ANALYSIS OF OVERLAPPING RESONANCES IN PROTON-CARBON SCATTERING

By D. Robson*† and W. Toutenhoofd*

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Summary

The R-matrix theory of nuclear reactions is discussed for the situation where distant levels must be included and is successfully applied to the elastic scattering of protons by ^{12}C in the energy range $E_{LAB} = 10$–$11 \cdot 5$ MeV, where the resonance levels are overlapping. Definite spin and parity assignments are made for three levels in this region by comparing the predicted excitation functions at several angles of scattering with experiment. Previously unexplained polarization data are explained by this analysis. The optical-model potential is found to be parity dependent for $E_{LAB} < 10$ MeV and yields a close fit to the available data at $E_{LAB} = 8 \cdot 5$ MeV.

I. Introduction

The elastic scattering of protons by ^{12}C has proved most fruitful in providing information on the spins, parities and widths of energy levels in ^{13}N. In an earlier paper by Shute *et al.* (1962), hereafter referred to as I, the elastic scattering of protons with energies less than $E_{LAB} = 9 \cdot 5$ MeV was successfully analysed in terms of complex phases based on optical-model phases plus Breit-Wigner forms which serve to describe the "isolated" resonance levels.

The present work is concerned with extending the earlier analysis to an energy region where the resonance levels are no longer isolated but overlapping. Fortunately, none of the levels analysed here have the same spin and parity so that it is possible to retain Breit-Wigner forms for the resonance levels. The theory appropriate to the situation where distant levels are accounted for is discussed briefly in Section II using R-matrix theory. Some discussion of the diagonal approximation for the distant level matrix R^0, which is invoked in order to "phase" the resonant amplitude, is also given in Section II.

Non-resonant complex phases are found in Section III by fitting non-resonant data at $E_{LAB} = 9 \cdot 7$ and $11 \cdot 9$ MeV. Trial phases at $9 \cdot 7$ MeV are obtained from the earlier work at $8 \cdot 5$ MeV where a parity-dependent optical potential has since been found necessary to explain the data. The agreement between theory and experiment for the excitation function at various angles of scattering for the energy range $E_{LAB} = 10$–$11 \cdot 5$ MeV found in Section IV is not as convincing as that obtained in I for $E_{LAB} < 9 \cdot 5$ MeV, but is sufficiently good to allow the extraction of resonance level parameters for three levels. Polarization data (Sanada 1961) in this region, although sparse, is discussed in Section IV and is explained by the present analysis. A brief discussion of the energy levels of ^{13}N is given in Section V and it is concluded that ^{13}N is in close agreement with shell-model predictions.

* School of Physics, University of Melbourne.

† Present address: Physics Department, Florida State University, U.S.A.

II. Theory

The relevant expressions for the differential cross section and polarization have been given in I in terms of two amplitudes, $A(\theta)$ and $B(\theta)$. These amplitudes are given incorrectly in I, although the calculations given there are correct. The true equations are

$$A(\theta) = -(\eta/2k)\mathrm{cosec}^2\tfrac{1}{2}\theta \exp[-\mathrm{i}\eta \log(\sin^2\tfrac{1}{2}\theta)] \\ -(2\mathrm{i}k)^{-1}\sum_{LJ}(J+\tfrac{1}{2})[\exp(2\mathrm{i}\omega_L)-U_{LJ}]P_L(\cos\theta), \tag{2.1}$$

$$B(\theta) = (2\mathrm{i}k)^{-1}\sum_{LJ}(-)^{J-L+\frac{1}{2}}[\exp(2\mathrm{i}\omega_L)-U_{LJ}]\mathrm{i}P^1_L(\cos\theta). \tag{2.2}$$

Following I and adopting a similar notation we separate the collision matrix U into two parts, i.e. $U = U^0+U^R$, the term U^0 describing the "potential" scattering due to "distant" levels and U^R arising from the nearby resonance levels. Exactly the same approximations for U^R used in I may be employed when the resonance levels are overlapping, provided none of the overlapping levels have the same spin and parity. This result follows from the fact that the collision matrix is diagonal in total angular momentum J and parity π, so that the level expansions for each J and π may be treated separately. Provided levels of the same spin and parity are well separated then a single-level approximation is appropriate for U^R.

Following Lane and Thomas (1958) the single-level formula (corresponding to a particular spin J_R and parity π_R) for U^R, when distant levels are taken into account, is given quite generally by

$$U^R_{\pi_R J_R} = \frac{2\mathrm{i}\Omega P^{\frac{1}{2}}[\alpha^{\pi_R J_R}_\lambda \times \alpha^{\pi_R J_R}_\lambda]P^{\frac{1}{2}}\Omega}{E_\lambda+\Delta_\lambda-E-\frac{1}{2}\mathrm{i}\Gamma^{\pi_R J_R}_\lambda}, \tag{2.3}$$

in which

$$\alpha^{\pi_R J_R}_\lambda = (1-R^0_{\pi_R J_R}L^0)^{-1}\,\gamma^{\pi_R J_R}_\lambda, \tag{2.4}$$

where

$$U^0_{\pi J} = \Omega P^{\frac{1}{2}}(1-R^0_{\pi J}L^0)^{-1}(1-R^0_{\pi J}L^{0*})P^{-\frac{1}{2}}\Omega \tag{2.5}$$

defines $R^0_{\pi J}$. The diagonal matrices, P, Ω, and L^0 are given explicitly by Lane and Thomas (1958). We assume here that the choice of boundary conditions which defines the eigenvalues E_λ is such that L^0 is directly proportional to penetrability, that is,

$$L^0 = \mathrm{i}P. \tag{2.6}$$

In obtaining (2.6) we have assumed that the shift factor S referred to by Lane and Thomas is almost independent of energy within the resonance region; a reasonable approximation in the present work due to the small level widths involved.

In order to evaluate the matrix inversion involved in equation (2.4) we adopt the diagonal approximation for the submatrix $R^0_{\pi_R J_R}$, which corresponds to assuming that the diagonal elements of the submatrix $U^0_{\pi_R J_R}$ are dominant. Although this method was used in I no justification was given there for employing the diagonal

approximation. We now point out that the diagonal approximation is required only for those submatrices involving a resonance level, i.e. those submatrices $U^0_{\pi J}$ with $J = J_R$, $\pi = \pi_R$ of the resonance level. Moreover, the diagonal approximation is invoked only as an approximate way of finding the relative phase between the resonant and non-resonant amplitudes. Indeed the diagonal approximation should not be applied to U^0 itself in the case of p–^{12}C scattering due to the size of the non-resonant inelastic cross section (McKenna and Shute 1961) in the energy region considered here.

The diagonal terms of U^0 are defined, as in I, by

$$U^0_{LJ} = \exp[2i(\delta_{LJ}+\omega_L)], \tag{2.7}$$

where the nuclear phase δ_{LJ} is in general complex, the imaginary part arising from absorption of particles out of the elastic channel into non-elastic channels. In the diagonal approximation the diagonal elements of U^R are given with $L^0 = iP$ by

$$U^R_{L_RJ_R} = U^0_{L_RJ_R}\, i\Gamma_{\lambda,L_RJ_R}[E_R - E - \tfrac{1}{2}i\Gamma_\lambda]^{-1}, \tag{2.8}$$

in which $E_R = E_\lambda + \Delta_\lambda$ is regarded as the observed resonance energy and the level shift Δ_λ is assumed to be independent of energy. Denoting a set of channel quantum numbers by c we find,

$$\Gamma_\lambda = \sum_c \Gamma_{\lambda,c} = 2\sum_c P_c \cos^2(\delta_c+\phi_c)\gamma^2_{\lambda c}, \tag{2.9}$$

$$\Delta_\lambda = \sum_c \Delta_{\lambda,c} = \tfrac{1}{2}\sum_c P_c \sin 2(\delta_c+\phi_c)\gamma^2_{\lambda c}, \tag{2.10}$$

where P_c and $-\phi_c$ are the penetrability and "hard sphere" phase respectively as evaluated at a chosen radius a_c.

It is interesting to note that, since $\cos^2(\delta_c+\phi_c) \leqslant 1$, the width of a level occurring in the presence of the "tails" of distant levels is in general expected to be narrower than the corresponding resonances when the effects of distant levels are negligible (i.e. when $\delta_c \simeq -\phi_c$). Physically this corresponds to the resonant cross section being reduced owing to the loss of flux into the non-resonant channel. Not much can be said about the level shift Δ_λ because the factors $\sin 2(\delta_c+\phi_c)$ may have positive or negative signs depending on the value of $(\delta_c+\phi_c)$. In obtaining Γ_λ or $\Gamma_{\lambda,c}$ we shall neglect the small imaginary part of δ_c so that Γ_λ, $\Gamma_{\lambda,c}$ are real parameters.

III. Non-resonant Analysis

Since the elastic scattering cross section between 9·4 and 10·0 MeV is reasonably flat, an analysis of the data (McKenna 1961) at $E_{\mathrm{LAB}} = 9{\cdot}7$ MeV was carried out in order to find the complex non-resonant phases δ_{LJ}. Since δ_{LJ} varies slowly with energy, the phases at 9·7 MeV were at first approximated to by the phases obtained at 8·5 MeV in I. Systematic variations of these trial phases resulted in the close fit to the differential cross section, as shown in Figure 1.

Above $E_{\mathrm{LAB}} = 11{\cdot}5$ MeV the cross section again becomes non-resonant and phases at $E_{\mathrm{LAB}} = 11{\cdot}9$ MeV were obtained by using the results of the optical model analysis (Nodvik, Duke, and Melkanoff 1961) at this energy. The parameters used

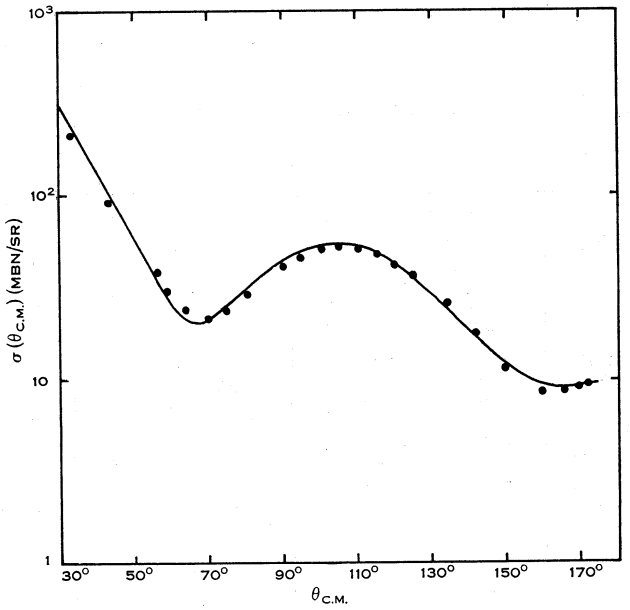

Fig. 1.—Angular distribution for the elastic scattering of protons by ^{12}C at $E_{\mathrm{LAB}} = 9{\cdot}7$ MeV. The solid curve corresponds to the non-resonant phases δ_{LJ} that are used in Section IV for the resonance analysis. The experimental points were obtained on the Melbourne cyclotron and are taken from McKenna (1961).

here are $V = 55$ MeV, $W_V = 0$, $W_S = 20{\cdot}1$ MeV, $R = 2{\cdot}98$ f, $a = 0{\cdot}45$ f, $V_{s0} = 6{\cdot}6$ MeV and $b = 0{\cdot}25$ f, which are almost identical to the set of parameters used by Nodvik, Duke, and Melkanoff. The notation for these parameters is the same as in I

TABLE 1

PARITY-DEPENDENT OPTICAL MODEL PARAMETERS

$V^+ = 55$ MeV,	$V^- = 50$ MeV	$V_{s0}^+ = 6$ MeV	$V_{s0}^- = 8$ MeV
$W_V^+ = 2$ MeV,	$W_V^- = 2$ MeV	$a^+ = 0{\cdot}7$ f	$a^- = 0{\cdot}3$ f
$W_s^+ = 0$ MeV,	$W_s^- = 0$ MeV	$R^+ = 2{\cdot}97$ f	$R^- = 2{\cdot}82$ f

except that the form factor associated with W_S is now taken to be the Gaussian shape as used by Nodvik, Duke, and Melkanoff. The fits obtained to the experimental curves are indistinguishable from the curves predicted by Nodvik, Duke, and Melkanoff and served as a useful check on the present calculations. "Non-resonant"

phases δ_{LJ} were then found for the whole energy range by simply interpolating between the set of phases at $E_{\mathrm{LAB}} = 9{\cdot}7$ MeV and the set of phases at $E_{\mathrm{LAB}} = 11{\cdot}9$ MeV. Such a procedure is expected to be sufficiently accurate, since the two sets of phases are similar.

Since the work reported in I was completed, calculations have been carried out which show that the phases at $E_{\mathrm{LAB}} = 8{\cdot}5$ MeV can be obtained by the use of a parity-dependent optical model potential. The parameters used are given in Table 1, where positive and negative parity parameters are denoted by plus and minus respectively.

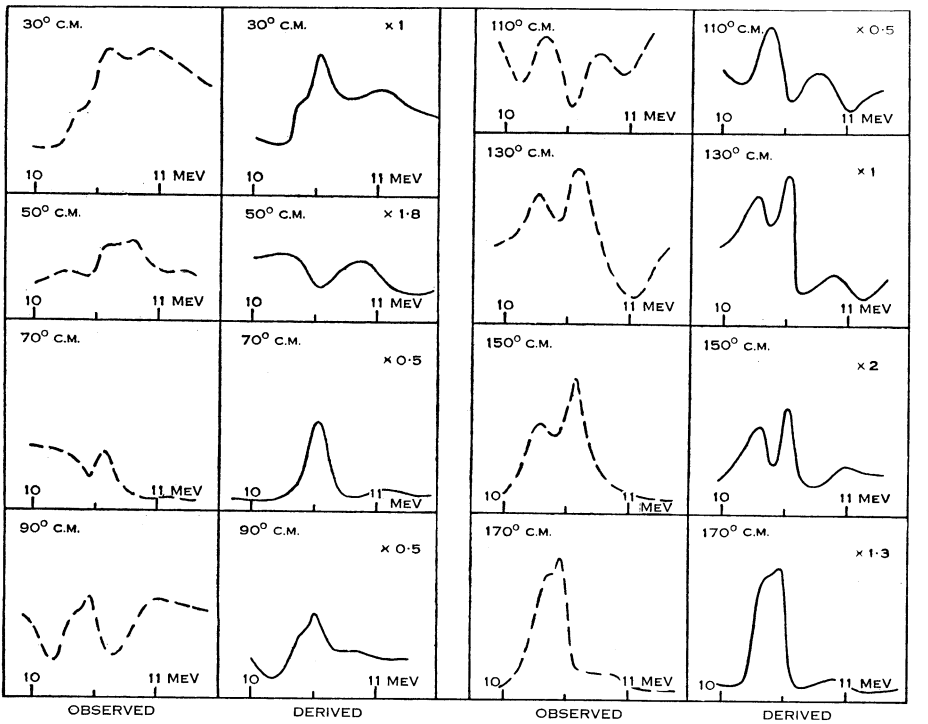

Fig. 2.—Dashed curves are experimental yield curves at various angles of scattering which were obtained on the Melbourne cyclotron and are taken from McKenna (1961). The solid curves are the theoretical predictions using the level parameters given in Table 2 and are only intended to illustrate the main characteristics of the excitation function. The multiplying factor given on each theoretical diagram is the factor used to normalize the theory to the experimental values.

Clearly the positive parity potential is much longer "tailed" than the negative parity potential. The difference in the strengths VR^2, that is,

$$V^{+}R^{+2} \sim \tfrac{5}{4} V^{-}R^{-2},$$

is large enough to explain why the parity-independent potential usually used (Hoare, Robbins, and Greenlees 1960; Hodgson, personal communication 1961; Shute *et al.* 1962) proved inadequate as far as a close fit is concerned. The reaction

cross section predicted by the parity-dependent potential is 180 mbn, which is in qualitative agreement with the inelastic cross section (McKenna and Shute 1961) at this energy.

The parity dependence of the optical potential is not unexpected if coupling to the first excited state channels is important. For p–^{12}C scattering the positive parity elastic waves are coupled only to the positive parity inelastic waves and similarly for negative parity waves.

If, for some reason, the coupling is stronger for one parity than the other, the effective potentials describing the effects of this coupling are unlikely to be equal. Unfortunately, the use of a parity-dependent potential is not likely to be of much general interest because of the requirement of twice the number of parameters.

TABLE 2

LEVEL PARAMETERS

E_R^{LAB} (MeV)	$E(^{13}N)$ (MeV)	J^π	Γ_{LAB} (keV)	a_{LJ}	θ_{LJ}^2
10·30	11·45	$5/2^+$	200	0·75	0·007
10·50	11·64	$7/2^-$	120	0·75	0·014
10·95	12·05	$5/2^-$	320	0·55	0·028

IV. Resonance Analysis

The phases δ_{LJ} obtained in the previous section make it possible for the resonance levels occurring between 10 and 11·5 MeV to be analysed in terms of Breit-Wigner formulae, as given in Section II. The final analysis was carried out using the three dominant levels which occur at $E_{LAB} = 10{\cdot}3$, $10{\cdot}5$, and $10{\cdot}95$ MeV. Figure 2 shows excitation functions at several angles, which were calculated using the level parameters given in Table 2. Comparing these results with the experimental curves (McKenna 1961) shows that the main features of the excitation functions are predicted and, except for some disagreement at $\theta = 50°$, leads us to believe that the spin and parity assignments are correct. All possible values of L and J up to $L = 4$ and $J = 9/2$ were tried for each level, but only the values given reproduced the correct behaviour at most angles.

On Figure 2 there is no vertical scale because we have concentrated on fitting the *shapes* of the excitation curves rather than their sizes. It is a difficult problem to fit the sizes of the excitation curves owing to the large number of level parameters involved when the levels are overlapping. For this reason the values of the widths and elastic fractions $a_{LJ} = \Gamma_{LJ}/\Gamma$ given in Table 2 are qualitative. The dimensionless reduced widths are given by

$$\theta_{LJ}^2 = \gamma_{LJ}^2(2Ma^2/3\hbar^2), \tag{4.1}$$

where γ^2_{LJ} is defined by equation (2.10), M is the reduced mass and a is the conventional separation parameter; in this case $a = 4{\cdot}71$ f. We take this opportunity to correct the values of θ^2_{LJ} given in I for the levels at $E_{\mathrm{LAB}} = 7{\cdot}53$ and $8{\cdot}17$ MeV, which were calculated by assuming $\cos^2(\delta_c+\phi_c) = 1$. The correct values are $\theta^2_{LJ} = 0{\cdot}04$ ($7{\cdot}53$) and $\theta^2_{LJ} = 0{\cdot}002$ ($8{\cdot}17$). The value quoted in I for the $9{\cdot}14$ level is unaffected, owing to the smallness of δ_{LJ} and ϕ_L for f-waves.

There is some evidence (Mitchell, personal communication 1961; Adams *et al.* 1961) for a level at $E_{\mathrm{LAB}} = 10{\cdot}75$ and also for one or more levels above 11 MeV. These levels if they exist do not show up very well in the elastic scattering data

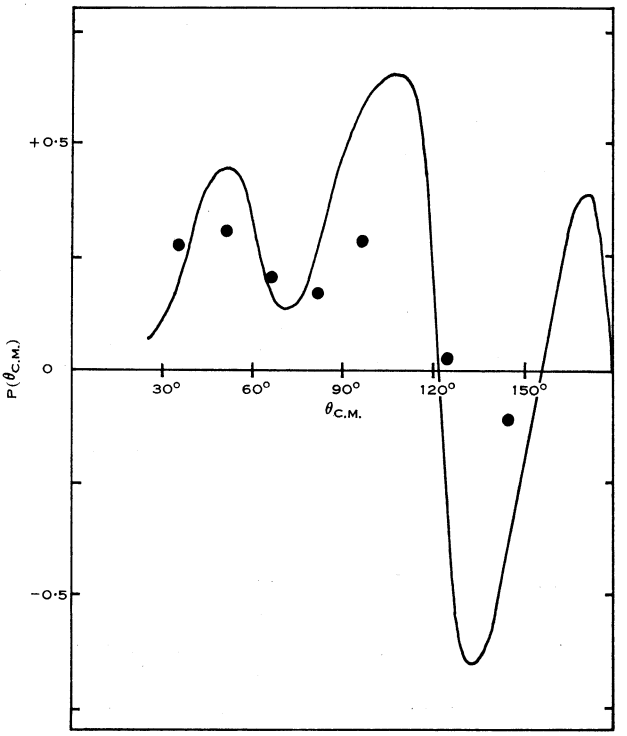

Fig. 3.—Angular distribution of the polarization at $E_{\mathrm{LAB}} = 10{\cdot}5$ MeV. The solid curve is the theoretical prediction and the experimental points are taken from Sanada (1961). Note that the experimental energy has a spread of $\pm 0{\cdot}5$ MeV.

obtained at Melbourne, although this may in part be due to the 100 keV energy spread (due essentially to target thickness) of the incident proton beam. Some attempt was made to include levels at $E_{\mathrm{LAB}} = 10{\cdot}75$ and $11{\cdot}35$ MeV in the theoretical calculations, but no definite improvement was obtained for the excitation curves for any particular values of L and J. For this reason the final analysis was carried out in terms of three levels only.

Further evidence for the level assignments made above is provided by the polarization data of Sanada (1961) in this energy region. Figure 3 shows the predicted

polarization at $E_{\mathrm{LAB}} = 10{\cdot}5$ MeV as compared with the experimental results. The agreement is good when one considers that the energy spread of the proton beam was $\pm\frac{1}{2}$ MeV, so that some averaging is contained in the experimental results. More convincing perhaps is the energy dependence of the polarization, which is shown in Figure 4. The experimental result that $P(50°_{\mathrm{LAB}})$ is positive for $E_{\mathrm{LAB}}{\sim}10$ MeV and negative for $E_{\mathrm{LAB}}{\sim}11$ MeV is explained by the theory which predicts positive polarization for the levels at $10{\cdot}3$ and $10{\cdot}5$ MeV and large negative polarization for the $10{\cdot}95$ level. This result is most likely associated with the fact that $B_R(\theta)$ takes opposite signs depending on whether $J_R = L_R + \frac{1}{2}$ or $J_R = L_R - \frac{1}{2}$.

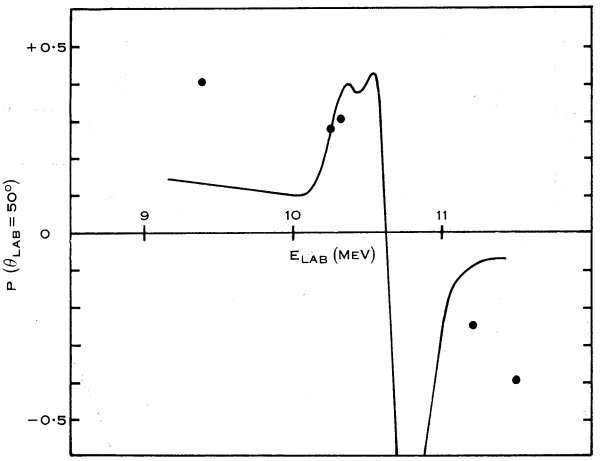

Fig. 4.—Energy dependence of the polarization for $\theta_{\mathrm{LAB}} = 50°$. The solid curve is the theoretical prediction and the experimental points are taken from Sanada (1961). Note that the experimental energy has a spread of $\pm 0{\cdot}5$ MeV.

V. Discussion

It is interesting to see if these new levels are in agreement with shell-model predictions (Kurath 1956; Barker 1961, 1963; Kurath and Lawson 1961) in this region. The $5/2^+$ level at $11{\cdot}5$ MeV excitation is in close agreement with the $5/2^+$ level predicted at about this excitation by both Barker (1961) and Kurath and Lawson (1961). Except for the lack of a $9/2^+$ level in the experimental level scheme shown in Figure 5, there is a one-to-one correspondence between theoretical and experimental levels for the positive parity levels on ^{13}N. We note that the level at $7{\cdot}42$ MeV excitation which was assigned as $5/2^+$ in I (and therefore an embarrassment to the theory) is now considered (Barker 1963) to be the missing $5/2^-$ level discussed in I.

Barker (1963) has also found that it is possible to move the position of the $5/2^-$ level (which according to Kurath occurs at 4–5 MeV excitation) up to about 7 MeV excitation by altering shell-model parameters without appreciably altering the positions of the other negative parity levels except the $7/2^-$ level previously predicted at $10{\cdot}4$ MeV excitation. This level is moved up in excitation energy by a

similar amount to the shift in excitation energy of the $5/2^-$ level. It is plausible to associate this level with the level found here at 12·1 MeV excitation. The $7/2^-$ level found at 10·4 MeV excitation in I is now considered to arise from strong coupling

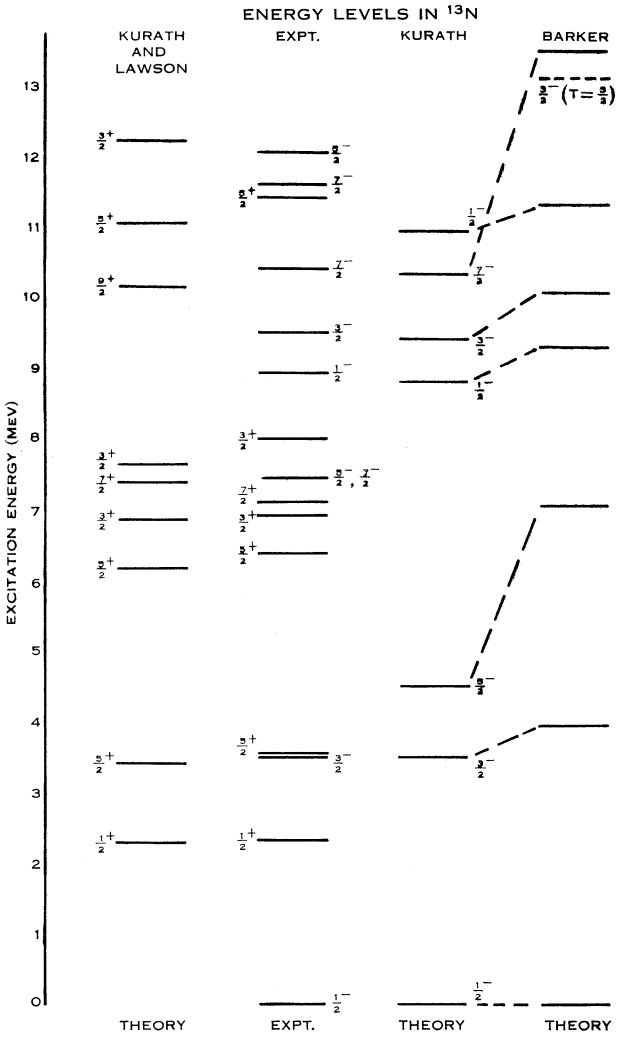

Fig. 5.—Experimental and theoretical level diagrams for ^{13}N. The positive parity predictions are due to Kurath and Lawson (1961) and the negative parity predictions to Kurath (1956) and Barker (1963).

(Okai and Tamura 1962) between the elastic and inelastic channels. If this is the case this level could not be predicted by a Kurath-type calculation since it would belong to a different configuration from those considered by Kurath. The $5/2^-$ level found here does not appear to have been predicted by any nuclear model of ^{13}N.

The success of the method used here for the analysis of resonances which are overlapping and also in I for the case of isolated levels suggests that this method of analysis has a widespread validity and constitutes a practical method of extracting level parameters when the effects of distant levels are not negligible. The application of this method to inelastic scattering is also feasible and is being considered at the present time.

VI. Acknowledgments

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