ELECTROMAGNETIC 2π DECAY OF K_2^0

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[Manuscript received April 22, 1965]

Summary

As alternatives to the observed $K_2^0 \rightarrow 2\pi$ decay we consider the processes (i) $K_2^0 \rightarrow 2\pi + \gamma$ and (ii) $K_2^0 \rightarrow K_1^0$ magnetic conversion. It is shown that the former process is quite unlikely with the photon energy as suggested from experiment, while the second one is not so unlikely. Also suitable experiments are suggested to observe the processes considered here.

INTRODUCTION

Experimentalists (Abashian *et al.* 1964; Christenson *et al.* 1964) have observed a process which could be $K_2^0 \rightarrow 2\pi$ decay with a branching ratio $\sim 2 \times 10^{-3}$, corresponding to a partial decay rate $(1/\tau)(K_2^0 \rightarrow 2\pi) \approx 3 \times 10^4 \text{ s}^{-1}$. If this is the case, then the process violates CP invariance. In the following we try to consider what they have found as 2π decay with one real or virtual photon which conserves CP invariance. There are two possibilities: (i) radiative decay and (ii) $K_2^0 \rightarrow K_1^0$ conversion on passage through inhomogeneities in a magnetic field. Mechanism (i) appears to be ruled out by the absence of observed γ -rays exceeding about 10 MeV; mechanism (ii) seems also improbable but is not so obviously excluded by the experimental conditions.

Both types of mechanism may be within possible observation in experiments specifically designed for the purpose.

RADIATIVE DECAY, $K_2^0 \rightarrow 2\pi + \gamma$

An approximate estimation of the partial decay rate can be made as follows. Comparison of $K_2^0 \rightarrow 2\pi + \gamma$ with $K_1^0 \rightarrow 2\pi$ gives

$$\frac{1}{\tau}(\mathbf{K}_2^0 \rightarrow 2\pi + \gamma) \sim \left(\frac{a}{2\pi}\right) \left\{ \frac{\rho(\mathbf{K}_2^0 \rightarrow 2\pi + \gamma)}{\rho(\mathbf{K}_1^0 \rightarrow 2\pi)} \right\} \frac{1}{\tau(\mathbf{K}_1^0 \rightarrow 2\pi)} \approx 10^6 \ \mathrm{s}^{-1},$$

where ρ represents the phase volume for the respective processes.

Similarly from the comparison with $K_2^0 \rightarrow 3\pi$, we get

$$\frac{1}{\tau}(\mathbf{K}_2^0 \rightarrow 2\pi + \gamma) \thicksim \left(\frac{a}{2\pi}\right) \left\{ \frac{\rho(\mathbf{K}_2^0 \rightarrow 2\pi + \gamma)}{\rho(\mathbf{K}_2^0 \rightarrow 3\pi)} \right\} \frac{1}{\tau(\mathbf{K}_2^0 \rightarrow 3\pi)} \approx 10^5 \, \mathrm{s}^{-1}.$$

The preceding estimates show that

$$\tau({
m K}_2^0 \to 2\pi + \gamma) \approx 10^{-5}$$
 to 10^{-6} s.

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We now consider a dynamical model, as shown in Figure 1, where the $K_2^0 \gamma K_1^{0*}$ vertex has the same structure as the $\pi \gamma \rho$ vertex from the unitary symmetry scheme. The effective interaction Hamiltonians for the two vertices are given by

$$H(\mathbf{K}_{2}^{0}\gamma\mathbf{K}_{1}^{0*}) = \frac{1}{2}(G/\kappa_{1})\epsilon_{a\beta\sigma\tau}F_{\alpha\beta}\Psi_{\alpha}\partial\Phi/\partial x_{\tau}$$
(1)

and

$$H(\mathcal{K}_1^{0*}\pi\pi) = 2^{-\frac{1}{2}}g\Psi_{\mu}(\partial_{\mu}\phi^*\phi - \phi^*\partial_{\mu}\phi), \qquad (2)$$

where $F_{\alpha\beta}$ is the electromagnetic field of the photon, Ψ the $K_1^{0^*}$ -meson amplitude, Φ the K_2^0 amplitude, ϕ the pion amplitude, κ_1 the $K_1^{0^*}$ mass, $G^2/4\pi \sim 0.02$ in accordance with a $\rho \rightarrow \pi + \gamma$ decay width of $\frac{1}{2}$ MeV (Adler and Drell 1964; Berman and Drell 1964), $g = 1.26 \times 10^{-11} M_0^2$ (M_0 has the dimension of mass in MeV) is the usual beta decay coupling.



Fig. 1.—Radiative 2π decay process for K_2^0 meson.

For the propagator of the intermediate boson we use the expression

$$\delta_{\mu\sigma} / \{ (k-q)^2 - \kappa_1^2 \}.$$
(3)

The decay rate for $K_2^0 \rightarrow 2\pi + \gamma$ in the K_2^0 rest frame is given by

$$(1/\tau)(\mathbf{K}_{2}^{0} \to 2\pi + \gamma) = \int (2\pi)^{4} \,\delta(k' + k'' - q) |\mathscr{M}|^{2} \rho_{E} \,\mathrm{d}\rho_{E}, \tag{4}$$

where by equations (1), (2), and (3)

$$\mathscr{M} = \frac{1}{\sqrt{(2M)}} \cdot \frac{1}{\sqrt{(2q)}} \cdot \frac{G}{\kappa_1} \epsilon_{a\beta\sigma\tau} e_a q_\beta k_\tau \frac{\delta_{\mu\sigma}}{(k-q)^2 - \kappa_1^2} \cdot \frac{g(k'-k'')_\mu}{\sqrt{(2)}\sqrt{(2\omega')}\sqrt{(2\omega'')}}$$

with four-vectors (M, 0), (q, \mathbf{q}) , (ω', \mathbf{k}') , and (ω'', \mathbf{k}'') for the K_2^0 , γ -ray, and two π mesons; furthermore,

$$\rho_E \mathrm{d}\rho_E = \frac{\mathrm{d}^3 k'}{(2\pi)^3} \cdot \frac{\mathrm{d}^3 q}{(2\pi)^3} \cdot \frac{\mathrm{d}^3 q}{(2\pi)^3} \delta(E_i - E_f), \qquad E_f = \omega' + \omega''.$$

Replacing $\mathbf{k}' + \mathbf{k}'' = \mathbf{q}$, $\mathbf{k}' - \mathbf{k}'' = 2\mathbf{p}$ with $d^3k' d^3k'' = d^3p d^3q$, we find

$$\frac{1}{\tau} = \int (2\pi)^{-5} \frac{g^2 G^2 M}{8\omega' \omega'' \kappa_1^2} \cdot \frac{p^4 q^3 \sin^2 \theta}{(M^2 - 2Mq - \kappa_1^2)^2} \left| \frac{\mathrm{d}p}{\mathrm{d}E_f} \right|_{E_i = E_f} \mathrm{d}q \mathrm{d}\Omega_p \mathrm{d}\Omega_q, \tag{5}$$

where we put

$$|\epsilon_{a\beta\sigma\tau}e_a q_\beta k_\tau (2p)_\sigma|^2 = 4p^2 q^2 M^2 \sin^2\theta$$

(θ is the angle between **p** and **q**).

Now

$$\frac{p^4}{\omega'\omega''} \left| \frac{\mathrm{d}p}{\mathrm{d}E_f} \right|_{E_i = E_f} = \frac{(M - q)^4 (M^2 - 2Mq - 4m^2)^{3/2}}{8(M^2 - 2Mq + q^2\sin^2\theta)^{5/2}},$$

where m is the π -meson mass. Putting

 $\mathrm{d}\Omega_p = 4\pi, \qquad \mathrm{d}\Omega_q = 2\pi\sin\theta\,\mathrm{d}\theta,$

and performing the θ -integration, we find from (5)

$$\frac{1}{\tau} = \int \frac{(2\pi)^{-3}g^2G^2}{24\kappa_1^2} \cdot \frac{M(M^2 - 2Mq - 4m^2)^{3/2}q^3 \,\mathrm{d}q}{(M^2 - 2Mq - \kappa_1^2)^2(M^2 - 2Mq)^{\frac{3}{2}}},$$

and, after the following substitution

$$q/M = x, \qquad m/M = y, \qquad ext{and} \ \kappa_1/M = z,$$

the decay rate $(K_2^0 \rightarrow 2\pi + \gamma)$ reduces to

$$\frac{1}{\tau} = \frac{g^2 G^2}{(2\pi)^3 \, 24 z^2 M} \int_0^{(0\cdot 5 - 2y^2)} \frac{x^3 (1 - 2x - 4y^2)^{3/2} \, \mathrm{d}x}{(1 - 2x - z^2)^2 (1 - 2x)^{\frac{1}{2}}} = 7 \times 10^4 \, \mathrm{s}^{-1},$$

where we have used

$$g = 1.5 \times 10^{-5}$$
, corresponding to $M_0 = 1.1$ GeV,
 $M = 496$ MeV, $y = 0.276$, and $z = 1.51$.

Thus the value of $\tau(K_2^0 \rightarrow 2\pi + \gamma) \approx 1.4 \times 10^{-5}$ s, as obtained from the present dynamical model, is consistent with the approximate estimation made at the beginning of the paper.

The photon energy spectrum in the K_2^0 rest frame is given by

$$f(x) = \frac{x^3(1-2x-4y^2)^{3/2}}{(1-2x-z^2)^2(1-2x)^{\frac{3}{2}}},$$
(6)

as shown in Figure 2.

From experiment (Abashian et al. 1964; Christenson et al. 1964) we have

$$(\mathrm{K_2^0}
ightarrow 2\pi)/(\mathrm{K_2^0}
ightarrow \mathrm{all\ decay\ modes}) = 2 imes 10^{-3}.$$

This ratio corresponds to about half the radiative events, according to the above estimates, or hence to an average photon energy of not quite 100 MeV. Thus, if the γ -ray energy is really $\lesssim 10$ MeV, the possibility for the interpretation that the observed event is $K_2^0 \rightarrow 2\pi + \gamma$ is very slight. Conversely, if the γ -ray energy could happen to be as large as 100 MeV, such interpretation may be possible.

This suggests the importance of the experimental study of $K_2^0 \rightarrow 2\pi + \gamma$. The energy distribution is approximately a parabola peaked at 120 MeV, and the angular correlation is approximately proportional to $\sin^2\theta$.

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Magnetic $K_2^0 \rightarrow K_1^0$ Conversion

The fact that $K_2^0 \rightarrow K_1^0 + \text{ odd } \gamma$ is in principle only inhibited but not forbidden (Peaslee and Vaughn 1960) suggests a possible interaction of the form

$$H_{\rm int} = eA_{\mu}f(\Box^2)[\phi_4\partial_{\mu}\phi_3 - \phi_3\partial_{\mu}\phi_4] \tag{7}$$

$$= ieA_{\mu} f(\Box^2) [\phi^* \partial_{\mu} \phi - \partial_{\mu} \phi^* \phi], \qquad (8)$$

where $\phi = (\phi_3 + i\phi_4)/\sqrt{2}$ represents a K⁰ meson.



Fig. 2.—Photon energy spectrum associated with $K_2^0 \rightarrow 2\pi + \gamma$, where x measures the photon energy in units of average baryon mass.

Electrical neutrality of the K⁰ meson assumes by equation (8) that f(0) = 0 but tells nothing about higher moments. If $f(\square^2)$ does not vanish identically, the possibility exists of $K_2^0 \rightarrow K_1^0$ conversion on passage through an external magnetic field. One-photon conversion can involve only scalar photons because the K⁰ meson has spin zero. We write

$$f(\underline{}^{2}) = f_{0} + f_{2}(\underline{}^{2}/M_{0}^{2}) + f_{4}(\underline{}^{4}/M_{0}^{4}) + \dots, \qquad (9)$$

where $f_0 = 0$ and $M_0 = 1 \cdot 1$ GeV is the baryon mass.

Now

 $\Box^2 A_{\mu} = -j_{\mu}, \text{ source of the magnetic field}, \dagger$ (10)

and from equations (7), (9), and (10) we find

$$H_{\text{int}} = \frac{ef_2}{M_0^2} \cdot \frac{\mathbf{k} \cdot \mathbf{j}(\kappa)}{\omega (2\pi)^{3/2}} \Psi^2(R),$$

where we use

$$\begin{split} \mathbf{k} &\approx \mathbf{k}' = \mathrm{K}^{0}\text{-meson momentum,} \\ \boldsymbol{\omega} &\approx \boldsymbol{\omega}' = \mathrm{K}^{0}\text{-meson energy,} \\ \mathbf{j}(x) &= (2\pi)^{-3/2}\int \mathbf{j}(\kappa) \,\mathrm{e}^{-\mathrm{i}\mathbf{K}\cdot\mathbf{x}} \,\mathrm{d}^{3}\kappa, \end{split}$$

[†] In the present calculation only the transverse fringe of the beam, which actually passes through the current coils of the magnets, produces regeneration. The entire beam is taken coherent in the transverse dimension, and the coil acts as the edges of a gigantic slit. with $\kappa =$ wave vector of magnetic field. Here $\Psi(R)$ is a function describing the spread of the beam, considered as a coherent wave packet in the transverse dimensions (cf. Appendix). The momentum of this wave packet is negligibly disturbed by the $K_2^0 \to K_1^0$ transition; therefore the $K_1^0 \to 2\pi$ decay occurs principally in the centre of the beam, although the converting interaction was located in the fringes. If the total path length through the magnetic field is L, the probability of $K_2^0 \to K_1^0$ conversion is given by

where

$$P = (L/\tau)(\omega/k), \tag{11}$$

$$\frac{1}{\tau} = \frac{1}{3} (2\pi)^{-2} \left(\frac{ef_2}{M_0^2} \right)^2 \frac{k^2}{\omega^2} j^2(\kappa) \left(\frac{\mathrm{d}^3 k'}{\mathrm{d} E} \right) \left| \Psi^2(R) \right|^2.$$

Integrating over all wave vectors κ present in the field, we get

$$P = \frac{1}{3} (2\pi)^{-2} \left(\frac{ef_2}{M_0^2}\right)^2 \frac{Lk}{\omega} \int j^2(\kappa) \,\mathrm{d}^3 \kappa \left(\frac{\mathrm{d}^3 k'}{\mathrm{d}E}\right) \left| \Psi^2(R) \right|^2.$$

$$\int j^2(\kappa) \,\mathrm{d}^3 \kappa = \int \kappa^2(\mathbf{j}, \mathbf{A}) \,\mathrm{d}^3 \kappa = \langle \kappa^2 \rangle 4\pi \epsilon_{\mathrm{t}},$$
(12)

Now put

where
$$\epsilon_{\rm t}$$
 is the total energy in the magnetic field $H(\approx 10^3 \text{ to } 10^4 \text{ gauss})$ and $\langle \kappa^2 \rangle = (2\pi n/l)^2$, with *l* the mean transverse wavelength in the magnetic field and *n* a numerical factor depending on the nonuniformity of the magnetic field. With $d^3k'/dE = 4\pi k\omega$, the expression for the probability of magnetic conversion becomes (after inserting the $\hbar c$ factors)

$$P = \frac{1}{3} (4\pi n)^2 f_2^2 \left(\frac{L}{l^2} \cdot \frac{e^2}{M_0 c^2} \right) \left(\frac{\epsilon_t}{M_0 c^2} \right) \left(\frac{p_\kappa}{M_0} \right)^2 \left| \Psi^2(R) \right|^2,$$
(13)

where $p_{\kappa} = \hbar k$, and each term in the brackets is dimensionless.

For L = 10 ft, $H = 10^4$ gauss, $\Psi^2(R) = 10^{-3}$, $p_{\kappa} = 1 \cdot 1$ GeV/c, $nf_2 = 3$, and $(L/l^2)\epsilon_t = (1/4\pi)(HL)^2$, we get from equation (13)

$$P = 10^{-5}$$
.

This is about the same as for radiative decay with $E_{\gamma} \lesssim 3$ MeV and is about 10^2 short of the observed value. On the other hand, $nf_2 \approx 30$ would explain the observations. (See Peaslee and Vaughn (1960) for a possible value of f_2 .)

The presence of magnetic conversion $K_2^0 \rightarrow K_1^0$ seems feasible to observe by changing the magnetic field, length of the field, and n. Such a measurement might be of interest even if the mechanism is not responsible for the previous observations (Abashian *et al.* 1964; Christenson *et al.* 1964).[†]

Failure of CP invariance as a local effect due to dissymmetry of the environment was also suggested by Bell and Perring (1964).

* According to the Appendix this corresponds to a magnetic field at a distance of order $R = 0.97R_0$ from a beam of radius R_0 .

[†] The results of Christenson *et al.* have been repeated independently at CERN and in England. In every case the decay fraction of $(K_2^0 \rightarrow 2\pi)/K_2^0$ was $\epsilon = 2 \times 10^{-3}$, although the experimental set-ups must have differed greatly. All electromagnetic regeneration effects vary as $(p_{\kappa}/M_0)^2 \rightarrow \gamma^2 = 1/(1-\beta^2)$. Christenson *et al.* had already made preliminary measurements showing the *absence* of any γ^2 effect.

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Acknowledgment

The author is grateful to Professor D. C. Peaslee for initiating the investigation and for many helpful discussions.

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Appendix

Transverse beam function $\Psi(R)$, corresponding to a uniform momentum distribution, is

$$\begin{split} \Psi(R) &= \frac{1}{2\pi} \int_0^{K_0} \{ \exp(iKR\cos\theta) \} K \, \mathrm{d}K \, \mathrm{d}\theta \\ &= \int_0^{K_0} K \, \mathrm{d}K \mathrm{J}_0(KR) \\ &= (K_0/R) \mathrm{J}_1(K_0R). \end{split}$$

Thus

$$\Psi(R) = C(R_0/R) \mathbf{J}_1(3 \cdot 83R/R_0),$$

where $R_0 =$ beam radius, and $3 \cdot 83 =$ 1st zero of J_1 , the Bessel function of order one. The constant C is fixed by the condition

$$2\pi \int_0^{R_0} \frac{R dR}{R_0^2} \Psi^2(R) = 1.$$

Hence, after numerical integration, we get

$$C = 0.64$$

and accordingly

$$\Psi(R) = 0.64(R/R_0)^{-1} J_1(3.83R/R_0).$$