ± 10 $s^{\frac{1}{2}}$),

(iii) the duration (to the nearest 0·05 s) of the response on channel 4 (11·98 kc/s), channel 8 (7·10 kc/s), channel 12 (2·78 kc/s), and channel 16 (1·43 kc/s), or of such of those as showed a response,

(iv) the maximum and minimum frequencies present in the whistler.

The *diffuseness* (F) of a whistler is now defined as the average duration, in units of 0·05 s, on channels 4, 8, 12, and 16, or of such of these as responded ; thus a whistler present on channels 4, 8, and 12 lasting 0·1 s, on each of these would be assigned a diffuseness of two. The F-value divided by the number of traverses of the whistler path is called the *reduced diffuseness* and denoted by F_1. A further parameter, $F'=1000F/D$, called the *diffuseness-dispersion ratio*, is also introduced.

III. Experimental Results

(*a*) *Comparison of Diffuseness of Short and Long Whistlers*

The mean values of F for the four stations, together with the standard errors of the mean (S) and the number of whistlers used (n), is shown in Table 1 for the both short ($\overline{F}_s$) and long ($\overline{F}_L$) whistlers.

The value of $\overline{F}$ for long whistlers is approximately twice that for short whistlers except for Brisbane where long whistlers are rare events. The assumption that $\overline{F}_L$ is twice $\overline{F}_s$ was tested by using the test due to Welch and Aspin (Bennett and Franklin 1954). A larger difference (i.e. $2\overline{F}_s-\overline{F}_L$) than that actually observed has probabilities of occurrence as follows: Adelaide 7%, Hobart 40%, Macquarie Island 6%. The hypothesis that $\overline{F}_L=2\overline{F}_S-0\cdot1$ was similarly tested and gave probabilities as follows : Adelaide 25%, Hobart 30%, Macquarie Island 7%. It thus seems likely that the diffuseness for a double traverse of the whistler path is approximately twice that for a single traverse and accordingly the reduced diffuseness F_1 is adopted as a working parameter.

(*b*) *Occurrence of Diffuseness*

A plot of the number of whistlers with a given value of F_1 against the value of F_1 is shown in Figure 2 for the four stations, making use of data over a year. (The figures for Adelaide are scaled down by a factor of two.) The distributions for Brisbane, Adelaide, and Hobart are all skew with the mode at small values of diffuseness and with spreads up to about five times the modal value. For Macquarie Island a less peaked and more symmetrical distribution is obtained. The curves suggest that the modal value of diffuseness increases as the latitude increases.

Table 1

COMPARISON OF SHORT AND LONG WHISTLER DIFFUSENESS

Station	Short Whistlers			Long Whistlers		
	$\overline{F}_s$	S	n	$\overline{F}_L$	S	n
Brisbane ..	1·50	0·06	145	1·29	0·30	7
Adelaide ..	2·20	0·035	1536	4·12	0·14	150
Hobart	2·72	0·10	396	5·75	0·34	111
Macquarie I. ..	3·70	0·26	53	5·80	0·65	36

(*c*) *Relationship between Diffuseness and Dispersion*

A plot of the average diffuseness of whistlers with a given dispersion, $(\overline{F})_D$ versus the dispersion is shown in Figure 3 for the four stations. The results for Adelaide and Hobart and to a lesser extent those for Macquarie Island (for which fewer whistlers were recorded) suggest, within the limits of experimental

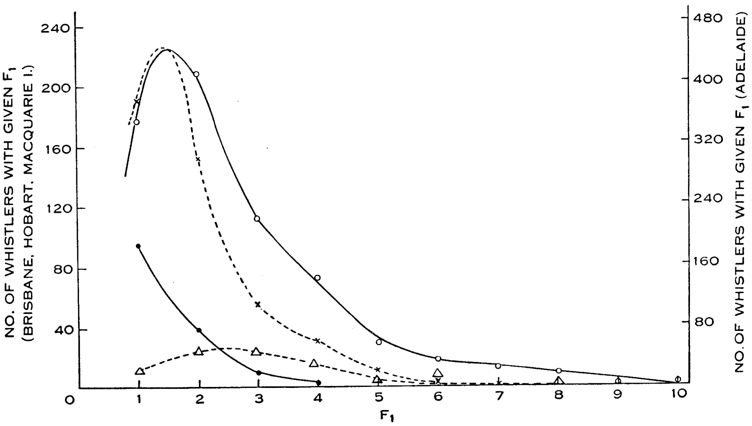

Fig. 2.—The distribution of occurrence of reduced diffusion. ● Brisbane, × Adelaide, ○ Hobart, △ Macquarie Island.

error, a proportionality between diffuseness and dispersion. For Brisbane the range of the variables is small, the diffuseness, being small, is more difficult to estimate, and the two outer points of the graph were obtained from only a small number of readings, thus the line is uncertain. This approximate proportionality between diffuseness and dispersion and the fact that the dispersion is a property of the path (including the number of traverses) suggests the use of F/D as an alternative parameter to describe the spread of the whistler. For numerical convenience the diffuseness-dispersion ratio is multiplied by 1000 to give F'.

(*d*) *Diurnal and Month-to-month Variations*

The average diurnal variations of F_1 and F' (over one year) are shown in Figures 4 (*a*) and 4 (*b*) for hours at which sufficient whistlers were recorded. There is no evidence of a systematic variation, other than a possible small decrease in F_1 and F' after midnight at Hobart.

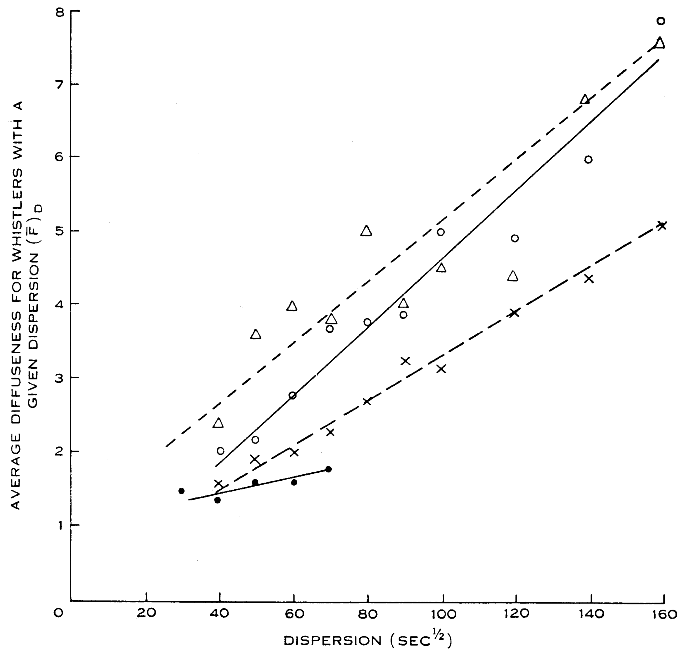

Fig. 3.—The variation of diffuseness with dispersion. ● Brisbane, × Adelaide, ○ Hobart, △ Macquarie Island.

The month-to-month changes in average reduced diffuseness and average diffuseness-dispersion ratio are plotted in Figures 5 (*a*) and 5 (*b*), where, because of the seasonal variations in the occurrence of whistlers (Part I) the points have different weights. There is no evidence for a seasonal variation in F_1 or F'. (A similar plot of diffuseness did show a seasonal variation corresponding to the variations in the relative numbers of long and short whistlers.)

(*e*) *Variation of Reduced Diffuseness and Diffuseness-Dispersion Ratio with Latitude*

Figure 6 shows a plot of mean reduced diffuseness and mean diffuseness-dispersion ratio against geomagnetic latitude. For both quantities the points lie close to a straight line passing through latitude 12°. Also shown are points obtained by using only short whistlers, which points lie close to those obtained using all whistlers except for Macquarie Island (geomagnetic latitude 61 °S.) and even here the departure of the short whistler points from the lines is less than twice the standard error of the mean. The intercept at latitude 12° is

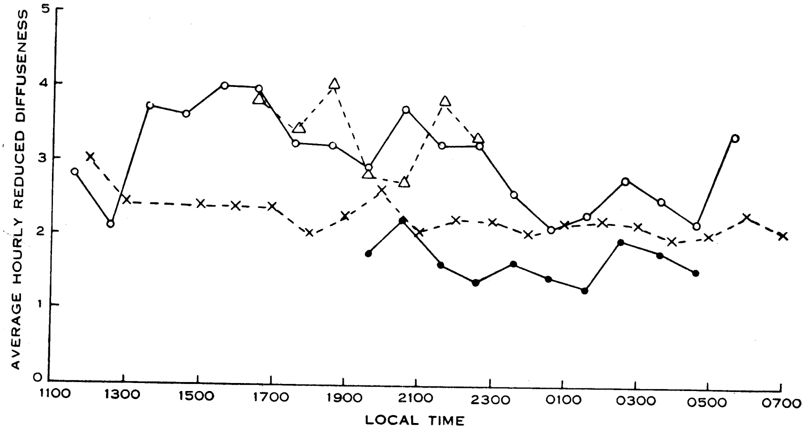

Fig. 4 (*a*).—Diurnal variation of reduced diffusion. ● Brisbane, × Adelaide, ○ Hobart, △ Macquarie Island.

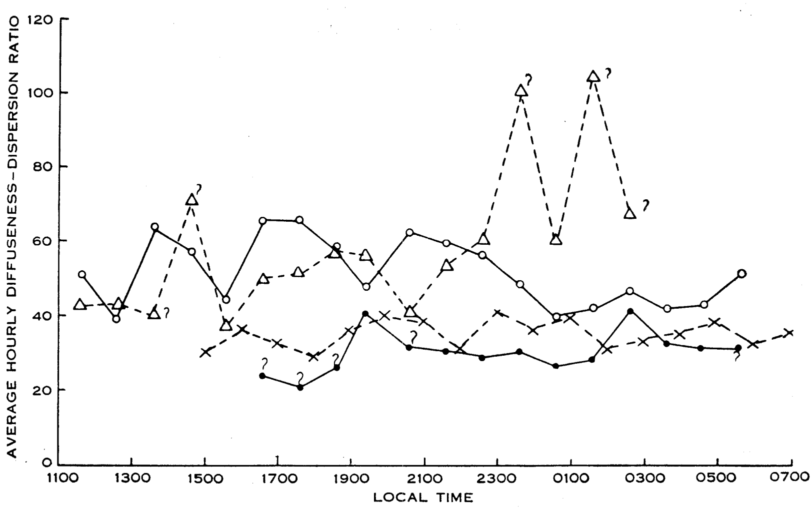

Fig. 4 (*b*).—Diurnal variation of diffuseness-dispersion ratio. ● Brisbane, × Adelaide, ○ Hobart, △ Macquarie Island.

significant as this approximately marks the boundary between lines of force which intercept the ionosphere and those which pass entirely below it.

The variation of F_1 with geomagnetic latitude may be represented approximately by the equation

$$F_1 = 0 \cdot 066(\lambda - 12),$$

where λ is in degrees.

(*f*) *Dependence of Reduced Diffuseness on the Initiating Discharge*

In seeking a relationship between the duration of the initiating lightning discharge and the diffuseness or diffuseness-dispersion ratio 59 long whistlers recorded at Hobart were studied. The whistlers were classified according to the values of F or F' and the atmospherics as sharp (less than $0 \cdot 1$ s long), broad, or

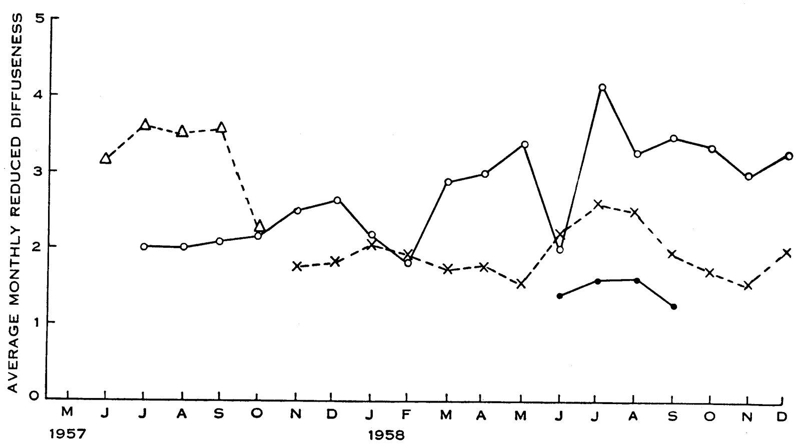

Fig. 5 (*a*).—Month-to-month variation of reduced diffuseness. ● Brisbane, × Adelaide, ○ Hobart, △ Macquarie Island.

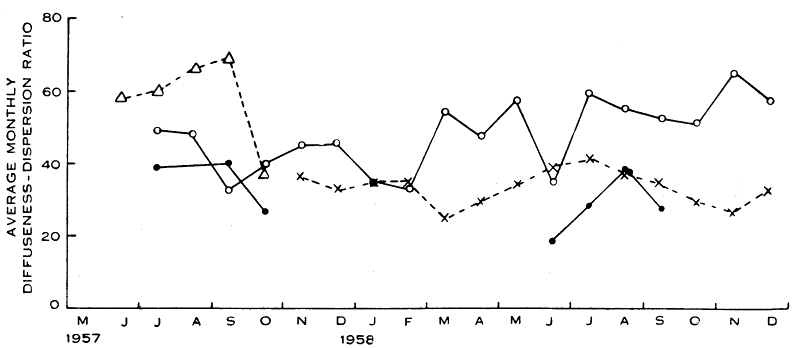

Fig. 5 (*b*).—Month-to-month variation of diffuseness-dispersion ratio. ● Brisbane, × Adelaide, ○ Hobart, △ Macquarie Island.

indefinite according to their duration on the film from the analyser. Table 2 (*a*) shows the number of whistlers of given F-values for the three classes of atmospherics and Table 2 (*b*) is a similar classification in terms of F'-values.

The assumption that the distribution of values so obtained is independent of the classification of the atmospherics was examined by means of the chi-squared test. For this purpose the tables as presented, thus giving twelve degrees of freedom (ν), were used; and also, since the number of values in the individual cells is small, a contracted table of only two columns (obtained by summing the first four values in a row and the last three values in a row) and hence two degrees of freedom was tested. The value of chi-squared so obtained and also the probability of obtaining larger values on this assumption are shown in Table 2 (*c*).

It thus seems unlikely that the diffuseness or diffuseness-dispersion ratio is dependent on the apparent time duration of the initiating discharge. Moreover, it was noted that the time duration of the atmospherics was not necessarily the same on all frequencies. There was not, however, any correspondence between the duration of a frequency in the atmospheric and of the same frequency in the whistler. Indeed on occasions a frequency present in one was apparently absent in the other.

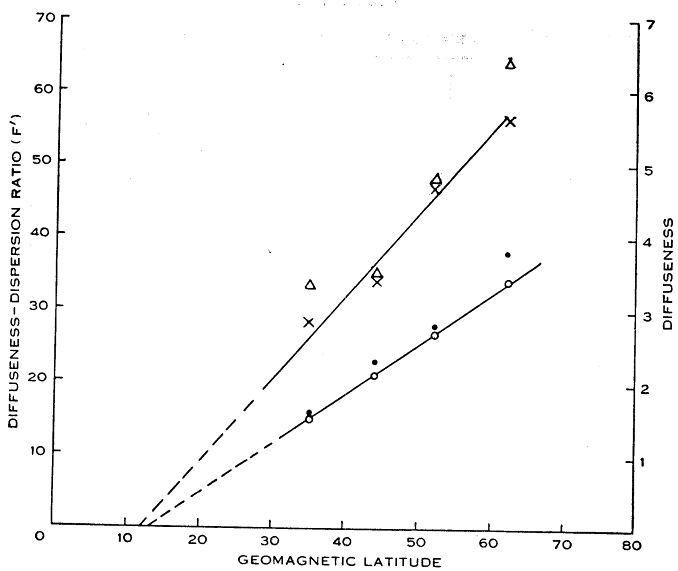

Fig. 6.—Latitude variation of diffuseness, reduced diffuseness, and diffuseness-dispersion ratio. ○ Average reduced diffuseness of all whistlers, ● average diffuseness of short whistlers, × average *F′* of all whistlers, △ average *F′* of short whistlers.

The apparent length of an atmospheric, as judged from film, will depend on its amplitude, the ringing time of the tuned circuit, and how closely it is followed by another discharge. Broad atmospherics show, in general, considerable structure, suggesting that they are due to a number of separate discharges. Any individual discharge may be expected to give a detectable trace on the film for some milliseconds after it has ceased because of the ringing of the tuned circuits, and the time to fall below the threshold of observability will depend on the strength of the discharge. It seems unlikely therefore that the clipping of the strong atmospherics (Part I) has introduced any ambiguity in classification, as the effect of this would be to tend to equalize the response produced by atmospherics of different strengths but equal time durations.

TABLE 2 (*a*)

DIFFUSENESS AND DURATION OF INITIATING ATMOSPHERIC

Spheric \ F	1	2	3	4	5	6	>6
Sharp ..	1	3	1	2	5	3	3
Broad ..	1	4	5	2	1	2	4
Indefinite ..	0	3	0	9	5	2	3

TABLE 2 (*b*)

DIFFUSENESS-DISPERSION RATIO AND ATMOSPHERIC DURATION

Spheric \ F'	0–10	10–20	20–30	30–40	40–50	50–60	>60
Sharp ..	1	1	3	3	6	1	3
Broad ..	0	4	3	4	4	3	1
Indefinite ..	1	2	1	7	7	1	3

TABLE 2 (*c*)

STATISTICAL ANALYSIS OF TABLES 2 (*a*) AND 2 (*b*)

Table	χ^2	ν	Probability (%)
F-*values*			
Whole table	18·4	12	≈10
Contracted table	2·2	2	≈35
F′-*values*			
Whole table	9·3	12	≈50
Contracted table	0·78	2	≈50

(*g*) *Dependence of Diffuseness on Ionospheric Spread-*F *and Sporadic-*E *Ionization*

An association between spread-F at Hobart (where an ionosonde operates) and F, F_1, and F' was searched for, by comparing the average values of F, F_1, and F' with values of a spread-F index which were obtained as follows. If the ionospheric observatory reports showed the presence of spread-F half-an-hour before and half-an-hour after the whistler schedule (i.e. at the regular observing times) the index 2 was assigned, if at only one of those times the index 1, and if spread-F was not present at either time the index 0 was assigned. The mean values of F, F_1, and F', together with the standard errors (S) of these means and the number of whistlers considered (n), are shown in Table 3 (a).

The differences of the means for spread-F index 0 and 2 was tested by means of the "Aspin-Welch test" (Bennett and Franklin 1954), and the values of "t" so calculated, together with the probability of exceeding these values of t in samples from the same population, are shown in Table 3 (*b*).

It seems likely that all three parameters are higher when spread-F is prevalent.

TABLE 3 (*a*)

DIFFUSENESS, REDUCED DIFFUSENESS, AND DIFFUSENESS-DISPERSION RATIO AND SPREAD-F

Spread-F index ..	0			1			2		
	Mean	S	n	Mean	S	n	Mean	S	n
F	2·9	(0·23)	109	3·5	(0·24)	60	3·8	(0·25)	79
F_1	2·35	(0·205)	109	2·08	(0·222)	60	3·10	(0·218)	79
F'	39·5	(2·6)	109	36·1	(3·0)	60	47·5	(2·9)	79

The procedure was extended to include spread-F one and one-half hours before and after the whistler schedule with, again, the indication that whistlers are somewhat more diffuse when spread-F is present before and after the whistler recording-period.

No evidence of a similar association with sporadic-E ionization was obtained.

TABLE 3 (*b*)

STATISTICAL ANALYSIS OF TABLE 3 (*a*)

	t	Probability of Exceeding Calculated t (%)
$\overline{F}$	2·6	≈ 1
$\overline{F_1}$	2·5	≈ 1
$\overline{F'}$	2·1	≈ 4

(*h*) *Dependence of Reduced Diffuseness on Other Geophysical Phenomena*

An autocorrelogram of average daily values of F' at Adelaide for lags of up to 65 days is shown in Figure 7. A peak in the values of the correlation coefficient around 25–27 days' lag suggests a possible recurrence-tendency in F' as in whistler occurrence (Part I) but no evidence of association with Zurich Provisional Relative Sunspot Numbers (R_z) was obtained. No indications of periodicities or of association with R_z were obtained for the Hobart data. For both stations the first few correlation coefficients of the autocorrelograms were small, suggesting that changes in F' on one day are not apparent on the following day.

Cross-correlograms between average daily values of F' for Adelaide and for Hobart with daily sums of magnetic K-index (both planetary and K-Toolangi, the nearest magnetic observatory) and Macquarie Island cosmic ray intensities showed no features of significance.

(i) *Variation of Whistler Duration with Frequency*

The majority of the whistlers studied proved to have a rather limited range of frequencies and only some 20% were present on both channels 4 and 8 while a different 10% were present on both channels 8 and 12. These two groups will be referred to respectively as upper frequency whistlers and lower frequency

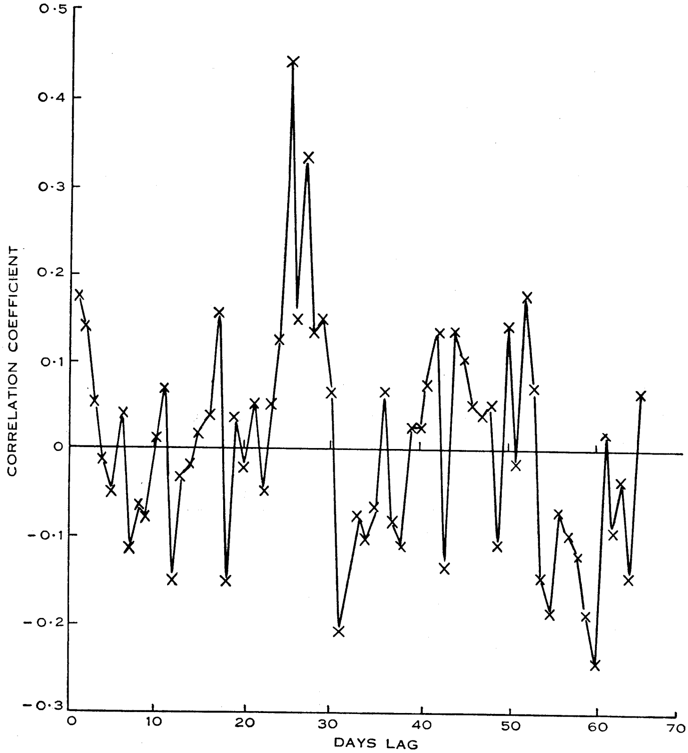

Fig. 7.—Autocorrelogram of daily average dispersion ratio for Adelaide (1958). (No. of values approximately 70.)

whistlers. The mean value of the time durations, $\overline{\delta t}$ (in seconds), for a single traverse of the path, together with the standard error of the mean and the number of whistlers involved, is shown for the two groups of whistlers and for Adelaide and Hobart in Table 4.

The assumption that the two values of $\overline{\delta t}$ at 7 kc/s are caused by taking different samples from the same population was tested by the method of Welch and Aspin. In the case of Adelaide differences greater than the observed have a probability of occurrence of about 0·03 and this probability for Hobart is less than 0·005. It thus seems likely that there is a real difference between the two groups.

TABLE 4

AVERAGE DURATIONS OF WHISTLERS AT DIFFERENT FREQUENCIES

Frequency	12·0 kc/s			7·1 kc/s			2·8 kc/s		
Station	$\overline{\delta t}$ (sec)	S.E.	n	$\overline{\delta t}$ (sec)	S.E.	n	$\overline{\delta t}$ (sec)	S.E.	n
Adelaide									
Upper frequency ..	0·087	0·003	309	0·104	0·004	309			
Lower frequency ..				0·148	0·019	34	0·128	0·015	34
Hobart									
Upper frequency	0·117	0·019	110	0·138	0·012	110			
Lower frequency ..				0·260	0·015	130	0·265	0·019	130

IV. DISCUSSION

It would be expected that the diffuseness of a whistler would be due either to the initiating discharge lasting for times comparable to those observed for the duration of individual frequencies in a whistler, or to some feature of the whistler path, or to a combination of these two factors. Since the diffuseness shows no apparent connexion with the duration of the atmospheric but is related to (i) the number of traverses of the path, (ii) the dispersion, and (iii) latitude, and the last two factors are related to path length, it seems likely that the path is the important factor. Furthermore, the fact that double-hop whistlers have approximately twice the diffuseness of single-hop whistlers (at least for middle latitudes) suggests that the reflection mechanism does not contribute markedly to the diffuseness and, thus, that the energy after reflection returns along a path close to the forward path.

In the initial discussion of whistler path Storey (1953) showed that very-low-frequency electromagnetic radiation from a lightning flash would experience a strong refraction at the base of the ionosphere and that hereafter " all the rays will follow the line of force of the earth's magnetic field fairly closely ". The time of travel (t_f) of a frequency f over a path along which the electron density N and magnetic field strength H vary was found to be

$$t_f \simeq f^{-\frac{1}{2}}\left(\frac{e}{2}\right)^{\frac{1}{2}}\int\left(\frac{N}{H}\right)^{\frac{1}{2}}\mathrm{d}l, \tag{1}$$

and is thus dependent on electron density, magnetic field strength, and path length (l). More recently various workers (Helliwell and Gehrels 1958 ; Northover 1959 ; Smith, Helliwell, and Yabroff 1960) have suggested that guiding tubes or ducts, i.e. regions (surrounding a line of force) in which the electron density is significantly different from the surroundings, are responsible for the propagation of whistlers. The maxima and minima commonly observed in the response of the tuned circuits to a whistler thus suggest propagation of energy from one lightning flash over several ducts.

On either of the above suppositions the values of δt_f would be expected to depend on the distribution of electrons along the paths. This is still uncertain and accordingly two different models will be discussed: (*a*) one derived from experimental values of dispersion, and (*b*) one derived from the assumption that N/H is constant over a whistler path.

Model (*a*)

Treating this first from the point of view of a smooth distribution of electrons rather than a duct type of propagation, we may expect that both the finite area of "illumination" of the lower ionosphere and imperfect guiding will cause energy from one lightning flash to traverse a range of paths. The effect of both factors may be approximated to by assuming that all the energy is propagated

TABLE 5

FREQUENCY DEPENDENCE OF δx_f

Frequency	12·0 kc/s	7·1 kc/s	2·8 kc/s
Station	δx_f		
Adelaide			
Upper frequency	435 km	420 km	
Lower frequency		600 km	340 km
Hobart			
Upper frequency	580 km	550 km	
Lower frequency		1080 km	700 km

in a region bounded in latitude by two lines of force. Unless there is a considerably greater displacement of the rays in the tangent plane to a line of force than in the diametral plane, the maximum time spread will be determined by the two rays which determine the spread in latitude, as there is no evidence of dispersion being dependent upon longitude. On these assumptions and using the equations

$$t_f = Df^{-\frac{1}{2}} \quad \text{(Dispersion law)},$$
$$D \simeq 2 \cdot 2(\lambda - 12) \quad \text{(from graph in Allcock 1959)},$$

where λ=geomagnetic latitude, we may derive the equation

$$\delta t_f = 0 \cdot 021 f^{-\frac{1}{2}} \delta x_f$$

for the difference in propagation time δt_f of a frequency f over two lines of force separated by δx_f km along the meridian of longitude. Substitution of values from Table 4 leads to estimates of δx_f for Adelaide and Hobart as in Table 5. For each station the estimates of δx_f using the higher frequency whistlers are quite close but the lower frequency whistlers require larger values of δx_f to explain the observed spread in time.

An estimate of the variation of δx with latitude may be made from the equation for the variation of $\overline{F}_1$ with latitude if the assumption is made that $\overline{F}_1$

values (which are effectively average values of δt_f for three frequencies) apply to an " average frequency " of $7 \cdot 1$ kc/s. The values are shown in Table 6.

Since the illuminated area at the base of the ionosphere at the source end of the path may reasonably be assumed to have a radius of at least 100 km and the field lines range in length from 9000 km (30°) to 58,000 km (60°) these results suggest that the departure from perfect guiding is much less than one degree.

If, however, the propagation of whistlers is due to the presence of ducts, then the above reasoning may be considered to apply to the ducts rather than to a continuous medium, for the observed average dispersions must then be an average property of the guiding ducts. The diffuseness would then be expected to be determined by the range of ducts " illuminated " by energy from a lightning stroke. At low latitudes the diffuseness would be expected to be small because of the relatively large enhancements of electron density needed to produce effective ducts and the consequent relative decrease in the number of ducts (Smith, Helliwell, and Yabroff 1960), and also because of the limitation of the region of illumination suggested by Iwai and Outsu (1956). At higher latitudes a larger number of ducts may be expected to be illuminated, with a consequent increase in diffuseness. The supposition of a duct type of propagation also facilitates the explanation of the occasional very sharp whistlers which are observed at high latitudes. On these premises the diameter of ducts must be considered to be only a fraction of the dimensions listed in Table 6.

TABLE 6

LATITUDE DEPENDENCE OF δx

Latitude ..	30°	40°	50°	60°
δx (km) ..	240	380	510	650

Model (*b*)

A model of the upper atmosphere in which the quotient of electron density and magnetic field strength is constant along a line of force has been used by Gallet (1959) in studies of V.L.F. emissions. If it is assumed that such a condition is satisfied throughout the region in which a whistler propagates, then the time of travel is, by virtue of equation (1), proportional to path length and

$$\delta t_f = (\delta l/l) D f^{-\frac{1}{2}},$$

where δl is the difference in length between the two extreme paths and l the length of the " middle path ". The quotient, $\delta l/l$, depends on latitude and also upon the separation in latitude (at the surface of the Earth) of the two extreme paths. The appropriate values may be readily estimated from tables of path length (Chapman and Sugiura 1956). These data, together with the variation of $\overline{F}_1$ with λ, the experimental values of dispersion, and the use of an " average frequency " of $7 \cdot 1$ kc/s, gave estimates of δx ranging from 240 km at latitude 30° to 190 km at latitude 60°. The values for higher latitudes are thus appreciably smaller than for model (*a*).

Again, if propagation is by means of ducts within which the electron density is comparable with the surroundings, it seems reasonable (failing any more detailed knowledge of the variation in properties from duct to duct) to regard the values of δx calculated above to be an approximate measure of the extent of illumination of the lower ionosphere.

For both models the numerical values calculated are dependent on the values assigned to the dispersion. The experimental values of dispersion, which are average values for a large number of whistlers observed at several different latitudes, are likely to be somewhat biased because of the variation in the occurrence of whistlers with latitude (Part I) and the distance over which a whistler may be heard (Storey 1953; Crary, Helliwell, and Chase 1955). It thus seems likely that the values used for D are too large for lower latitudes and too small for high latitudes and are least in error for middle latitudes where whistlers are most common. However, if such is the case, the diffuseness figures will be in error in a similar fashion and the one error will tend to reduce the other.

The smaller diffuseness of upper frequency whistlers (Section III (a)) is not explicable in terms of either of the above models. The difference noted in $\overline{\delta t}$ on the 7 kc/s channel for the two groups suggests a difference in the mode of propagation either over the ground or in the outer atmosphere. If the former were the case, then the smaller diffuseness of the upper frequency whistlers might be explained in terms of these arriving at the ground a considerable distance from the observing station and undergoing sufficient attenuation to reduce the time for which any frequency is observable. The absence of the lower frequencies might then be due to the increased absorption discussed by both theoretical and experimental workers (Wait 1957; Obayashi 1959). The lower frequency whistlers would then have to be considered to be those which reached the ground near to the observing station, but no explanation for the absence of the higher frequencies is apparent on this basis. Alternatively, it might be postulated that the upper frequency whistlers are guided by ducts with a lower frequency cut-off of about 5 kc/s and that the lower frequency ones are propagated, as described by Storey, in a region without ducts. The values of $\overline{\delta t}$ for 3 kc/s, which for either model suggest smaller values for δx at this frequency than at 7 kc/s, are in qualitative agreement with Storey's work which predicted " the spreading will be least for the lowest frequencies ". While a difference in the spectrum of the source could explain the difference in frequency range of the two groups of whistlers, it would not explain the difference in diffuseness.

If the data of Table 4 are considered to indicate a cut-off frequency of the order of 5 kc/s, then one may attempt to estimate the radius of the ducts (considered to be circular in cross section) in the lower atmosphere by considering them to be of the nature of a dielectric waveguide. For such a guide of radius b, permittivity ε_1, permeability μ_1, in a medium ε_2, μ_2, the cut-off frequency (Bronwell and Beam 1947)

$$f_0=k_1b/2\pi b(\varepsilon_1\mu_1-\varepsilon_2\mu_2)^{\frac{1}{2}},$$

where k_1b is approximately $2\cdot 4$. This equation may be transformed to

$$f_0=2\cdot 4v_1/2\pi b(1-N_2/N_1)^{\frac{1}{2}},$$

where v_1 is the velocity in medium 1 and N the electron density. Taking a value of 100 for the refractive index of the F layer (Storey 1953) and assuming $N_1=1\cdot 1N_2$ for latitudes of about 50° (Smith, Helliwell, and Yabroff 1960) leads to an estimate of approximately 800 m for the radius of the duct in the F layer; furthermore, this value is reduced by a factor of only about three times when N_1 is much larger than N_2.

The explanation of the increase in whistler diffuseness observed when ionospheric spread-F is prevalent must depend on the interpretation of the latter phenomenon. However, various workers have found an association between spread-F and the patches of ionization, elongated along magnetic field lines, which are considered to be responsible for radio star scintillations (Briggs 1958), and a correlation has been reported between spread-F and the occurrence of field-aligned ionization (Leadabrand 1955). Thus, it seems plausible to regard the reported increase in diffuseness as being due to there being an increase in the number of ducts illuminated when ionospheric spreading is observed.

The values calculated for δx are mean values which, for Brisbane, Adelaide, and Hobart, will be somewhat larger than the modal values as indicated by Figure 2. Values of δx several times larger than these must be postulated to exist on occasions to account for the very diffuse whistlers sometimes observed.

V. Conclusions

The diffuseness of whistlers is primarily dependent on the path over which the energy travels. It may be explained either by assuming imperfect guiding in a medium with smoothly varying properties or in terms of propagation along ducts with parameters somewhat different to those of the surrounding medium. The experimental evidence from the present study is inadequate to differentiate between the two models but suggests that ducts may be operative at higher frequencies and inoperative at lower frequencies. No significant seasonal or diurnal variations in diffuseness have been observed but diffuseness increases at times when ionospheric spread-F is prevalent.

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