IV. The Curvature of "Nose" Whistlers

At increasingly great heights above the Earth's surface the magnetic field becomes progressively weaker so that near the top of a ray trajectory the condition $Y \gg 1$ may no longer hold. So far as the dispersion curve is concerned, this gives rise to the so-called "nose" whistlers (Helliwell *et al.* 1956).

Here we shall consider the ray paths in such a medium for which Y is greater than, but of the order of, unity, assuming that the condition $X \gg 1$ still holds.

Propagation is possible for angles between the wave-normal and the field for which

$$|\theta| < \cos^{-1}(1/Y),$$

and the quasi-longitudinal approximation is valid in the form

$$\mu^2 = X/(Y_L - 1),$$

so that

$$\tan\alpha = -\frac{1}{\mu}\frac{\partial\mu}{\partial\theta} = \frac{Y_T}{2(1-Y_L)}, \tag{17}$$

where $Y_L = Y\cos\theta$, $Y_T = Y\sin\theta$. We find also

$$\tan\varphi = \frac{2\sin\theta - Y\sin\theta\cos\theta}{2\cos\theta - Y(1+\cos^2\theta)}. \tag{18}$$

It follows that the values of φ corresponding to the extreme values of θ, namely to $\pm\cos^{-1}(1/Y)$, are $\pm\{\frac{1}{2}\pi - \cos^{-1}(1/Y)\}$. Also if $Y > 2$, φ is zero for $\theta = \pm\cos^{-1}(2/Y)$ and θ is a three-valued function of φ. For $Y < 2$, θ is a single-valued function of φ. The relation of φ and θ is illustrated in Figure 3 for several values of Y. It will be seen that for values of Y not much above unity the direction of the ray may diverge very greatly from that of the magnetic field.

The coefficients of curvature are:

$$\left.\begin{aligned} Z_L^X &= \zeta Y_T(1-Y_L)\Gamma X^{-1}, \\ Z_T^X &= 2\zeta(1-Y_L)^2\Gamma X^{-1}, \\ Z_L^Y &= (2+\zeta Y_L)\Gamma\sin\theta, \\ Z_T^Y &= 2\zeta(1-Y_L)\Gamma\cos\theta, \\ Z_L^\theta &= 4\zeta(1-Y_L)^2\Gamma - 1, \\ Z_T^\theta &= -2\zeta Y_T(1-Y_L)\Gamma, \end{aligned}\right\} \tag{19}$$

where

$$\Gamma = \{4(1-Y_L)^2 + Y_T^2\}^{-1},$$
$$\zeta = 1 + 2(Y_L - Y^2)\Gamma.$$

It will be seen that these are not only more complex algebraically than the expression (12) for the usual whistler mode, but *give rise to frequency-dependent terms in the curvature.* Thus the ray paths in a region of "nose" propagation will be different for rays at different frequencies.

V. Conclusion

The main conclusion is that *there is in general no tendency for a true longitudinal ray (both ray and wave-normal parallel to the field: $\varphi = 0$ and $\theta = 0$) to follow the lines of force closely.*

Two possibilities thus remain: either the ray swings back and forth about the direction of the magnetic field, or $|\theta|$ increases up to its limiting value θ_c, when the ray is again parallel to the field. The expressions (12) indicate that, when this happens, the gradients of X and Y are without effect on the trajectory, and that *the ray will follow the lines of force.* Also by (7) the group delay is small.

Work on ray plotting, and work in which explicit analytic expressions for ray paths have been obtained, suggest that both types of trajectory are possible. For the type with $|\theta|$ increasing towards the limit θ_c, it may be necessary to go a long way along the ray before the limit is approached closely. Since the wave-normal will then be almost transverse to the field, the approximation (2) for μ will be at its worst, as has been pointed out, and some caution may be needed in applying our results to the further plotting of the ray, especially if the inequalities (1) do not hold very strongly.

Finally, when the field is weak ("nose" whistlers) the ray paths exhibit a more complex behaviour; the paths depend markedly on the frequency, and the directions of the rays may be inclined at large angles to the field.

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