SHORT COMMUNICATIONS

ECCENTRIC DIPOLE COORDINATES*

By K. D. Cole†

Recently, Bond and Jacka (1962) sought to order the average frequency of occurrence of visible auroras in eccentric dipole latitude. It is considered that eccentric dipole co-latitudes (θ'), longitudes (ϕ'), and times for places on the surface of the Earth (here assumed spherical) may be of interest to workers in various fields. Mapped grids of θ' and ϕ' are reproduced here.

The Working Data and Formulae

(*a*) *The coordinate transformations*

One approximation to the magnetic field of the Earth is that of the *eccentric* dipole. It is defined so that the direction of its axis is that of the centred dipole approximation to the geomagnetic field (Chapman and Bartels 1951) and in which all but the sectorial terms of second order in the magnetic potential vanish (Bartels 1936).

Working data for the present computations were taken from Parkinson and Cleary (1958). They find the geographic co-latitude (θ) and longitude (ϕ) of the poles (a' for austral, b' for boreal) of the eccentric dipole to be

$$\theta_{a'} = 165\cdot0, \quad \phi_{a'} = 120\cdot4,$$

$$\theta_{b'} = 9\cdot0, \quad \phi_{b'} = -84\cdot7,$$

whilst the location (D) of the eccentric dipole is $0\cdot0685$ Earth radii from the centre (C) of the Earth in the direction $\theta_1 = 74\cdot4$, $\phi_1 = 150\cdot9$. This refers to observations of the geomagnetic field for epoch $1955\cdot0$.

For present purposes the transformation from geographic to eccentric dipole coordinates is effected in stages as follows.

(1) A right-handed system of geographic coordinates (x, y, z) is selected so that the origin is the centre of the Earth, the z axis points to the north geographic pole, and the x axis is in the meridian plane of Greenwich.

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(2) An eccentric dipole coordinate system (x', y', z') is chosen so that the origin is at D, the z' axis points in the direction of Db' and the x' axis is in the direction of the vector product $Cb' \times Ca'$. It follows that

$$\left.\begin{aligned} x' &= l_1 x''+m_1 y''+n_1 z'', \\ y' &= l_2 x''+m_2 y''+n_2 z'', \\ z' &= l_3 x''+m_3 y''+n_3 z'', \end{aligned}\right\} \tag{1}$$

where

$$\left.\begin{aligned} x'' &= x-X, \\ y'' &= y-Y, \\ z'' &= z-Z, \end{aligned}\right\} \tag{2}$$

where (x'', y'', z'') represent an intermediate set of coordinates given by parallel transfer of the x, y, z system to D, so that X, Y, Z are the (x, y, z) coordinates of D.

With the aid of equations (1) and (2) the eccentric dipole longitude (ϕ_0) and co-latitude (θ_0) of any particular observatory may be calculated directly by

$$\phi_0' = \tan^{-1} (y_0'/x_0'), \tag{3}$$

and

$$\theta_0' = \tan^{-1} \{(x_0'^2+y_0'^2)^{\frac{1}{2}}/z_0'\}. \tag{4}$$

Expressions for the direction cosines l, m, n may be obtained from the positions of the eccentric dipole poles and the position of the eccentric dipole. Thus,

$$\begin{aligned} l_3 &= (\sin \theta_{b'} \cos \phi_{b'}-\sin \theta_{a'} \cos \phi_{a'})/L, \\ m_3 &= (\sin \theta_{b'} \sin \phi_{b'}-\sin \theta_{a'} \sin \phi_{a'})/L, \\ n_3 &= (\cos \theta_{b'}-\cos \theta_{a'})/L, \end{aligned}$$

where

$$l_3^2+m_3^2+n_3^2 = 1$$

(from which L is derived).

$$\begin{aligned} l_1 &= (\sin \theta_{b'} \sin \phi_{b'} \cos \theta_{a'}-\sin \theta_{a'} \sin \phi_{a'} \cos \theta_{b'})/\sin \beta, \\ m_1 &= (\sin \theta_{a'} \cos \phi_{a'} \cos \theta_{b'}-\sin \theta_{b'} \cos \phi_{b'} \cos \theta_{a'})/\sin \beta, \\ n_1 &= [\sin \theta_{b'} \sin \theta_{a'} \sin(\phi_{a'}-\phi_{b'})] /\sin \beta, \end{aligned}$$

where

$$\sin \beta = \{1-[\sin \theta_{b'} \sin \theta_{a'} \cos(\phi_{b'}-\phi_{a'})+\cos \theta_{b'} \cos \theta_{a'}]^2\}^{\frac{1}{2}}.$$

From these l_2, m_2, n_2 can be found from the well-known relations between the l's, m's, and n's.

The direction cosines may also be evaluated using the positions of the geomagnetic pole (Finch and Leaton 1957) and the position of the eccentric dipole (Parkinson and Clearly 1958). Thus

$$\begin{aligned} l_3 &= \sin \theta_b^* \cos \phi_b^*, \\ m_3 &= \sin \theta_b^* \sin \phi_b^*, \\ n_3 &= \cos \theta_b^*, \end{aligned}$$

where θ_b^*, ϕ_b^* are the geographic co-latitude and longitude of the geomagnetic dipole boreal pole. Also,

$$\begin{aligned} l_1 &= (\sin\theta_b^* \sin\phi_b^* \cos\theta_1 - \sin\theta_1 \sin\phi_1 \cos\theta_b^*)/\sin\gamma, \\ m_1 &= (\sin\theta_1 \cos\phi_1 \cos\theta_b^* - \sin\theta_b^* \cos\phi_b^* \cos\theta_1)/\sin\gamma, \\ n_1 &= [\sin\theta_b^* \sin\theta_1 \sin(\phi_1 - \phi_b^*)]/\sin\gamma, \end{aligned}$$

where $l_1^2+m_1^2+n_1^2 = 1$, from which, $\sin\gamma$ may be obtained. The values of l_2, m_2, n_2 follow simply.

For the purpose of computation the following values of the transformation constants were evaluated by the first method from the data given by Parkinson and Cleary.

$$\begin{aligned} l_1 &= -0\cdot5146, & m_1 &= -0\cdot8481, & n_1 &= -0\cdot1262, \\ l_2 &= 0\cdot8543, & m_2 &= -0\cdot4947, & n_2 &= -0\cdot1596, \\ l_3 &= 0\cdot0728, & m_3 &= -0\cdot1894, & n_3 &= 0\cdot9791, \\ X &= -0\cdot0577, & Y &= 0\cdot0320, & Z &= 0\cdot0184. \end{aligned}$$

The radius of the Earth is taken as the unit of length.

(*b*) *The latitude curves*

The curves of *eccentric* dipole co-latitude (θ') on the surface of the Earth are the intersections of the cones $z' = \cot\theta' \; [(x')^2+(y')^2]^{\frac{1}{2}}$ with the unit spheres $x^2+y^2+z^2 = 1$. Thus they are determined by the equation

$$\begin{aligned} l_3(\sin\theta\cos\phi - X) + m_3(\sin\theta\sin\phi - Y) + n_3(\cos\theta - Z) \\ = \cot\theta' \; \{[l_1(\sin\theta\cos\phi - X) + m_1(\sin\theta\sin\phi - Y) + n_1(\cos\theta - Z)]^2 \\ + [l_2(\sin\theta\cos\phi - X) + m_2(\sin\theta\sin\phi - Y) + n_2(\cos\theta - Z)]^2\}^{\frac{1}{2}}. \end{aligned} \tag{5}$$

(*c*) *The longitude curves*

The curves of *eccentric* dipole longitude on the surface of the Earth are the intersections of the planes $y' = x' \tan\phi'$ with the unit sphere $x^2+y^2+z^2 = 1$. Thus they are defined by the equation

$$\begin{aligned} l_2(\sin\theta\cos\phi - X) + m_2(\sin\theta\sin\phi - Y) + n_2(\cos\theta - Z) \\ = \tan\phi' \; [l_1(\sin\theta\cos\phi - X) + m_1(\sin\theta\sin\phi - Y) + n_1(\cos\theta - Z)]. \end{aligned} \tag{6}$$

Using a set of values of (θ, ϕ) distributed over the globe, equations (5) and (6) were used to determine the corresponding set (θ', ϕ'). From these, isolines of eccentric dipole latitude and longitude lines were drawn on the maps (Figs. 1, 2, and 3).

Finally the eccentric dipole longitude zero was defined by that eccentric dipole longitude plane which passes through the south geographic pole. In the (θ', ϕ') system of coordinates the longitude (ϕ_S') of the south geographic pole is $61\cdot02°$ E. Figures 1, 2, and 3 show (θ, ϕ) and $(\theta', \phi' - \phi_S')$ grids.

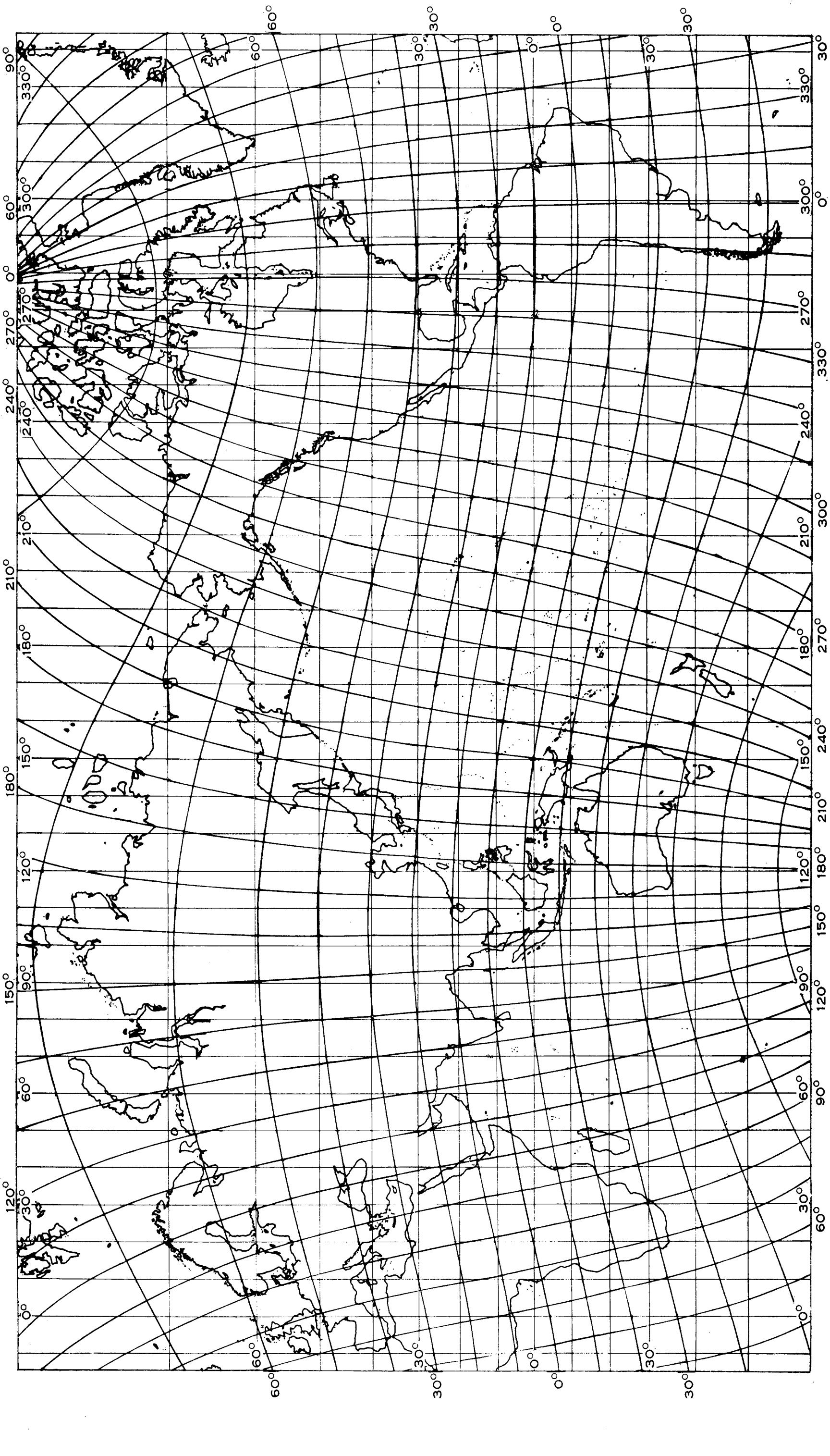

Fig. 1.—Eccentric dipole geomagnetic latitude and longitude superimposed on the geographic grid of a Mercator's projection. Land masses are approximate.

(d) Eccentric dipole time

Let *eccentric dipole local solar time* (α_e) be defined as the difference of the eccentric dipole longitudes of the observatory (ϕ'_0) and the Sun (ϕ'_{sun}), being measured eastward from the Sun to the observatory, i.e. $\phi'_0 - \phi_{sun}$. The direction of the Sun in geographic coordinates is ($\cos\delta\cos H$, $\cos\delta\sin H$, $\sin\delta$) where H is the longitude

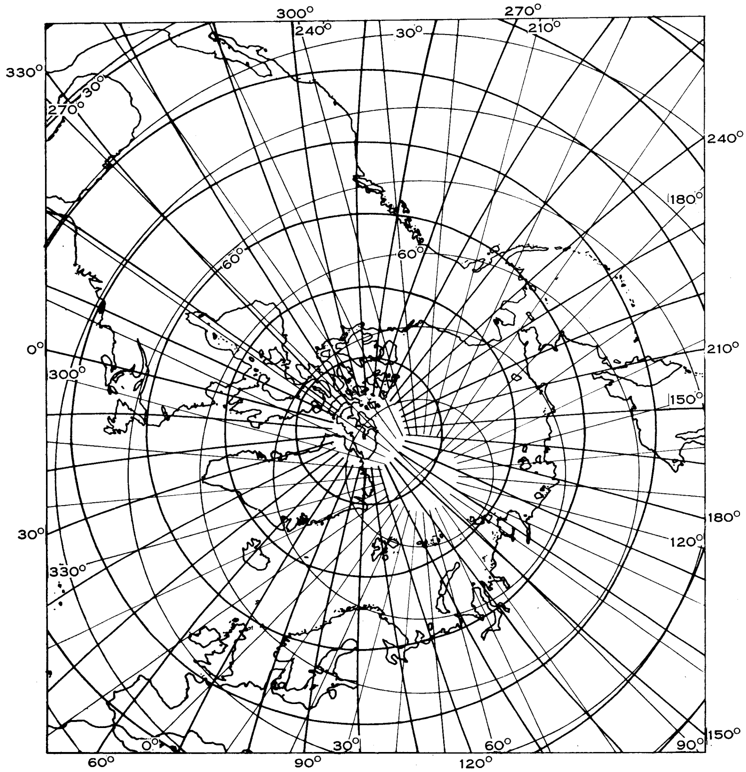

Fig. 2.—Eccentric dipole geomagnetic latitude and longitude superimposed on the geographic grid of an azimuthal equidistant projection (north pole). Land masses are approximate.

east of Greenwich of the Sun and δ its declination. From the transformations (1) and (2) above it follows that

$$\phi'_{sun} = \tan^{-1}\frac{l_2(D_s\cos\delta\cos H - X) + m_2(D_s\cos\delta\sin H - Y) + n_2(D_s\sin\delta - Z)}{l_1(D_s\cos\delta\cos H - X) + m_1(D_s\cos\delta\sin H - Y) + n_1(D_s\sin\delta - Z)}, \quad (7)$$

where D_s is the distance of the Sun from the Earth. Since $D_s \gg OD$, ϕ'_{sun} may be computed with precision from the formula

$$\phi'_{sun} = \tan^{-1}\frac{l_2\cos\delta\cos H + m_2\cos\delta\sin H + n_2\sin\delta}{l_1\cos\delta\cos H + m_1\cos\delta\sin H + n_1\sin\delta}. \quad (8)$$

Being independent of X, Y, and Z, (8) gives also the longitude of the Sun in centred dipole coordinates. Likewise the ϕ_0' of an observatory (co-latitude θ_0, longitude ϕ_0) on the Earth's surface is given by

$$\phi_0' = \tan^{-1}\frac{l_2(\sin\theta_0\cos\phi_0 - X) + m_2(\sin\theta_0\sin\phi_0 - Y) + n_2(\cos\theta_0 - Z)}{l_1(\sin\theta_0\cos\phi_0 - X) + m_1(\sin\theta_0\sin\phi_0 - Y) + n_1(\cos\theta_0 - Z)}. \tag{9}$$

Finally,

$$\alpha_e = \phi_0' - \phi_{\text{sun}}'. \tag{10}$$

The local centred dipole solar time (α_c) is given by

$$\alpha_c = [\phi_0']_{X=Y=Z=0} - \phi_{\text{sun}}'. \tag{11}$$

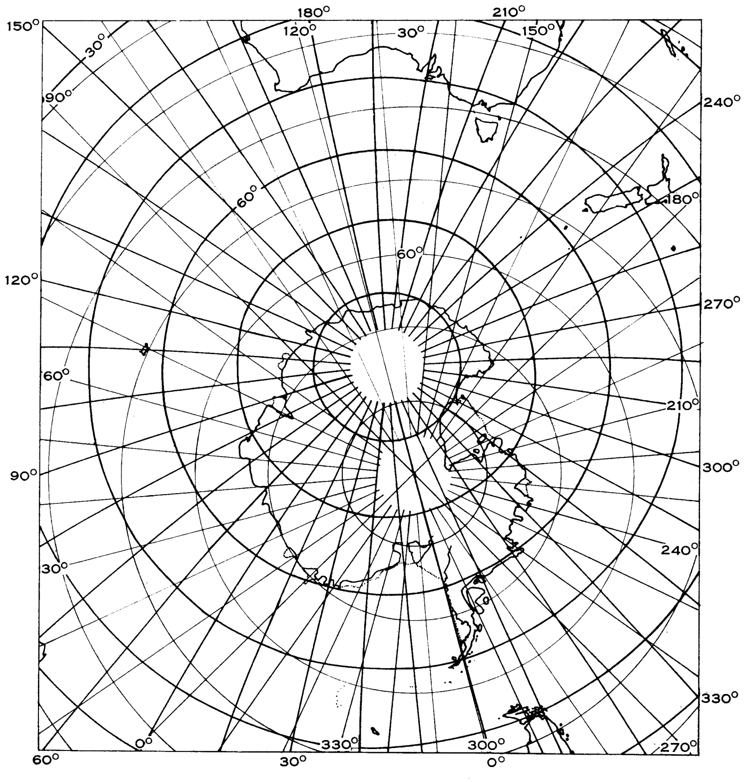

Fig. 3.—Eccentric dipole geomagnetic latitude and longitude superimposed on the geographic grid of an azimuthal equidistant projection (south pole).

It is clear from (7)–(10) that local eccentric dipole solar time (α_e) for a particular observatory runs parallel to local centred dipole solar time (α_c). The constant difference between the two,

$$\alpha_e - \alpha_c = \phi_0' - [\phi_0']_{X=Y=Z=0}, \tag{12}$$

is a function of (θ_0, ϕ_0, X, Y, Z). Geomagnetic times are discussed elsewhere by Chapman and Sugiura (1956), Hultqvist and Gustafsson (1960), and Simonow (1963).

(*e*) *Errors*

If one computes the values of (l, m, n) by the second method described in (*a*) one finds, naturally, slightly different values. The two values of l_3 so found differ by about 1 part in 1000; and since l_3 largely governs the accuracy of latitude determination, it is considered that the latitude calculations are accurate to the order of $0 \cdot 1$ degree. The two values of l_1 (and also the values of m_1) differ by about 1 part in 100; and since l_1 and m_1 largely govern the accuracy of longitude it is considered that the longitude calculations are acurate to about $0 \cdot 5$ degree.

The calculations of eccentric dipole latitudes and longitudes were done at the request of Major F. R. Bond for use in studies of auroral morphology. He supervised the graphical work and much of the numerical computing. Messrs. R. Eggleton, Mun Ping Shu, and R. Little assisted in the numerical computing. The author thanks Dr. G. V. Simonow and Dr. F. Jacka, Chief Physicist, for some critical comments.

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