THE PHASE SPEED OF A TRAVELLING DISTURBANCE IN THE FREGION OF THE IONOSPHERE AND ITS COMPARISON WITH GROUP VELOCITY

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Summary

Three methods of measuring the phase speed of a disturbance in the F region, that is, the speed of a peak or trough in the isoionic contours, are given using ionograms records from one station. The analysis has been applied to one such disturbance whose group velocity was known to be 10 km/min. All three methods gave the phase speed to be about half of this. The theoretical explanation of this observation is discussed.

I. INTRODUCTION

Travelling ionospheric disturbances pertinent to this discussion have been previously described by Munro and Heisler (1956), and cause gross distortions on ionogram records, due to ripples formed in isoionic contours. The velocity of the disturbances has been measured to be about 10 km/min (Heisler 1958) by observations at stations spaced the order of 1000 km apart. As the stations were spaced much more widely than a wavelength of the ripples, this velocity is evidently the group velocity of the ripples, that is, the velocity with which a patch of large distortion of the contours moves regardless of the detailed movement within the patch. Recently, however, Heisler (1959) and Heisler and Whitehead (1960) found that travelling disturbances are associated with the appearance of sporadic E. This relationship is all the more puzzling when it is realized that the velocity of sporadic E clouds measured at relatively closely spaced stations (Harvey 1955) is only about half the velocity of F disturbances. However, sporadic E clouds may be associated not with the travelling disturbance as a whole, but with a particular phase (i.e. a peak or trough) of the isoionic ripples, as suggested by an observation by Bowman (1960a). Such an association would be feasible if the *phase velocity* of the ripples was equal to that of the sporadic E clouds.

The purpose of this paper is to describe three methods used to measure this phase velocity, or rather speed, for we do not determine the direction of the movement, and to show that the results support the hypothesis, as previously reported by Heisler and Whitehead (1961).

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Fig. 1.—Isoionic curves showing true path against time for each electron density.

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II. DERIVATION OF THE ISOIONIC CURVES AND THEIR RELATION TO THE ISOIONIC CONTOURS

Ionogram records taken every two minutes were used in the analysis and suitable records were chosen on the following basis.

(1) Ionogram records existed for Canberra $(35^{\circ} 18' \text{ S.}, 149^{\circ} \text{ E.})$, Sydney $(33^{\circ} 52' \text{ S.}, 151^{\circ} 11' \text{ E.})$, Brisbane $(27^{\circ} 30' \text{ S.}, 153^{\circ} \text{ E.})$, and Townsville $(19^{\circ} 10' \text{ S.}, 146^{\circ} 58' \text{ E.})$ and gave a non-ambiguous group velocity.

(2) The Brisbane records which were used in the phase speed analysis had a well-defined second echo.

(3) The first echo on these records showed little or no splitting (implying a relatively small disturbance) so that a true height reduction could be carried out without ambiguity.

After inspecting several thousand ionogram records, we consider ourselves fortunate in finding one disturbance which satisfied all three conditions.



Fig. 2.—The geometry of off-vertical reflections.

The next step was to reduce the ionograms to electron density : true path curves by Duncan's (1958) method which takes the Earth's magnetic field into account. Note the term "true path" rather than "true height", because in general the echoes do not return from the vertical. Errors due to this deviation (which changes the angle between the direction of propagation and magnetic field) and to the fact that all echoes recorded on one ionogram do not come from the same direction are likely to be quite small because the deviation is only a few degrees and the group delay is determined principally by the electron density gradient at the reflection level.

It is now possible to construct a diagram showing how the true path P for each value of electron density varies with time. These isoionic curves are shown in Figure 1 and are simply related to the isoionic *contours* in space once the phase velocity is known. For instance, in Figure 2, (x, z) is the reflecting point when the ionosonde is at a distance x_0 along the ground, z being the distance of the point above the ground. Then we have

$$P^{2} = 4\{z^{2} + (x_{0} - x)^{2}\}, \tag{1}$$

and

$$x_0 = z \mathrm{d}z/\mathrm{d}x + x. \tag{2}$$

Differentiating equation (1) with respect to x_0 ,

$$P \cdot \frac{\mathrm{d}P}{\mathrm{d}x_0} = 4 \left[z \cdot \frac{\mathrm{d}z}{\mathrm{d}x} \cdot \frac{\mathrm{d}x}{\mathrm{d}x_0} + (x_0 - x) \left(1 - \frac{\mathrm{d}x}{\mathrm{d}x_0} \right) \right]$$

=4(x_0 - x), (3)

from (2). Therefore

$$x = x_0 - \frac{1}{4}P \cdot \mathrm{d}P/\mathrm{d}x_0,\tag{4}$$

and

$$z^{2} = \frac{1}{4}P^{2} \{1 - \frac{1}{4} (\mathrm{d}P/\mathrm{d}x_{0})^{2}\}.$$
 (5)

But

$$\mathrm{d}P/\mathrm{d}x_0 = (1/V)\mathrm{d}P/\mathrm{d}t,$$

where V = phase velocity. Hence from equations (4) and (5)

$$x = x_0 - (P/4V) \mathrm{d}P/\mathrm{d}t,\tag{6}$$

and

$$z^{2} = \frac{1}{4}P^{2} \{ 1 - (1/4V^{2})(dP/dt)^{2} \}.$$
(7)

(8)

Equations (6) and (7) enable us to calculate the isoionic contours (x, z) once V is known. In particular, the radius of curvature R of the contours (positive when concave downwards) is given by

$$1/R \approx d^2 z/dx^2$$
 as $(dz/dx)^2 \ll 1$.

But from equations (2) and (3)

$$z dz/dx = \frac{1}{4}P dP/dx_0$$
.

Therefore

$$z \mathrm{d}^2 z / \mathrm{d} x^2 \! pprox \! rac{1}{4} P (\mathrm{d}^2 P / \mathrm{d} x_0^2) \mathrm{d} x_0 / \mathrm{d} x \ = \! rac{1}{4} P rac{\mathrm{d}^2 P}{\mathrm{d} x_0^2} \cdot rac{1}{1 - \! rac{1}{4} P \mathrm{d}^2 P / \mathrm{d} x_0^2}$$

from equation (4). But $z \approx \frac{1}{2}P$, therefore

$$R = \frac{Pd^{2}P/dt^{2} - 4V^{2}}{2d^{2}P/dt^{2}},$$
$$V^{2} = \frac{1}{4}(P - 2R)d^{2}P/dt^{2}.$$

or

III. THE PHASE SPEED FROM A KNOWLEDGE OF R

The first method of calculating V relies on the fact that R may be estimated from the splitting of the various order echoes. If R is greater than the height of the layer, the first echo will not be split (Munro 1953) but if it is less than

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twice this, the second echo will be split. This situation occurred during the travelling disturbance under discussion. Hence when the second echo is split $\frac{1}{2}P < R < P$ and $V^2 < \frac{1}{4}P(-d^2P/dt^2)$ the quantity d^2P/dt^2 being negative when $R > \frac{1}{2}P$. At the lowest frequency at which splitting of the second echo occurred during the disturbance, $d^2P/dt^2 \approx -0.22$ km/min², giving V < 4.5 km/min.

IV. CALCULATING V FROM THE RELATIVE GROUP PATHS OF THE FIRST AND SECOND ECHOES

The second method of calculating the phase speed is based on the fact that the second echo is not in general reflected from the same point as the first. It is usually more off-vertical than the first and therefore even more unreliable than the first as regards the conditions directly overhead—in contrast to the assumption by Wright (1959) that the second echo gives the more reliable data



Fig. 3.—The paths of first and second echoes when the isoionic contours are straight lines.

under these circumstances. Thus the second echo has true and group paths not equal to twice those of the first echo. Figure 3 shows the simple situation when the isoionic contours are straight and parallel but not horizontal (when $d^2P/dt^2=0$ but $dP/dt\neq 0$). If the first echo path inclined at angle θ to the vertical is P_1 and the group path is P'_1 , and the second echo group path is P'_2 , we have

$$P_{2} = \frac{P_{1}}{\cos \theta} + \frac{P_{1} \cos 2\theta}{\cos \theta} + \frac{2(P_{1} - P_{1})}{\cos \theta},$$

or approximately

$$2P_1' - P_2' = \theta^2 (2P_1 - P_1'),$$

allowing θ to be calculated. V may then be found from the relation

$$V = (1/2\theta) \cdot (dP/dt).$$

The analysis was extended to the case where $d^2P/dt^2 \neq 0$ by assuming that the contours form concentric circles : the validity of this assumption depends on there being only a small change in the curvature of the contours over a horizontal distance of $\frac{1}{2}P_1\theta$. During the disturbance, this amounted to as much as 25 km, making single measurements unreliable. A systematic error seems unlikely from this course, but may occur in the measurement of $2P'_1 - P'_2$, should the height marks on the ionograms have a zero error. By noting the variations with time and probing frequency, this zero error was found to be about 3 km, making a difference of 6 km in $2P'_1 - P'_2$ which had a value not greater than 60 km. The random error in $2P'_1 - P'_2$ was about the same order.

This method gave a mean value for V of $4 \pm 1 \text{ km/min}$.

V. CALCULATING V FROM THE SLOPE OF THE FRONT

This method makes use of the evidence that wave fronts of travelling disturbances tend to lie along the Earth's magnetic field (Munro and Heisler 1956; Bowman 1960b). It must be admitted that the present disturbance is probably much larger in amplitude than those investigated. However, it is known that the wave front slopes forward for disturbances moving towards the equator, that is, in the same sense as the magnetic field slope, and thus it appears not unlikely that the slope is along the magnetic field lines. From the isoionic curves, it is possible to trace out the sloping wavefronts (shown as dotted lines in Fig. 1). If the fronts are to lie along the magnetic field then $V = 5 \pm 2$ km/min.

VI. THE MEAN PHASE VELOCITY

Whilst it could be argued that each one of the three methods given for measuring the phase speed has some uncertainty about it, yet the fact that all three gave similar values for V is very strong evidence in favour of their veracity. It is not suggested that these methods could be used on a systematic basis and there is need for further experimental verification. Indeed, the difficulty introduced by having to choose suitable records and analysis involving many true path calculations precludes the general application of the method. The purpose in describing the methods is to display their validity in arriving at the final result that the phase speed of the disturbance was 4.5 km/min whereas its group velocity was about twice this at 10 km/min travelling 6° E. of N.

This result confirms the indication we had of two velocities relevant to travelling disturbances, and thus removes one stumbling block to the understanding of the disturbance : sporadic E association.

Furthermore, inspection of the methods used to estimate the velocity of "fronts", which cause "satellite" traces on ionogram records, shows that this is the phase velocity. Its magnitude of ~ 4 km/min (McNicol, Webster, and Bowman 1956) agrees well with our value and makes it more certain that the satellite traces are just another manifestation of travelling disturbances.

Waves moving with this large dispersion will change form rapidly. Indeed, the lifetime of a single ripple would only be about one period—the order of half an hour. Such a lifetime was found by Gusev *et al.* (1960) for observations of radio phase path changes at fairly closely spaced stations. The changes in form should not be completely random and further stations more widely spaced could be used to follow this change of form.

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VII. THEORETICAL IMPLICATIONS OF THE OBSERVED DISPERSION

The reported dispersion does not agree with that given by Hines (1960) for internal gravity waves which he regards as the cause of F-region disturbances. His dispersion in fact has an opposite sign.

One explanation is, however, possible which retains Hines's gravity waves in the E region as the basic cause of the F-region disturbance, that is, if they give rise to ionization movements in the F region by virtue of the electric fields they generate in the E region rather than by neutral air movements in the F region. The electric fields in the E region are generated by movements of the neutral air which drive ionized material across the Earth's magnetic field. The details are discussed by Whitehead (1961), from which paper it may be seen that the electric fields are proportional to a mean horizontal velocity \overline{U} of the neutral air in the E region given by

 $\overline{U} = \int UNf(\mathbf{v}_i) \mathrm{d}h / \int Nf(\mathbf{v}_i) \mathrm{d}h,$

where N = electron density,

 v_i = ionic collision frequency,

U = horizontal velocity of neutral air,

and $f(v_i)$ is a function of v_i which becomes small above 130 km.

For a particular electron density distribution in the E region, the total electric field generated by two wind systems is the sum of the electric fields generated by each. However, each wind system itself changes the electron density distribution and thus so far as the electric field is concerned, and hence F-region ionization movements, there is considerable interaction between the systems.

Now suppose a short gravity wave only generates an appreciable electric field for the electron density distribution appearing at a particular phase of a long gravity wave. Then the corresponding disturbance in the F region will have a "phase velocity" equal to the actual velocity of the short wave but a "group velocity" equal to that of the long wave. Hines predicts a velocity increasing with wavelength, and so this interaction mechanism may provide an explanation of an apparent phase velocity in the F region being less than the apparent group velocity.

VIII. GENERAL COMMENT

The fact that two distinct velocities have been shown to be associated with the same disturbance illustrates that great care must be exercised in comparing velocities of movements using different methods. For instance, Thomas (1959) estimated height gradients of horizontal velocities to be the order of 2×10^{-3} /s in the F region from four average velocities found by using three different methods variously at three places. Clearly this height gradient is inconsistent with the observation that wavefronts of travelling disturbances maintain their slopes fairly constant for several hours, for Thomas's height gradient would lead to a front rotating at about one revolution per hour. Our result shows that one might expect different velocities when using different.

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methods of observation even though this may be only a matter of the spacing of the observing stations. Single-station observations by Heisler (1960) would suggest that such a height gradient is non-existent.

IX. CONCLUSIONS

It has been shown that the phase speed of a particular travelling disturbance was 4.5 km/min whereas its group velocity was 10 km/min. The validity of the result rests principally on the consistency of the three independent methods used in deriving it. One possible explanation of the result is given which retains gravity waves in the *E* region as the prime cause of travelling disturbances in the *F* region.

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