SHORT COMMUNICATIONS

THE RELATION BETWEEN ISOPHOTAL DIAMETERS AND REDSHIFT OF EXTRA GALACTIC NEBULAE*

By R. Van der Borght†

In a recent paper, Sandage (1961) considers the problem of using the angular diameters of extra galactic nebulae in order to decide between various world models. In his approach Sandage uses formulae derived from the field equations, as given by Mattig (1958), under the assumption that both the pressure and cosmological constant are zero.

It is the purpose of the present paper to derive the relation between isophotal diameter and redshift, using the method of series expansions, and to compare this formula with the corresponding one for metric diameters.

The emitting area $\mathrm{d}A$ of a galaxy, corresponding to a solid angle of 1 sec^2, is given by

$$\mathrm{d}A=\xi^2/P \tag{1}$$

where ξ is the distance by apparent size, and $P=3283\times(3600)^2$.

The corresponding area $\mathrm{d}A'$ on the plate is given by

$$\mathrm{d}A'=\mu^2/P \tag{2}$$

where μ is a constant depending on the telescope used.

Let $B(y,\lambda,\tau)\mathrm{d}\lambda$ be the energy ; measured in ergs cm^{-2}, emitted in the range of wavelength $\mathrm{d}\lambda$, at a distance y from the centre of the galaxy at the time t, where $\tau=t_0-t$ is the time of travel of the light emitted at the time t and observed at the time t_0.

The energy, in ergs cm^{-2}, received from $\mathrm{d}A$ on the corresponding area $\mathrm{d}A'$ of the plate, in the range λ to $\lambda+\mathrm{d}\lambda$ of received wavelength is

$$\mathrm{d}h=\frac{\mathrm{d}A'\mathrm{d}A}{4\pi D^2}B\left(y,\ \frac{\lambda}{1+\delta},\ \tau\right)\frac{\mathrm{d}\lambda}{1+\delta}, \tag{3}$$

or using (1) and (2)

$$\mathrm{d}h=\frac{\mu^2}{4\pi P^2}\cdot\frac{\xi^2}{D^2}\cdot B\left(y,\ \frac{\lambda}{1+\delta},\ \tau\right)\frac{\mathrm{d}\lambda}{1+\delta}, \tag{4}$$

where D is the luminosity distance.

We shall consider a function $B(y,\lambda,\tau)$ of the form

$$B(y,\lambda,\tau)=\varphi(y)\mathscr{B}(\lambda,\tau), \tag{5}$$

* Manuscript received October 25, 1961.

† Department of Mathematics, School of General Studies, The Australian National University, Canberra, A.C.T.

for example, de Vaucouleurs (1959) gives

$$\ln B = H + F y^{\alpha}, \tag{6}$$

with $\alpha = 1$ for spiral and lenticular galaxies and 1/4 for ellipticals.

Comparing (5) and (6) we have

$$\varphi(y) = \mathrm{e}^{Fy^{\alpha}}, \tag{7}$$

$$\mathscr{B}(\lambda,\tau) = \mathrm{e}^{H}. \tag{8}$$

Let $\mathrm{d}h_0$ be the energy in ergs $\sec^{-1}$ received on an area of the plate, at the centre of the image, corresponding to a solid angle of 1 $\sec^2$, for a standard source ($\delta = 0$).

We know (McVittie 1956) that

$$\xi^2/D^2 = (1+\delta)^{-4}, \tag{9}$$

that is, if $\delta = 0$

$$\xi = D.$$

Then

$$\mathrm{d}h_0 = \frac{\mu^2}{4\pi P^2}\varphi(0)\mathscr{B}(\lambda,0)\mathrm{d}\lambda. \tag{10}$$

If m_0 and m are the apparent magnitudes corresponding to $\mathrm{d}h_0$ and $\mathrm{d}h$ respectively we have

$$m_0 - m = 2 \cdot 5 \log_{10} \frac{\mathrm{d}h}{\mathrm{d}h_0}.$$

Suppose we measure the radius a of the image to a surface brightness of $m = m_1$ (e.g. $21 \cdot 0$ mg $\sec^{-2}$ at fixed monochromatic wavelength of 6800 Å). The corresponding dimension y of the galaxy is given by

$$y = a\xi/\mu, \tag{11}$$

and it follows that

$$m_0 - m_1 = 2 \cdot 5 \log_{10} \left\{ \frac{\xi^2}{D^2} \cdot \frac{\varphi[(a/\mu)\xi]}{\varphi(0)} \cdot \frac{\mathscr{B}[\lambda/(1+\delta),\ \tau]1/(1+\delta)}{\mathscr{B}(\lambda,0)} \right\}. \tag{12}$$

It can be shown (Van der Borght 1961) that

$$\frac{2 \cdot 5}{E} \ln \frac{\mathscr{B}[\lambda/(1+\delta),\ \tau]1/(1+\delta)}{\mathscr{B}(\lambda,0)} = -(K_1 + W_1)\delta - \left(K_2 + W_2 + C_1 - \frac{2h_1^2 - h_2}{2h_1^2} W_1\right)\delta^2, \tag{13}$$

where

$$W_1 = \frac{2 \cdot 5}{E h_1} \mathscr{B}_{\tau}/\mathscr{B},$$

$$K_1 = \frac{2 \cdot 5}{E} \cdot \frac{1}{\mathscr{B}}(\mathscr{B} + \lambda\mathscr{B}_{\lambda}),$$

$$W_2 = \frac{-2 \cdot 5}{E} \cdot \frac{1}{2h_1^2} \cdot \frac{1}{\mathscr{B}^2}(\mathscr{B}\mathscr{B}_{\tau\tau} - \mathscr{B}_{\tau}^2),$$

$$K_2 = \frac{2 \cdot 5}{E} \cdot \frac{1}{\mathscr{B}^2}\left(-\tfrac{1}{2}\mathscr{B}^2 + \tfrac{1}{2}\lambda^2\mathscr{B}_{\lambda}^2 - \lambda\mathscr{B}\mathscr{B}_{\lambda} - \tfrac{1}{2}\lambda^2\mathscr{B}\mathscr{B}_{\lambda\lambda}\right),$$

$$C_1 = \frac{2 \cdot 5}{E h_1} \cdot \frac{1}{\mathscr{B}^2}(\lambda\mathscr{B}_{\tau}\mathscr{B}_{\lambda} - \lambda\mathscr{B}\mathscr{B}_{\tau\lambda}),$$

where

$$\mathscr{B}=\mathscr{B}(\lambda,0),\quad \mathscr{B}_\tau=\left(\frac{\partial\mathscr{B}}{\partial\tau}\right)_{\lambda,0},\quad \mathscr{B}_\lambda=\left(\frac{\partial\mathscr{B}}{\partial\lambda}\right)_{\lambda,0},$$

with corresponding notations for the second derivatives. Also

$$\frac{2\cdot5}{E}\ln\left(\frac{\xi^2}{D^2}\right)=-\frac{10}{E}(\delta-\tfrac{1}{2}\delta^2\ldots). \tag{14}$$

Using (7) it is seen that

$$\frac{2\cdot5}{E}\ln\frac{\varphi[(a/\mu)\xi]}{\varphi(0)}=\frac{2\cdot5}{E}F\left(\frac{a}{\mu}\xi\right)^\alpha.$$

But (McVittie 1956),

$$\xi=\frac{c}{h_1}\delta\left(1-\frac{3h_1^2-h_2}{2h_1^2}\delta\right),$$

therefore,

$$\frac{2\cdot5}{E}\ln\frac{\varphi[(a/\mu)\xi]}{\varphi(0)}=\frac{2\cdot5}{E}F\left\{\frac{a}{\mu}\cdot\frac{c\delta}{h_1}\left(1-\frac{3h_1^2-h_2}{2h_1^2}\delta\right)\right\}^\alpha. \tag{15}$$

Substituting (13), (14), and (15) in (12) we have

$$\frac{1}{a}=\left(\frac{2\cdot5}{E}F\right)^{1/\alpha}(m_0-m_1)^{-1/\alpha}\,\frac{c}{\mu h_1}\cdot\delta\left(1-\frac{3h_1^2-h_2}{2h_1^2}\delta\right)(1+N_1\delta+N_2\delta^2)^{-1/\alpha}, \tag{16}$$

where

$$\left.\begin{aligned}N_1&=\left(\frac{10}{E}+K_1+W_1\right)/(m_0-m_1),\\ N_2&=\left(-\frac{5}{E}+K_2+W_2+C_1-\frac{2h_1^2-h_2}{2h_1^2}W_1\right)/(m_0-m_1).\end{aligned}\right\} \tag{17}$$

But

$$(1+N_1\delta+N_2\delta^2)^{-1/\alpha}=1-\frac{1}{\alpha}N_1\delta+\left[-\frac{N_2}{\alpha}+\frac{1}{2\alpha}\left(1+\frac{1}{\alpha}\right)N_1^2\right]\delta^2, \tag{18}$$

and substituting this in (16) we have to the second order in δ

$$\frac{1}{a}=\left(\frac{2\cdot5}{E}F\right)^{1/\alpha}\cdot\frac{c}{\mu h_1}(m_0-m_1)^{-1/\alpha}\left\{\delta-\left(\frac{3h_1^2-h_2}{2h_1^2}+\frac{1}{\alpha}N_1\right)\delta^2\right\},$$

or

$$\frac{1}{\theta_i}=\frac{1}{2}\left(\frac{2\cdot5}{E}F\right)^{1/\alpha}\cdot(m_0-m_1)^{-1/\alpha}\cdot\frac{c}{h_1}\left\{\delta-\left(\frac{3h_1^2-h_2}{2h_1^2}+\frac{1}{\alpha}N_1\right)\delta^2\right\}, \tag{19}$$

where θ_i is the apparent angular isophotal diameter.

Comparing (19) with the following formula for the apparent angular metric diameter (McVittie 1956)

$$\frac{1}{\theta_m}=\frac{c}{h_1 l}\left\{\delta-\left(\frac{3h_1^2-h_2}{2h_1^2}\right)\delta^2\right\}, \tag{20}$$

where l is the average linear diameter of the galaxies, we have

$$\frac{\theta_m}{\theta_i}=\left(\frac{2\cdot5}{E}F\right)^{1/\alpha}\cdot(m_0-m_1)^{-1/\alpha}\cdot\tfrac{1}{2}l\left(1-\frac{N_1}{\alpha}\delta\right). \tag{21}$$

Since N_1 is negative, it follows from (21) that the ratio of metric to isophotal diameter increases with the redshift. This was already pointed out by Sandage (1961).

Formula (21) again emphasizes the fact that in order to distinguish between various world models, i.e. in order to determine the value of $q_0 = -h_2/h_1^2$, very accurate determinations of the luminosity distribution inside galaxies, of the K-correction and of the evolutionary effects have to be made. These three factors correspond in fact to an accurate determination of the function $B(y,\lambda,\tau)$ used in this paper.

References

de Vaucouleurs, G. (1959).—*Astr. J.* **64**: 397.
Mattig, W. (1958).—*Astr. Nachr.* **284**: 109.
McVittie, G. C. (1956).—" General Relativity and Cosmology." (Chapman & Hall: London.)
Sandage, A. (1961).—*Astrophys. J.* **133**: 355.
Van der Borght, R. (1961).—*Mon. Not. R. Astr. Soc.* **122**: 177.