# ERRORS DUE TO DIFFUSION IN DRIFT VELOCITY MEASUREMENTS

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#### Summary

An analysis is made of errors which occur due to diffusion in measurements of the drift velocity of electrons in gases using the methods of Bradbury and Nielsen, Pack and Phelps, and Hornbeck. This analysis includes an account of the influences of back diffusion and the effect of the boundary condition imposed by the receiving electrode. It is shown that, in the case of the method of Bradbury and Nielsen, effects due to diffusion can lead to spuriously high measurements of the drift velocity introducing a relative error of the order of 3/h(W/D) (h being the distance between the shutters and W/D the ratio of the drift speed to the diffusion coefficient).

Experimental results obtained by using the method of Bradbury and Nielsen support the theoretical predictions. For E/p=0.4 V cm<sup>-1</sup> mmHg<sup>-1</sup>, the measured value of the drift velocity rose by 13% in helium and 8% in nitrogen as the coefficient of diffusion was increased.

### I. INTRODUCTION

This paper discusses errors introduced, due to diffusion, when measurements of the drift velocity of electrons are made using the shutter methods of Bradbury and Nielsen (1936) and Pack and Phelps (Phelps, Pack, and Frost 1960; Pack and Phelps 1961) and also the method used by Hornbeck (1951). The analysis is an extension of the investigation by Duncan (1957) to take into account the boundary conditions imposed on the electron density. It is shown that by using large chamber lengths and high gas pressures these errors can be made less than other experimental errors. For the method of Bradbury and Nielsen, evidence is presented that the order of the relative error in the measured value of the drift velocity is given by 3/h(W/D) (h being the distance between the shutters and W/D the ratio of the drift velocity W to the diffusion coefficient D).

Drift velocity results are given which were obtained using the method of Bradbury and Nielsen. These results illustrate the effect of varying the coefficient of diffusion of the electrons in a gas for a constant value of E/p (E being the field strength in  $V \text{ cm}^{-1}$  and p the gas pressure in mmHg) by taking drift velocity readings different gas pressures. Experimental variations  $\operatorname{at}$ of asmuch as 13% in helium and 8% in nitrogen have been obtained at  $E/p = 0.4 \text{ V cm}^{-1} \text{ mmHg}^{-1}$ . These results represent more accurate measurements of the effect first investigated by Crompton, Hall, and Macklin (1957), who showed that the measured value of the drift velocity becomes lower for higher values of the gas pressure, approaching an asymptotic limit at high pressure, as would be expected from the theoretical predictions.

The expressions derived for the relative error introduced owing to diffusion in the measurements of the drift velocity of electrons are also directly applicable

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to measurements of the mobility of positive ions. Such measurements have been made by Biondi and Chanin (1954) (using a modification of Hornbeck's method for electrons) and by Crompton and Elford (1959) (using the method of Bradbury and Nielsen). In both of these investigations the predicted errors due to diffusion are less than the experimental errors because the value of W/D is much higher for positive ions than for electrons under equivalent conditions of pressure and electric field.

## II. METHOD OF BRADBURY AND NIELSEN

In this method electrons emitted from a filament F (Fig. 1) drift through the gas to the collecting electrode B under the influence of a uniform electric field which is maintained between the electrodes A and B. Two shutters,  $S_1$ and  $S_2$ , each consisting of a plane grid of parallel wires, of which alternate wires are connected together, are placed in the path of the electrons, the distance h



Fig. 1.-Schematic diagram of Bradbury and Nielsen apparatus.

between the two shutters being accurately known. An a.c. signal, whose frequency can be varied while maintaining a constant voltage, is applied to the two halves of  $S_1$ . The voltages applied to each half of the shutter are exactly 180° out of phase. Similar voltages exactly in phase with those applied to  $S_1$ are applied to  $S_2$ . All a.c. signals have superimposed on them the appropriate d.c. voltage required to maintain the uniform d.c. field between A and B. The frequency of the a.c. signal is then varied and the current collected at B is recorded. When a graph is drawn of current against frequency a series of maxima will be obtained as shown in Figures 7 and 8.

The action of  $S_1$  is to divide the stream of electrons into a series of pulses, as electrons will be admitted through the shutter only when the a.c. signal on the shutter wires approaches zero. If f/2 is the frequency of the a.c. signal, f will be the frequency with which the shutters open.  $S_2$  will have a similar influence on the electron current and the first maximum of the current/frequency curve will correspond to the frequency  $f_1$  for which pulses of electrons produced by  $S_1$  arrive at  $S_2$  when the shutters are again about to open. Other current maxima will occur at integral multiples of  $f_1$ .

The drift velocity is then generally determined from  $W = hf_1$ . Use of this formula assumes, however, that the time required for the centre of an undisturbed pulse to travel the distance h is  $1/f_1$ . This assumption is not exactly true for the following reasons :

- (1) Some electrons from each pulse will be absorbed owing to back diffusion by the first shutter after it is shut. This slightly alters the position of the maximum of the electron density for the pulse from that corresponding to the ideal case of an undisturbed pulse.
- (2) The pulse is disturbed by the absorption of electrons by the second shutter before it opens.
- (3) The frequency corresponding to maximum electron current is not equal to the frequency corresponding to maximum electron density of the undisturbed pulse at the second shutter because of diffusion current.
- (4) The number of electrons in each pulse varies as the frequency is varied.
- (5) Current/frequency curves will not be symmetrical about each current maximum because there is a continuous decay of the maximum electron density of a pulse while it is passing through the second shutter.
- (6) The shutters transmit electrons of high speed more readily than electrons of low speed. Thus errors will be introduced due to any variation of  $\tilde{c}$  along the length of the pulse ( $\tilde{c}$  being the mean agitational speed of the electrons).

The influence of factors (3), (4), and (5) was analysed by Duncan (1957) for the case of a point source of electrons with a small collecting electrode.

Throughout this paper the terms "relative error", represented by r, and "error", are used to imply variations in the measured results due to any of the above factors. Accidental experimental errors are referred to as "accidental errors".

## (a) Diffusion Errors other than the Error due to Back Diffusion

This subsection discusses the combined effect of factors (2), (3), (4), and (5) mentioned previously. The final expression for the order of the relative error applies only to the limiting case when the second shutter is open for a very small percentage of the time. In the Appendix the effect of the second shutter being open for a finite time interval is considered and from the equations that are derived it is possible to calculate the modified value of the relative error.

The differential equation governing the motion of the electrons is

$$\nabla^2 n = \frac{W}{D} \frac{\partial n}{\partial x} + \frac{1}{D} \frac{\partial n}{\partial t}, \tag{1}$$

where *n* is the electron density, *x* is the distance from the first shutter, and *t* is the time. As the mean agitational speed of the electrons  $\bar{c}$  is generally of the order of 100 times the drift speed *W*, it is justifiable to assume that the coefficient of diffusion parallel to the applied electric field is equal to the coefficient of diffusion perpendicular to the field.

It is assumed that the shutters can be considered as being either open or shut. This approximation of the experimental conditions is made to simplify the analysis. It is also assumed that the first shutter produces an infinitely thin plane source of electrons at x=0 when t=0 which diffuses as it passes down the chamber as in Figure 2.

Then

$$n = [N_0/(4\pi Dt)^{\frac{1}{2}}] \exp\left[-(x - Wt)^2/4Dt\right]$$
(2)

(where  $N_0$  is the number of electrons in the pulse per unit area of the yz plane) is a solution of the differential equation (1), satisfying the initial conditions.

At any time t, the maximum value of n is  $N_0/(4\pi Dt)^{\frac{1}{2}}$  which decays with time. The decay leads to the error described by factor (5) above.



Fig. 2.—Showing how the function for the electron density is modified by an image term so that the boundary condition at the second shutter is satisfied.

When the second shutter is shut, it acts as a metal plane absorbing electrons in its vicinity, imposing a boundary condition n=0 at x=h on the solution to the differential equation. This boundary condition can be satisfied for all t by introducing a negative image term, of appropriate strength, which represents a pulse starting at x=2h and also travelling with velocity W. The modified solution of the differential equation is then:

$$n = \frac{N_0}{(4\pi Dt)^{\frac{1}{2}}} \left\{ \exp\left[-\frac{(x-Wt)^2}{4Dt}\right] - \exp\left[\frac{hW}{D} - \frac{(x-2h-Wt)^2}{4Dt}\right] \right\}.$$
 (3)

The instantaneous current density J is given by

$$J = neW - De\frac{\partial n}{\partial x}$$
  
=  $-De\left(\frac{\partial n}{\partial x}\right)_{x=h}$  at the second shutter (4)  
=  $\frac{N_0 e}{(4\pi D t)^{\frac{1}{2}}} \frac{h}{t} \exp\left[-\frac{(h-Wt)^2}{4Dt}\right].$ 

To determine J as a function of frequency the following modifications to equation (4) are made.

- (i) Substitute t=1/f to change the variable from time to frequency.
- (ii) Substitute  $N_0 = N_1/f$  because the number of electrons admitted by the first shutter is proportional to the time that it is open (factor (d)). This substitution is valid only in case of high diffusion where the undisturbed diffused pulse at x=h can be closely represented by an exponential function of the form given in equation (2). In the case of low diffusion the electron density is more closely represented by a step function and the following analysis is not applicable.

On making these two substitutions the expression for J is

$$J = \frac{N_1 e h f^{\frac{1}{2}}}{(4\pi D)^{\frac{1}{2}}} \exp\left[-\frac{(h - W/f)^2 f}{4D}\right].$$
 (5)

As the time constant of the measuring circuit is large, the current measured will be independent of displacement currents, and will depend only on the number of electrons collected by the receiving electrode. The number of electrons received might be expected to be doubled when the frequency is doubled because of the increased number of times that the second shutter is open, but this is cancelled by the fact that, on each occasion, the interval of time that the second shutter is open is then halved. Thus I, the current measured, is simply given by  $I=C_1J$ , where  $C_1$  is a constant of proportionality. It is assumed that the second shutter is open for such a small interval of time that the effect of the removal of the boundary condition n=0 at x=h is negligible.

It is now required to find an expression to determine the approximate value of f for which the current I is a maximum. We change the variable in the expression for I from f to r where  $r=(f-f_0)/f_0$ ,  $f_0$  being given by  $W=hf_0$ . Then on substituting  $f=f_0(1+r)$  in equation (5) we obtain for I

$$I = \frac{C_1 N_1 e h f_0^{\frac{1}{2}} (1+r)^{\frac{1}{2}}}{(4\pi D)^{\frac{1}{2}}} \exp\left[-\frac{h^2 f_0 r^2}{4D(1+r)}\right].$$
(6)

Maximum current occurs when  $\partial I/\partial f=0$ , i.e. when  $\partial I/\partial r=0$ , i.e. when

$$\frac{1}{2} = h^2 f_0 r/2D - h^2 f_0 r^2/4D(1+r).$$

Thus r is the relative error in assuming that  $f_0$  is the frequency for which maximum current occurs. As r will be small, we can neglect the term in  $r^2$  and obtain

$$r = 1/h(W/D). \tag{7}$$

From this expression we can calculate the percentage error to be expected in the limiting case when the shutters are open for a very small time interval, since the values of W/D are known from independent diffusion measurements. For example, at E/p = 0.4 V cm<sup>-1</sup> mmHg<sup>-1</sup> at p=5 mm of nitrogen, the measured value of W/D is 6.35 (from unpublished measurements made by Dr. R. W. Crompton using Townsend's method of determining W/D). For a drift velocity apparatus where the distance between the shutters is 6 cm, formula (7) predicts the measured value of the drift velocity to be 2.6% high.

By plotting equation (29) derived in the Appendix, one can also predict the error when the second shutter is open for a finite time. Such calculations for the experimental conditions quoted above have been made when the bottom shutter is open for 0.01, 0.05, and 0.1 of the time and are given below.

 Fraction open
  $0 + 0 \cdot 01$   $0 \cdot 05$   $0 \cdot 1$  

 Predicted error
  $2 \cdot 6\%$   $2 \cdot 1\%$   $1 \cdot 6\%$   $1 \cdot 5\%$ 

For the case of the inert gases where the coefficient of diffusion is higher, the resultant errors will be greater.

## (b) Error due to Back Diffusion

This subsection discusses factor (1). In cases of high diffusion, after a pulse is formed by the first shutter, some of the electrons which are diffusing backwards from the pulse are swept up by the closed shutter, which again is considered to act as a conducting plane. The centre of the pulse is thereby displaced slightly forward, and in the following analysis we obtain an expression for this displacement. In this case the solution to the differential equation has the boundary condition n=0 at x=0. Owing to the difficulty of representing mathematically the initial conditions, the following analysis has been developed to approximate the experimental conditions.

We introduce the electron pulse as an infinitely thin plane at x=h/m from the first shutter at time  $t=t_0$ , where m is a number which may be made very large and  $t_0$  is the time required for the pulse to travel the distance h/m at velocity W, i.e.  $t_0=h/mW$ . It is found that the expression for the error approaches a limiting value as m is made very large. Again the boundary condition is satisfied by introducing an image term travelling in the same direction as the electron pulse, where in this case it starts at x=-h/m at  $t=t_0$ . The strength of the image term is chosen such that n=0 at x=0 for all  $t>t_0$  and the solution of the differential equation (1) is then

$$n = \frac{N_0}{[4\pi D(t-h/mW)]^{\frac{1}{2}}} \left\{ \exp\left[-\frac{(x-Wt)^2}{4D(t-h/mW)}\right] - \exp\left[-\frac{hW}{mD} - \frac{(x+2h/m-Wt)^2}{4D(t-h/mW)}\right] \right\}.$$
(8)

The value of n when the pulse is at the second shutter is given by substituting t=h/W; then

$$n = \frac{N_0 W^{\frac{1}{2}}}{[4\pi Dh(1-1/m)]^{\frac{1}{2}}} \left\{ \exp\left[-\frac{(x-h)^2 W}{4Dh(1-1/m)}\right] - \exp\left[-\frac{hW}{mD} - \frac{(x+2h/m-h)^2 W}{4Dh(1-1/m)}\right] \right\}.$$
(9)

The maximum of n will occur for a value of x slightly different from h owing to the presence of the image term. This value of x will satisfy  $\partial n/\partial x=0$ , i.e. x satisfies

$$x-h=(x+2h/m-h)\exp\left[-\frac{hW}{mD}-\frac{W(h^2/m^2+hx/m-h^2/m)}{Dh(1-1/m)}
ight]$$

The variable is again changed to r, where r = (x-h)/h is the relative error in taking the maximum of n to be at x=h. Omitting all terms of  $1/m^2$  and higher order and all terms in  $r^2$  and higher order, the above equation becomes

$$r = (r+2/m) \exp\left[-\frac{hW(1+r)}{Dm}\right],$$

$$r = \frac{2}{m[\exp(hW/mD) - 1]}.$$
(10)

i.e.

If m is so large that hW/mD is small, r is given by

$$r \simeq 2/h(W/D). \tag{11}$$

For the experimental conditions quoted earlier, equation (11) predicts an error of  $5 \cdot 2\%$  as a consequence of back diffusion. This prediction applies for the limiting case of the first shutter being open for a very small interval of time.

It is of interest to note the effect of different values of 1/m on the value of r as calculated from equation (10).

VALUES OF $r$ in relation to $1/m$ in equation (10)				
1/m	0+	0.01	$0 \cdot 05$	$0 \cdot 1$
Predicted error	5.2%	4.3%	1.8%	0.5%

TABLE 1

In the case of the first shutter being open for a finite time, the initial electron density can be approximated by a step function of width h/M. We consider this step function to be a series of infinitely thin elemental pulses which start at distance h/m from the first shutter where  $M < m < \infty$ . As can be seen from Table 1, the closer to the first shutter that an elemental pulse starts, the greater is the error due to back diffusion. Thus the total error for a pulse originally a step function of width say h/20 would generally be expected to be greater than that given in Table 1 when 1/m = 1/20.

It is assumed that, to a first approximation, the combined errors due to back diffusion and the errors considered in Section II (a) are additive. Since both of these errors lead to an increase in the measured value of the drift velocity, the total relative error in the limiting case of the shutters being open for a very small interval of time would be given by

$$r = 3/h(W/D).$$
 (12)

## (c) Error due to Variation of $\bar{c}$ within the Pulse

This subsection discusses factor (6). Townsend (1936) has shown that there will be a variation in the mean agitational speed of electrons within a pulse at any instant after its production in a gas, when it travels under the influence of a uniform electric field. The electrons in the leading section of a

pulse will have gained a greater amount of energy from the field than electrons in the trailing edge of the pulse because they have travelled a greater distance along the field. Now the electrical shutters admit electrons of high agitational speed more readily than electrons of low agitational speed because the faster moving electrons are less likely to be swept aside by the electric field produced between the shutter wires. Thus the number of electrons admitted by the second shutter for any section of an electron pulse will be dependent not only on the value of n and  $\partial n/\partial x$  but also on the value of  $\bar{c}$  for that section of the pulse. Consequently, the effect of the variation of  $\bar{c}$  within the pulse will be such as to increase the measured value of the drift velocity of a pulse above its true value. Although no quantitative expression for the order of this error has been attempted in this paper, it is evident that the error will tend to zero in cases where the width of the pulse tends to zero, i.e. where the gas pressure is made very large and thus the coefficient of diffusion very low.

# (d) Effect of having a Point Source of Electrons

The analysis of Sections II (a) and II (b) was based on the assumption of an infinite plane source of electrons. It is of interest to determine any change in the expressions for r on assuming that the electron pulse originates as a point source. The limiting values of r would then be known for any experimental conditions, which in general correspond neither to a point nor a plane source.

The expression for the electron density of a pulse of electrons from a point source satisfying the previous initial conditions is

$$n = \frac{N_0}{(4\pi Dt)^{3/2}} \exp\left[-\frac{(x - Wt)^2 + y^2 + z^2}{4Dt}\right].$$
 (13)

Equation (13) again satisfies the differential equation (1), and is such that

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}n\mathrm{d}x\mathrm{d}y\mathrm{d}z = N_0 \quad \text{for all } t > 0.$$

We consider first the case where the collecting electrode is infinite. The analysis of Section II (a) can then be repeated with the expression for n given by (13), again modified to include an image term. The value of the instantaneous current at the second shutter will then be

as before in (4). Consequently the relative error is still r=1/h(W/D). In a similar way the analysis for back diffusion in the case of a point source again gives r=2/h(W/D) so that the combined value of the relative error is still of the order of r=3/h(W/D).

However, the case of a point source with a collecting electrode which is not infinite does lead to a difference in the value of r from that obtained for a plane source. We consider the extreme case of a very small collecting electrode on

the X axis. It is no longer necessary to integrate over the collecting plane, for the instantaneous current at the second shutter may be regarded simply as the value of J at the point x=h, y=0, z=0. Thus from (14)

$$J = \frac{N_0 e}{(4\pi D t)^{3/2}} \frac{h}{t} \exp\left[-\frac{(h - W t)^2}{4Dt}\right].$$
 (15)

On making the substitutions of Subsection II (a), the current I is given by

$$I = C_2 (1+r)^{3/2} \exp\left[-\frac{hWr^2}{4D(1+r)}
ight]$$
 (C<sub>2</sub> is a constant).

When  $\partial I/\partial I = 0$ ,

$$r = 3/h(W/D).$$
 (16)

The analysis of Subsection II (b) appropriate to the case of a point source is not altered by having a small electrode instead of an infinite one insofar as it still gives a value of r for back diffusion of 2/h(W/D). Thus the appropriate expression for the combined error in the case of a point source used in conjunction with a very small collecting electrode on the X axis is given by r=5/h(W/D). This expression again only applies in the limiting case of the shutters being open for a very small interval of time.

#### (e) Summary

It is of interest to carry out the analysis of Section II (a) in the simpler cases of various combinations of the four factors numbered (2), (3), (4), and (5) listed at the beginning of Section II. From this analysis the contribution to the total error appropriate to each factor may be found. The results so obtained are consistent with the hypothesis that the individual errors from each factor are simply additive; the individual contributions are given below. The expressions for the errors apply for a plane source of electrons in the limiting case of the shutters being open for a very small time interval.

- (1) The relative error due to back diffusion as determined in Section II (b) is given by 2/h(W/D).
- (2) The influence of the boundary condition imposed by the second shutter gives a contribution to the relative error 1/h(W/D).
- (3) The inclusion of the term  $-D\partial n/\partial x$  to account for transport due to diffusion introduces a relative error of 1/h(W/D).
- (4) The varying number of electrons in each pulse with change of frequency (represented by the substitution  $N_0 = N_1/f$ ) changes the relative error by -2/h(W/D).
- (5) The coefficient  $1/(4\pi Dt)^{\frac{1}{2}}$  describing the decay with time of each pulse introduces a relative error of 1/h(W/D).
- (6) The varying value of  $\bar{c}$  within the pulse introduces an error the value of which is unknown.

The effect of having a point source with a small collecting electrode increases the relative error from that of a plane source by 2/h(W/D).

## III. METHOD OF PACK AND PHELPS

This method is similar to the method of Bradbury and Nielsen but with the following modifications. Instead of the electron pulses being formed by a shutter, the pulses are produced by photoelectric emission from a cathode by means of a pulsed source of ultraviolet light. The electron pulses then travel to a shutter, but this shutter is not operated by the application of a.c. sine wave voltages as in the Bradbury and Nielsen method. Instead, a constant d.c. bias is maintained between the two halves of the grid to close the shutter, and this bias is removed for a short time to open the shutter. The time interval between the flash of ultraviolet light and the opening of the shutter is varied, and the current from the electron pulses transmitted by the receiving shutter recorded. From the time at which maximum current occurs, the transit time of an electron pulse from the cathode to the shutter is determined. A second shutter is included in the apparatus further from the cathode than the first shutter, and the transit time of an electron pulse to the second shutter can similarly be determined. The difference in these two transit times should give the time taken for an electron pulse to travel the distance between the two shutters, independent of any end effects occurring at the cathode. On knowing the distance between the two shutters, the drift velocity is then calculated.

The calculation of diffusion errors for any transit time differs from that for the method of Bradbury and Nielsen. The number of electrons in each pulse is presumably proportional to the duration of each flash of ultraviolet light. This time is not varied when the time interval between the flash of ultraviolet light and the opening of the shutter is varied. Consequently, in the analysis appropriate to the method of Pack and Phelps, there is no substitution analogous to  $N_0 = N_1/f$  made in Section II (*a*), and an increase in the value of *r* results On the other hand, in the method of Bradbury and Nielsen, as the frequency of the signal applied to the shutters is increased, the first shutter is open for a shorter time and the number of electrons in each pulse is reduced. Consequently, there is a slight reduction in the frequency for which maximum current occurs which partially cancels other errors due to diffusion.

The expression for the instantaneous flux of particles at the receiving shutter is again given by J in equation (4). The variable may be changed from t to rwhere  $r=(t-t_0)/t_0$  and  $t_0$  is given by  $W=h/t_0$ . After substituting  $t=t_0(1+r)$ in (4), J becomes

$$J = C(1+r)^{-3/2} \exp\left[-\frac{hWr^2}{4D(1+r)}\right] \quad (C \text{ is a constant}).$$
(17)

Maximum current occurs when  $\partial J/\partial r = 0$ , i.e. when  $r = -3/\hbar(W/D)$  (neglecting powers in  $r^2$ ), where r is the relative error in assuming that  $t_0$  is the time at which maximum current occurs. If errors due to back diffusion also exist, the total relative error in any transit time measurement will be given by

$$r = -5/h(W/D).$$
 (18)

This expression again applies only in the limiting case of a very narrow width of the initial pulse and for the receiving shutter being open for a very small interval of time.

The error predicted by formula (18) is quite large for some of the experimental conditions used by Pack and Phelps. For example, consider the conditions for an apparatus with drift distances of  $2 \cdot 54$  cm and  $5 \cdot 08$  cm for neon at a pressure of 48 mmHg at 300 °K. For E/p = 0.5 V cm<sup>-1</sup> mmHg<sup>-1</sup>, W/D is approximately 7.1. (This figure is calculated using a value of Townsend's energy factor of 136 obtained by interpolation from the results of Bailey (1924).) The predicted errors in the two transit times are 28% for a drift distance of  $2 \cdot 54$  cm and 14% for a drift distance of  $5 \cdot 08$  cm. Differences should therefore exist in the drift velocities calculated from the two drift distances. Pack and Phelps have not reported any such differences so that it is possible that the errors due to diffusion have been masked by the finite width of the initial pulse and the finite time interval that the receiving shutters are open. These effects have not been considered in the above predictions.

The formulae for r that have been derived give the relative error as being inversely proportioned to h and so the absolute errors due to diffusion will be independent of h, to a first approximation. Thus when Pack and Phelps calculate drift velocities from the differences in the transit times for the two drift distances the absolute errors should largely cancel (see Pack and Phelps 1961, footnote 4). The errors, however, would still be expected to be greater than a second order effect, complete cancellation being unlikely as r is proportional to 1/h only in the limiting case of both the initial pulse width and the time interval that the shutters are open tending to zero.

## IV. HORNBECK'S METHOD

This method was used by Hornbeck (1951) to measure the drift velocity of electrons in helium, and has since been used by Bowe (1960) in an extensive and more accurate investigation. The method has also been used in a modified form by Biondi and Chanin to measure the mobility of positive ions. In the case of electrons, a flash of ultraviolet light produces an initial pulse of photoelectrons at the cathode, which then travels under the influence of a uniform electric field to the anode. The cathode and the anode are usually two large parallel circular plates. The resulting current pulse is then amplified and viewed oscillographically so that a photograph of the pulse can be obtained. Omitting influences due to diffusion the current I travelling in a Townsend gap is given by I = NeW/d, where N is the total number of electrons in the gap (assuming that there are no positive ions due to ionization), e is the electronic charge, W the drift velocity, and d the electrode separation. Thus, as the electrons are produced in the chamber the current rises to a constant level which is maintained while the electrons are traversing the gap. Then, as electrons are absorbed at the anode, the current will decrease, becoming zero when the pulse is completely absorbed. The form of a typical current trace is given in Figure 3.

The transit time of the pulse is assumed to be  $t_2-t_1$ , where  $t_1$  and  $t_2$  are the times at which the current is half of its maximum value. This procedure in determining say  $t_2$  is based on the assumption that when half of the electrons have been absorbed, the centre of the pulse is then at the collecting electrode. This assumption is not quite accurate when account is taken of the fact that the

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collecting electrode acts as a conducting plane reducing the electron concentration at its surface to zero, thus distorting the shape of the pulse. The time at which half of the electrons have been absorbed will occur slightly earlier than the time at which the centre of the undisturbed pulse would have reached the collecting electrode. As a consequence, values of the drift velocity which are slightly high will be recorded.

The analysis made in this section applies only to the evaluation of  $t_2$ . The position of  $t_1$  is determined by the rising characteristic of the current pulse which is dependent on the variation of the intensity of the light source with time and also on electron absorption due to back diffusion. The degree of back diffusion is further dependent on the energy with which the photoelectrons are emitted (see Theobold 1953).



Fig. 3.—Typical form of the oscillographic trace for a pulse of electrons crossing a Townsend gap.

The analysis of the position of  $t_2$  follows closely that of Section II (a). The first shutter in Figure 2 is replaced by a surface from which pulses of photoelectrons are emitted. Shutter 2 is replaced by the receiving electrode. As in Section II (a) the expression which satisfies the boundary condition n=0 at x=h is given by equation (3):

$$n = \frac{N_0}{(4\pi Dt)^{\frac{1}{2}}} \bigg\{ \exp\left[-\frac{(x-Wt)^2}{4Dt}\right] - \exp\left[\frac{hW}{D} - \frac{(x-2h-Wt)^2}{4Dt}\right] \bigg\}.$$

From elementary theory (as in Cobine 1941) the current in a Townsend gap is given by  $I_2$ , where

$$I_{2} = NeW/d - (n_{2} - n_{1})De/d, \qquad (19)$$

where N is the total number of electrons in the gap,

W the drift velocity,

- d the length of the gap,
- e the electronic charge,
- $n_2$  the electron density at the collecting electrode,

 $n_1$  the electron density at the electron emitting surface.

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When the pulse is collected by the anode, the electron density at the cathode is assumed to be zero, so  $n_1 = n_2 = 0$ . Equation (19) then becomes

$$\begin{split} I_{2} &= \frac{N_{0}eW}{d} \int_{-\infty}^{h} \frac{1}{(4\pi Dt)^{\frac{1}{2}}} \exp\left[-\frac{(x-Wt)^{2}}{4Dt}\right] \mathrm{d}x \\ &- \frac{N_{0}eW}{d} \int_{-\infty}^{h} \frac{1}{(4\pi Dt)^{\frac{1}{2}}} \exp\left[\frac{hW}{D} - \frac{(x-2h-Wt)^{2}}{4Dt}\right] \mathrm{d}x \\ &= \frac{N_{0}eW}{d\pi^{\frac{1}{2}}} \left[\frac{\pi^{\frac{1}{2}}}{2} + \operatorname{erf}\frac{h-Wt}{(4Dt)^{\frac{1}{2}}}\right] - \frac{N_{0}eW}{d\pi^{\frac{1}{2}}} \left[\frac{\pi^{\frac{1}{2}}}{2} - \operatorname{erf}\frac{h+Wt}{(4Dt)^{\frac{1}{2}}}\right] \exp\frac{hW}{D}, \quad (20) \end{split}$$

where  $\operatorname{erf} x = \int_0^x \exp\left[-u^2\right] du$ . If we ignore the effect of the collecting electrode, the image term in the expression for *n* is omitted and the expression for current as a function of time is given by  $I_1$ , where

$$I_{1} = \frac{N_{0}eW}{d\pi^{\frac{1}{2}}} \left[ \frac{\pi^{\frac{1}{2}}}{2} + \operatorname{erf} \frac{h - Wt}{(4Dt)^{\frac{1}{2}}} \right].$$
(21)

The shapes of the falling characteristics of  $I_1$  and  $I_2$  are shown in Figure 4 in which the difference between  $I_1$  and  $I_2$  is shown to be much larger than actually occurs under usual experimental conditions.



Fig. 4.—The theoretical current trace  $(I_1)$  for an undisturbed electron pulse differs from the trace  $(I_2)$  for a pulse which is disturbed by the presence of the collecting electrode.

When the pulse is at the collecting electrode, t=h/W and  $I_1$  becomes  $I_1=\frac{1}{2}N_0eW/d=\frac{1}{2}I_0$ , where  $I_0$  is the maximum current. However, because of the boundary condition imposed by the collecting electrode, the oscillographic trace gives  $I_2$  and not  $I_1$ . Thus, when  $t_2$  is estimated from the half current maximum

of  $I_2$  there will be an error introduced equal to  $\Delta t$  (see Fig. 4). The relative error r, in the determination of  $t_2$ , will be given by

$$r = \frac{\Delta t}{t_2} \simeq \frac{\Delta t^1}{t_2} \simeq \frac{\Delta I}{t_2} \bigg/ \bigg( \frac{\mathrm{d}I_1}{\mathrm{d}t} \bigg)_{t=t_2},\tag{22}$$

where

$$\left(\!\frac{\mathrm{d}I_1}{\mathrm{d}t}\!\right)_{t=t_2}\!\!=\!-\frac{N_0 eW^2}{\mathrm{d}(4\pi Dt_2)^{\frac{1}{2}}},$$

and

$$\Delta I = \frac{N_0 e W}{d\pi^{\frac{1}{2}}} \left[ \frac{\pi^{\frac{1}{2}}}{2} - \operatorname{erf}\left(\frac{hW}{D}\right)^{\frac{1}{2}} \right] \exp \frac{hW}{D}.$$

From a formula given in Comrie (1948) for large x,

$$\frac{\pi^{\frac{1}{2}}}{2} - \operatorname{erf} x = \frac{e^{-x^2}}{2x} \left[ 1 - \frac{1}{2x^2} + \frac{3}{(2x^2)^2} - \ldots \right]$$
$$\simeq e^{-x^2/2x} \text{ if } x > 2.$$

As experimental values of W/D and h are generally such that  $(hW)^{\frac{1}{2}}/2$ , we can use this formula to simplify  $\Delta I$ , and obtain

$$\Delta I \simeq \frac{N_0 e W D^{\frac{1}{2}}}{2d\pi^{\frac{1}{2}} (hW)^{\frac{1}{2}}}.$$
(23)

On substituting for  $\Delta I$  and  $(dI_1/dt)_{t=t_2}$ , equation (22) becomes r = -1/(hW/D). The error is such that the measured value of the drift velocity is too high.

Hornbeck (1951) has reported the appearance of spikes at the commencement of the oscillographic traces which indicate the loss of electrons due to back diffusion to the cathode. Biondi and Chanin (1954) have reported a similar effect in the case of positive ions. In the case under discussion the loss of some of the electrons from the trailing edge of the pulse to the cathode would lead to a shifting forward of the maximum electron density in the pulse. If the pulses are initially very narrow and are formed close to the surface, the analysis of Section II (b) is directly applicable giving an additional relative error of 2/h(W/D). This error may be cancelled or enhanced by further errors which may exist in the determination of  $t_1$ . However, the formulae for r that have been derived should indicate the order of the diffusion errors that are likely for any given experimental conditions.

For the measurements of Bowe the errors due to diffusion predicted from r=3/h(W/D) are in almost every case within the experimental error of 5%.

## V. EXPERIMENTAL RESULTS USING THE METHOD OF BRADBURY AND NIELSEN

In this section results obtained by using the method of Bradbury and Nielsen are presented in which the values of the electron drift velocity are pressure dependent for a constant value of E/p. This evidence supports the theoretical predictions of errors due to diffusion discussed above.

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The apparatus had a shutter separation of 6 cm. The shutters were constructed by threading "Nichrome" wire of diameter 0.003 in. through holes pierced in mica such that the distance between adjacent grid wires was 0.5 mm. Guard rings positioned by accurately ground glass spacers were placed every 0.5 cm to preserve a uniform electric field. All metal surfaces of the apparatus, including the shutter wires, were coated with gold to reduce errors due to contact potential differences.

The voltage applied to the shutters was supplied by means of an oscillator whose frequency was calibrated with a precision counter to an accuracy of better than  $\frac{1}{2}$ %. Grid voltages of up to 20 volts r.m.s. on each half of the grids could be obtained, this voltage remaining constant with change of frequency. Precautions were taken to ensure equality of the phases of the voltages applied to each shutter so that any phase differences present were certainly less than 1°. Errors due to any slight discrepancies in magnitude or phase of the voltages applied to each half of the shutters are such as to be self-cancelling, because the sign of all voltages changes every half cycle. These errors lead only to a decrease in resolving power because the current/frequency peaks become slightly broader. The appropriate d.c. voltage was applied to each shutter to maintain the uniform field within the drift tube.

Nitrogen was supplied by heating sodium azide in an outgassed tube; the helium used was stated to be spectroscopically pure. The apparatus was evacuated to  $10^{-4}$  mmHg and no rise in pressure could be detected when the apparatus was shut off over night. It is known that drift velocities, in monatomic gases in particular, are subject to errors due to small traces of impurity. However, it is felt that the pressure dependent measurements which were taken with one gas sample are still valid, even though the asymptotic limit of the measurements may be slightly in error due to traces of impurities. The measurements were repeated several times with different gas samples and in all cases the results were in agreement, within the experimental error.

Gas pressures were measured with two capsule type pressure gauges developed by Crompton and Elford (1957). The source of electrons was a tungsten filament. Errors due to the heating of the gas by the filament were calculated to be negligible. Corrections were made for the small deviations from 20 °C of the temperature at which measurements were taken. Thus all results given are for a gas density corresponding to 20 °C.

Results were taken at constant E/p (equal to  $0.4 \text{ V cm}^{-1} \text{ mmHg}^{-1}$ ) and are shown by circles in Figures 5 and 6. It is seen that as the pressure is reduced and thus diffusion increased, the measured values of the drift velocity increase by 13% in the case of helium and 8% in the case of nitrogen.\* The full and broken curves shown in Figures 5 and 6 are theoretical predictions of the variation of the measured values of the drift velocity from the asymptotic value, in the case

<sup>\*</sup> When the voltage applied to the shutters was reduced, the variation of the results from the asymptotic limit became less, as is predicted from theory, because the effective time that the shutters are open is increased.

of a plane source and point source respectively. These curves have been drawn by using the formulae r=3/h(W/D) and r=5/h(W/D), in which are substituted values of W/D obtained experimentally by Townsend and Bailey (1923) (for helium) and Crompton (unpublished results for nitrogen). The theoretical predictions apply only when the shutters are open for a very small time interval.

The effect of gas pressure on current/frequency curves is shown in Figures 7 and 8, both curves being taken in nitrogen for  $E/p = 0.4 \text{ V cm}^{-1} \text{ mmHg}^{-1}$  at 20 °C. Figure 8 shows the marked effect produced by the higher diffusion coefficient at the lower pressure.

Results to determine the pressure dependence of the results have been taken at many other values of E/p in nitrogen. In all cases results similar to those plotted in Figure 6 were obtained. In helium, measurements were made at only one value of E/p, being those given in Figure 5.



Fig. 5.—Variation of the measured value of the drift velocity in helium with change of gas pressure.  $(E/p=0.4 \text{ V cm}^{-1} \text{ mmHg}^{-1}, T=20 \text{ °C}).$ 

Crompton, Hall, and Macklin (1957) presented experimental evidence of the pressure dependence of drift velocity measurements. Their results have been checked using the original apparatus, and although the values of the asymptotic limit of the results at high pressure are unchanged, the variations from this limit at low pressures are considerably less than were reported previously. The discrepancy between the two sets of results has been attributed largely to the original electronic feeder network which supplied the a.c. voltages to the shutters, and which gave a falling voltage with increase of frequency. In the case of high diffusion when low pressures are used, the current/frequency peaks are very flat so that even a slight decrease in the shutter voltage with increasing frequency can significantly increase the frequency at which maximum current occurs.



Fig. 6.—Variation of the measured value of the drift velocity in nitrogen with change of gas pressure.  $(E/p=0.4 \text{ V cm}^{-1} \text{ mmHg}^{-1}, T=20 \text{ °C}).$ 



Fig. 7.—Electrometer current with change of frequency ( $N_2$ , E/p=0.4 V cm<sup>-1</sup> mmHg<sup>-1</sup>, p=100 mm, T=20 °C).



Fig. 8.—Electrometer current with change of frequency  $(N_2, E/p = 0.4 \text{ V cm}^{-1} \text{ mmHg}^{-1}, p = 5 \text{ mm}, T = 20 \text{ °C}).$ 

## VI. CONCLUSIONS

The experimental results that have been presented, by using the method of Bradbury and Nielsen, are strong evidence that diffusion effects of the type discussed theoretically can, under some conditions, introduce considerable errors into drift velocity measurements. If accurate results are to be obtained it is essential that an examination be made to determine if such errors due to diffusion are significant.

It is impossible to make any exact comparison of the theoretical predictions of errors with the experimental results because of two unknown factors. Firstly, the shutters are fed with sine wave voltages which do not conform to the theoretical model where the shutters open and close instantaneously. The theoretical predictions are dependent on the time interval that the shutters are open, and it is difficult to determine what the equivalent time interval for the experimental conditions will be. Secondly, the error due to the variation of  $\tilde{c}$ within the pulse is unknown since no quantitative expression has been attempted for this error. A further factor (which is usually unknown) is the extent to which the experimental source of electrons, the first shutter, conforms to the theoretical model of either a point or a plane source. In this case, however, the limits of the error that would be predicted theoretically are known.

Despite these unknown factors, the theory does explain the direction and order of magnitude of the errors that are observed in the case of the method of Bradbury and Nielsen. It is therefore reasonable to assume that the observed pressure dependence of the results does not indicate any fault in experimental technique. It should be noted that the error predicted from r=3/h(W/D) will on the one hand overestimate the true error because of the finite time that the shutters are open, and on the other hand underestimate the true error because of the error due to the variation of  $\tilde{c}$  within the pulse. The experimental results would seem to indicate that these two unknown variations largely cancel, giving the order of the total relative error as 3/h(W/D). Thus by choosing either the chamber length or the gas pressure to be sufficiently large, errors due to diffusion can be made insignificantly small.

Similar analysis predicts diffusion errors to be present in the drift velocity determinations using Hornbeck's method and in the transit time measurements using the method of Pack and Phelps. In all cases an awareness of the existence of errors due to diffusion is essential in assessing the accuracy of experimental results; these errors can be made less than experimental errors by the use of high gas pressures and long chamber lengths.

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### APPENDIX

Here equations are derived which enable the relative error in Section II (a) to be calculated when account is taken of the finite time that the bottom shutter is open.

Let T be the time that the bottom shutter opens. Then for t < T the expression for the electron density will be given by equation (3),

$$n = \frac{N_0}{(4\pi Dt)^{\frac{1}{2}}} \bigg\{ \exp\bigg[-\frac{(x-Wt)^2}{4Dt}\bigg] - \exp\bigg[\frac{hW}{D} - \frac{(x-2h-Wt)^2}{4Dt}\bigg] \bigg\},$$

which satisfies the boundary condition n=0 at x=h. When t=T the shutter opens and the boundary condition is removed. For all t>T we consider the electron density function to be the sum of an infinite number of elementary pulses at x=X where X goes from  $-\infty$  to h. The strength of each pulse is given by the value of n at x=X and t=T. Then for t>T the expression for n is given by

$$n = \int_{-\infty}^{h} \frac{N_{0}}{(4\pi DT)^{\frac{1}{2}}} \left\{ \exp\left[-\frac{(X-WT)^{2}}{4DT}\right] - \exp\left[-\frac{(X-2h-WT)^{2}}{4DT} + \frac{hW}{D}\right] \right\} \\ \times \frac{1}{[4\pi D(t-T)]^{\frac{1}{2}}} \exp\left[-\frac{(x-X-W[t-T])^{2}}{4D(t-T)}\right] dX.$$
(24)

On rearranging the terms involving the exponentials we obtain

$$n = \int_{-\infty}^{h} \frac{N_{0}}{4\pi DT^{\frac{1}{2}}(t-T)^{\frac{1}{2}}} \left\{ \exp\left[-\frac{t(X-xT/t)^{2}}{4DT(t-T)} - \frac{(x-Wt)^{2}}{4Dt}\right] - \exp\left[\frac{hW}{D} - \frac{t(X-xT/t-2h(t-T)/t)^{2}}{4DT(t-T)} - \frac{(x-2h-Wt)^{2}}{4Dt}\right] \right\} dX.$$
(25)

We now substitute a = t/4DT(t-T), b = xT/t, and c = xT/t + 2h - 2hT/t. Then on further simplification n becomes

$$n = \frac{N_0}{2\pi D^{\frac{1}{2}t^{\frac{1}{2}}}} \left\{ \left\{ \frac{\pi^{\frac{1}{2}}}{2} + \operatorname{erf} \left[ (h-b)a^{\frac{1}{2}} \right] \right\} \exp\left[ -\frac{(x-Wt)^2}{4Dt} \right] - \left\{ \frac{\pi^{\frac{1}{2}}}{2} + \operatorname{erf} \left[ (h-c)a^{\frac{1}{2}} \right] \right\} \exp\left[ \frac{hW}{D} - \frac{(x-2h-Wt)^2}{4Dt} \right] \right],$$
(26)  
$$x = \int^x \exp\left[ -u^2 \right] du.$$

where  $\operatorname{erf} x = \int_0^x \exp\left[-u^2\right] \mathrm{d}u.$ 

The electron density at x = h is given by

$$(n)_{x=h} = \frac{N_0}{2\pi D^{\frac{1}{2}t^{\frac{1}{2}}}} 2 \operatorname{erf} \left[ (h-b)a^{\frac{1}{2}} \right] \exp \left[ -\frac{(h-Wt)^2}{4Dt} \right].$$

On differentiating n to find  $\partial n/\partial x$  and substituting x=h, we obtain

$$\left(\frac{\partial n}{\partial x}\right)_{x=h} = \frac{N_0}{2\pi D^{\frac{1}{2}t^{\frac{1}{2}}}} \left\{\frac{W}{D} \operatorname{erf}\left[(h-b)a^{\frac{1}{2}}\right] - \frac{h\pi^{\frac{1}{2}}}{2Dt}\right\} \exp\left[-\frac{(h-Wt)^2}{4Dt}\right]. \quad (27)$$

The instantaneous electron flux at the second shutter is given by J where

$$J = neW - De\frac{\partial n}{\partial x}, \quad \text{at } x = h$$
$$= \frac{N_0 e}{2\pi D^{\frac{1}{2}} t^{\frac{1}{2}}} \left\{ W \operatorname{erf} \left[ \frac{h}{2} \left( \frac{t - T}{DTt} \right)^{\frac{1}{2}} \right] + \frac{h}{t} \frac{\pi^{\frac{1}{2}}}{2} \right\} \exp \left[ -\frac{(h - Wt)^2}{4Dt} \right]. \quad (28)$$

As in equation (5) we now express J in terms of frequency by changing the variable from t to f and substituting  $N_0 = N_1/f$ . If we assume that for any given frequency f, the shutter is open for 1/10th of the time for one cycle, we choose T = 19/20f as the opening time of the second shutter and t will equal (19 + a)/20f, where a will vary from 0 to 2 while the shutter is open. Then from reasoning similar to that presented in Section II (a) the current I measured will be proportional to J, and I will be given by

$$I = \int_{0}^{2} \frac{AS^{\frac{1}{2}}}{k} \left\{ \operatorname{erf}\left[\frac{h(af_{0}S)^{\frac{1}{2}}}{2(19D)^{\frac{1}{2}}}\right] + \frac{S\pi^{\frac{1}{2}}}{2} \right\} \exp\left[-\frac{hW}{4D}(1-1/S)^{2}S\right] \mathrm{d}a,$$
(29)

where A is a constant,  $k=f/f_0$ , and  $S=\frac{20k}{19+a}$ .

This integration can be done numerically for each value of f, and by plotting I as a function of f, the frequency for which maximum current occurs can be found. In a similar way the frequency for the maximum current can be determined when the shutters are open for 0.05 and 0.01 of the time. Results employing such calculations are quoted at the end of Section II (a).