# ELECTROMAGNETIC $2 \pi$ DECAY OF K ${ }_{2}^{0}$ 

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## Summary

As alternatives to the observed $\mathrm{K}_{2}^{0} \rightarrow 2 \pi$ decay we consider the processes (i) $\mathrm{K}_{2}^{0} \rightarrow 2 \pi+\gamma$ and (ii) $\mathrm{K}_{2}^{0} \rightarrow \mathrm{~K}_{1}^{0}$ magnetic conversion. It is shown that the former process is quite unlikely with the photon energy as suggested from experiment, while the second one is not so unlikely. Also suitable experiments are suggested to observe the processes considered here.

## Introduction

Experimentalists (Abashian et al. 1964; Christenson et al. 1964) have observed a process which could be $\mathrm{K}_{2}^{0} \rightarrow 2 \pi$ decay with a branching ratio $\sim 2 \times 10^{-3}$, corresponding to a partial decay rate $(1 / \tau)\left(\mathrm{K}_{2}^{0} \rightarrow 2 \pi\right) \approx 3 \times 10^{4} \mathrm{~s}^{-1}$. If this is the case, then the process violates CP invariance. In the following we try to consider what they have found as $2 \pi$ decay with one real or virtual photon which conserves CP invariance. There are two possibilities: (i) radiative decay and (ii) $\mathrm{K}_{2}^{0} \rightarrow \mathrm{~K}_{1}^{0}$ conversion on passage through inhomogeneities in a magnetic field. Mechanism (i) appears to be ruled out by the absence of observed $\gamma$-rays exceeding about 10 MeV ; mechanism (ii) seems also improbable but is not so obviously excluded by the experimental conditions.

Both types of mechanism may be within possible observation in experiments specifically designed for the purpose.

## Radiative Decay, $\mathrm{K}_{2}^{0} \rightarrow 2 \pi+\gamma$

An approximate estimation of the partial decay rate can be made as follows. Comparison of $\mathrm{K}_{2}^{0} \rightarrow 2 \pi+\gamma$ with $\mathrm{K}_{1}^{0} \rightarrow 2 \pi$ gives

$$
\frac{1}{\tau}\left(\mathrm{~K}_{2}^{0} \rightarrow 2 \pi+\gamma\right) \sim\left(\frac{\alpha}{2 \pi}\right)\left\{\frac{\rho\left(\mathrm{K}_{2}^{0} \rightarrow 2 \pi+\gamma\right)}{\rho\left(\mathrm{K}_{1}^{0} \rightarrow 2 \pi\right)}\right\} \frac{1}{\tau\left(\mathrm{~K}_{1}^{0} \rightarrow 2 \pi\right)} \approx 10^{6} \mathrm{~s}^{-1}
$$

where $\rho$ represents the phase volume for the respective processes.
Similarly from the comparison with $\mathrm{K}_{2}^{0} \rightarrow 3 \pi$, we get

$$
\frac{1}{\tau}\left(\mathrm{~K}_{2}^{0} \rightarrow 2 \pi+\gamma\right) \sim\left(\frac{\alpha}{2 \pi}\right)\left\{\frac{\rho\left(\mathrm{K}_{2}^{0} \rightarrow 2 \pi+\gamma\right)}{\rho\left(\mathrm{K}_{2}^{0} \rightarrow 3 \pi\right)}\right\} \frac{1}{\tau\left(\mathrm{~K}_{2}^{0} \rightarrow 3 \pi\right)} \approx 10^{5} \mathrm{~s}^{-1} .
$$

The preceding estimates show that

$$
\tau\left(\mathrm{K}_{2}^{0} \rightarrow 2 \pi+\gamma\right) \approx 10^{-5} \text { to } 10^{-6} \mathrm{~s} .
$$

[^0]We now consider a dynamical model, as shown in Figure 1, where the $\mathrm{K}_{2}^{0} \gamma \mathrm{~K}_{1}^{0^{*}}$ vertex has the same structure as the $\pi \gamma \rho$ vertex from the unitary symmetry scheme. The effective interaction Hamiltonians for the two vertices are given by

$$
\begin{equation*}
H\left(\mathrm{~K}_{2}^{0} \gamma \mathrm{~K}_{1}^{0^{*}}\right)=\frac{1}{2}\left(G / \kappa_{1}\right) \epsilon_{\alpha \beta \sigma \tau} F_{a \beta} \Psi_{\sigma} \partial \Phi / \partial x_{\tau} \tag{1}
\end{equation*}
$$

and

$$
\begin{equation*}
H\left(\mathrm{~K}_{1}^{0^{*}} \pi \pi\right)=2^{-\frac{1}{2}} g \Psi_{\mu}\left(\partial_{\mu} \phi^{*} \phi-\phi^{*} \partial_{\mu} \phi\right) \tag{2}
\end{equation*}
$$

where $F_{\alpha \beta}$ is the electromagnetic field of the photon, $\Psi$ the $\mathrm{K}_{1}^{0^{*}}$-meson amplitude, $\Phi$ the $\mathrm{K}_{2}^{0}$ amplitude, $\phi$ the pion amplitude, $\kappa_{1}$ the $\mathrm{K}_{1}^{0^{*}}$ mass, $G^{2} / 4 \pi \sim 0.02$ in accordance with a $\rho \rightarrow \pi+\gamma$ decay width of $\frac{1}{2} \mathrm{MeV}$ (Adler and Drell 1964; Berman and Drell 1964), $g=1 \cdot 26 \times 10^{-11} M_{0}^{2}$ ( $M_{0}$ has the dimension of mass in MeV ) is the usual beta decay coupling.


Fig. 1.-Radiative $2 \pi$ decay process for $\mathrm{K}_{2}^{0}$ meson.
For the propagator of the intermediate boson we use the expression

$$
\begin{equation*}
\delta_{\mu \sigma}\left\{\left\{(k-q)^{2}-\kappa_{1}^{2}\right\} .\right. \tag{3}
\end{equation*}
$$

The decay rate for $\mathrm{K}_{2}^{0} \rightarrow 2 \pi+\gamma$ in the $\mathrm{K}_{2}^{0}$ rest frame is given by

$$
\begin{equation*}
(1 / \tau)\left(\mathrm{K}_{2}^{0} \rightarrow 2 \pi+\gamma\right)=\int(2 \pi)^{4} \delta\left(k^{\prime}+k^{\prime \prime}-q\right)|\mathscr{M}|^{2} \rho_{E} \mathrm{~d} \rho_{E} \tag{4}
\end{equation*}
$$

where by equations (1), (2), and (3)

$$
\mathscr{M}=\frac{1}{\sqrt{ }(2 M)} \cdot \frac{1}{\sqrt{ }(2 q)} \cdot \frac{G}{\kappa_{1}} \epsilon_{a \beta \sigma \tau} e_{a} q_{\beta} k_{\tau} \frac{\delta_{\mu \sigma}}{(k-q)^{2}-\kappa_{1}^{2}} \cdot \frac{g\left(k^{\prime}-k^{\prime \prime}\right)_{\mu}}{\sqrt{ }(2) \sqrt{ }\left(2 \omega^{\prime}\right) \sqrt{ }\left(2 \omega^{\prime \prime}\right)}
$$

with four-vectors $(M, 0),(q, \mathbf{q}),\left(\omega^{\prime}, \mathbf{k}^{\prime}\right)$, and $\left(\omega^{\prime \prime}, \mathbf{k}^{\prime \prime}\right)$ for the $\mathrm{K}_{2}^{0}, \gamma$-ray, and two $\pi$ mesons; furthermore,

$$
\rho_{E} \mathrm{~d} \rho_{E}=\frac{\mathrm{d}^{3} k^{\prime}}{(2 \pi)^{3}} \cdot \frac{\mathrm{~d}^{3} k^{\prime \prime}}{(2 \pi)^{3}} \cdot \frac{\mathrm{~d}^{3} q}{(2 \pi)^{3}} \delta\left(E_{i}-E_{f}\right), \quad E_{f}=\omega^{\prime}+\omega^{\prime \prime}
$$

Replacing $\mathbf{k}^{\prime}+\mathbf{k}^{\prime \prime}=\mathbf{q}, \mathbf{k}^{\prime}-\mathbf{k}^{\prime \prime}=2 \mathbf{p}$ with $\mathrm{d}^{3} k^{\prime} \mathrm{d}^{3} k^{\prime \prime}=\mathrm{d}^{3} p \mathrm{~d}^{3} q$, we find

$$
\begin{equation*}
\left.\frac{\mathbf{1}}{\tau}=\int(2 \pi)^{-5} \frac{g^{2} G^{2} M}{8 \omega^{\prime} \omega^{\prime \prime} \kappa_{1}^{2}} \cdot \frac{p^{4} q^{3} \sin ^{2} \theta}{\left(M^{2}-2 M q-\kappa_{1}^{2}\right)^{2}} \right\rvert\, \frac{\mathrm{d} p}{\left.\mathrm{~d} E_{f}\right|_{E_{i}=E_{f}}} \mathrm{~d} q \mathrm{~d} \Omega_{p} \mathrm{~d} \Omega_{q} \tag{5}
\end{equation*}
$$

where we put

$$
\left|\epsilon_{a \beta \sigma \tau} e_{\alpha} q_{\beta} k_{\tau}(2 p)_{\sigma}\right|^{2}=4 p^{2} q^{2} M^{2} \sin ^{2} \theta
$$

( $\theta$ is the angle between $\mathbf{p}$ and $\mathbf{q}$ ).
Now

$$
\frac{p^{4}}{\omega^{\prime} \omega^{\prime \prime}}\left|\frac{\mathrm{d} p}{\mathrm{~d} E_{f}}\right|_{E_{i}=E_{f}}=\frac{(M-q)^{4}\left(M^{2}-2 M q-4 m^{2}\right)^{3 / 2}}{8\left(M^{2}-2 M q+q^{2} \sin ^{2} \theta\right)^{5 / 2}},
$$

where $m$ is the $\pi$-meson mass. Putting

$$
\mathrm{d} \Omega_{p}=4 \pi, \quad \mathrm{~d} \Omega_{q}=2 \pi \sin \theta \mathrm{~d} \theta
$$

and performing the $\theta$-integration, we find from (5)

$$
\frac{1}{\tau}=\int \frac{(2 \pi)^{-3} g^{2} G^{2}}{24 \kappa_{1}^{2}} \cdot \frac{M\left(M^{2}-2 M q-4 m^{2}\right)^{3 / 2} q^{3} \mathrm{~d} q}{\left(M^{2}-2 M q-\kappa_{1}^{2}\right)^{2}\left(M^{2}-2 M q\right)^{\frac{1}{2}}}
$$

and, after the following substitution

$$
q / M=x, \quad m / M=y, \quad \text { and } \kappa_{1} / M=z
$$

the decay rate $\left(\mathrm{K}_{2}^{0} \rightarrow 2 \pi+\gamma\right)$ reduces to

$$
\frac{1}{\tau}=\frac{g^{2} G^{2}}{(2 \pi)^{3} 24 z^{2} M} \int_{0}^{\left(0 \cdot 5-2 y^{2}\right)} \frac{x^{3}\left(1-2 x-4 y^{2}\right)^{3 / 2} \mathrm{~d} x}{\left(1-2 x-z^{2}\right)^{2}(1-2 x)^{\frac{2}{2}}}=7 \times 10^{4} \mathrm{~s}^{-1}
$$

where we have used

$$
\begin{aligned}
g & =1 \cdot 5 \times 10^{-5}, \text { corresponding to } M_{0}=1 \cdot 1 \mathrm{GeV}, \\
M & =496 \mathrm{MeV}, y=0 \cdot 276, \text { and } z=1.51
\end{aligned}
$$

Thus the value of $\tau\left(\mathrm{K}_{2}^{0} \rightarrow 2 \pi+\gamma\right) \approx 1 \cdot 4 \times 10^{-5} \mathrm{~s}$, as obtained from the present dynamical model, is consistent with the approximate estimation made at the beginning of the paper.

The photon energy spectrum in the $\mathbf{K}_{2}^{0}$ rest frame is given by

$$
\begin{equation*}
f(x)=\frac{x^{3}\left(1-2 x-4 y^{2}\right)^{3 / 2}}{\left(1-2 x-z^{2}\right)^{2}(1-2 x)^{\frac{1}{2}}}, \tag{6}
\end{equation*}
$$

as shown in Figure 2.
From experiment (Abashian et al. 1964; Christenson et al. 1964) we have

$$
\left(\mathrm{K}_{2}^{0} \rightarrow 2 \pi\right) /\left(\mathrm{K}_{2}^{0} \rightarrow \text { all decay modes }\right)=2 \times 10^{-3} .
$$

This ratio corresponds to about half the radiative events, according to the above estimates, or hence to an average photon energy of not quite 100 MeV . Thus, if the $\gamma$-ray energy is really $\lesssim 10 \mathrm{MeV}$, the possibility for the interpretation that the observed event is $\mathrm{K}_{2}^{0} \rightarrow 2 \pi+\gamma$ is very slight. Conversely, if the $\gamma$-ray energy could happen to be as large as 100 MeV , such interpretation may be possible.

This suggests the importance of the experimental study of $\mathrm{K}_{2}^{0} \rightarrow 2 \pi+\gamma$. The energy distribution is approximately a parabola peaked at 120 MeV , and the angular correlation is approximately proportional to $\sin ^{2} \theta$.

## Magnetic $\mathrm{K}_{2}^{0} \rightarrow \mathrm{~K}_{1}^{0}$ Conversion

The fact that $\mathrm{K}_{2}^{0} \rightarrow \mathrm{~K}_{1}^{\mathbf{0}}+$ odd $\gamma$ is in principle only inhibited but not forbidden (Peaslee and Vaughn 1960) suggests a possible interaction of the form

$$
\begin{align*}
H_{\mathrm{int}} & =e A_{\mu} f\left(\square^{2}\right)\left[\phi_{4} \partial_{\mu} \phi_{3}-\phi_{3} \partial_{\mu} \phi_{4}\right]  \tag{7}\\
& =\mathrm{i} e \mathrm{~A}_{\mu} f\left(\square^{2}\right)\left[\phi^{*} \partial_{\mu} \phi-\partial_{\mu} \phi^{*} \phi\right], \tag{8}
\end{align*}
$$

where $\phi=\left(\phi_{3}+\mathrm{i} \phi_{4}\right) / \sqrt{ } 2$ represents a $\mathrm{K}^{0}$ meson.


Fig. 2.-Photon energy spectrum associated with $\mathrm{K}_{2}^{0} \rightarrow$
$2 \pi+\gamma$, where $x$ measures the photon energy in units of average baryon mass.

Electrical neutrality of the $\mathrm{K}^{0}$ meson assumes by equation (8) that $f(0)=0$ but tells nothing about higher moments. If $f\left(\square^{2}\right)$ does not vanish identically, the possibility exists of $\mathrm{K}_{\mathbf{2}}^{\mathbf{0}} \rightarrow \mathrm{K}_{1}^{0}$ conversion on passage through an external magnetic field. One-photon conversion can involve only scalar photons because the $\mathrm{K}^{0}$ meson has spin zero. We write

$$
\begin{equation*}
f\left(\square^{2}\right)=f_{0}+f_{2}\left(\square^{2} / M_{0}^{2}\right)+f_{4}\left(\square^{4} / M_{0}^{4}\right)+\ldots, \tag{9}
\end{equation*}
$$

where $f_{0}=0$ and $M_{0}=1 \cdot 1 \mathrm{GeV}$ is the baryon mass.
Now

$$
\begin{equation*}
\square^{2} A_{\mu}=-j_{\mu} \text {, source of the magnetic field, } \dagger \tag{10}
\end{equation*}
$$

and from equations (7), (9), and (10) we find

$$
H_{\mathrm{int}}=\frac{e f_{2}}{M_{0}^{2}} \cdot \frac{\mathbf{k} \cdot \mathbf{j}(\kappa)}{\omega(2 \pi)^{3 / 2}} \Psi^{2}(R)
$$

where we use

$$
\begin{aligned}
\mathbf{k} \approx \mathbf{k}^{\prime} & =\mathrm{K}^{0} \text {-meson momentum, } \\
\omega \approx \omega^{\prime} & =\mathrm{K}^{0} \text {-meson energy, } \\
\mathbf{j}(x) & =(2 \pi)^{-3 / 2} \int \mathbf{j}(\kappa) \mathrm{e}^{-\mathrm{i} \cdot \mathbf{x}} \mathrm{~d}^{3} \kappa,
\end{aligned}
$$

$\dagger$ In the present calculation only the transverse fringe of the beam, which actually passes through the current coils of the magnets, produces regeneration. The entire beam is taken coherent in the transverse dimension, and the coil acts as the edges of a gigantic slit.
with $\kappa=$ wave vector of magnetic field. Here $\Psi(R)$ is a function describing the spread of the beam, considered as a coherent wave packet in the transverse dimensions (cf. Appendix). The momentum of this wave packet is negligibly disturbed by the $\mathrm{K}_{2}^{0} \rightarrow \mathrm{~K}_{1}^{0}$ transition; therefore the $\mathrm{K}_{1}^{0} \rightarrow 2 \pi$ decay occurs principally in the centre of the beam, although the converting interaction was located in the fringes. If the total path length through the magnetic field is $L$, the probability of $\mathrm{K}_{2}^{\mathbf{0}} \rightarrow \mathrm{K}_{1}^{0}$ conversion is given by

$$
\begin{equation*}
P=(L / \tau)(\omega / k) \tag{11}
\end{equation*}
$$

where

$$
\frac{1}{\tau}=\frac{1}{3}(2 \pi)^{-2}\left(\frac{e f_{2}}{M_{0}^{2}}\right)^{2} \frac{k^{2}}{\omega^{2}} j^{2}(\kappa)\left(\frac{\mathrm{d}^{3} k^{\prime}}{\mathrm{d} E}\right)\left|\Psi^{2}(R)\right|^{2}
$$

Integrating over all wave vectors $\boldsymbol{\kappa}$ present in the field, we get

$$
\begin{equation*}
P=\frac{1}{3}(2 \pi)^{-2}\left(\frac{e f_{2}}{M_{0}^{2}}\right)^{2} \frac{L k}{\omega} \int j^{2}(\kappa) \mathrm{d}^{3} \kappa\left(\frac{\mathrm{~d}^{3} k^{\prime}}{\mathrm{d} E}\right)\left|\Psi^{2}(R)\right|^{2} . \tag{12}
\end{equation*}
$$

Now put

$$
\int j^{2}(\kappa) \mathrm{d}^{3} \kappa=\int \kappa^{2}(\mathbf{j} \cdot \mathbf{A}) \mathrm{d}^{3} \kappa=\left\langle\kappa^{2}\right\rangle 4 \pi \epsilon_{\mathrm{t}}
$$

where $\epsilon_{\mathrm{t}}$ is the total energy in the magnetic field $H\left(\approx 10^{3}\right.$ to $10^{4}$ gauss) and $\left\langle\kappa^{2}\right\rangle=$ $(2 \pi n / l)^{2}$, with $l$ the mean transverse wavelength in the magnetic field and $n$ a numerical factor depending on the nonuniformity of the magnetic field. With $\mathrm{d}^{3} k^{\prime} / \mathrm{d} E=$ $4 \pi k \omega$, the expression for the probability of magnetic conversion becomes (after inserting the $\hbar c$ factors)

$$
\begin{equation*}
P=\frac{1}{3}(4 \pi n)^{2} f_{2}^{2}\left(\frac{L}{l^{2}} \cdot \frac{e^{2}}{M_{0} c^{2}}\right)\left(\frac{\epsilon_{t}}{M_{0} c^{2}}\right)\left(\frac{p_{\kappa}}{M_{0}}\right)^{2}\left|\Psi^{2}(R)\right|^{2}, \tag{13}
\end{equation*}
$$

where $p_{\kappa}=\hbar k$, and each term in the brackets is dimensionless.
For $L=10 \mathrm{ft}, H=10^{4}$ gauss, $\Psi^{2}(R)=10^{-3}$,* $p_{\kappa}=1 \cdot 1 \mathrm{GeV} / c, n f_{2}=3$, and $\left(L / l^{2}\right) \epsilon_{\mathrm{t}}=(1 / 4 \pi)(H L)^{2}$, we get from equation (13)

$$
P=10^{-5} .
$$

This is about the same as for radiative decay with $E_{\gamma} \lesssim 3 \mathrm{MeV}$ and is about $10^{2}$ short of the observed value. On the other hand, $n f_{2} \approx 30$ would explain the observations. (See Peaslee and Vaughn (1960) for a possible value of $f_{2}$.)

The presence of magnetic conversion $\mathrm{K}_{2}^{0} \rightarrow \mathrm{~K}_{1}^{0}$ seems feasible to observe by changing the magnetic field, length of the field, and $n$. Such a measurement might be of interest even if the mechanism is not responsible for the previous observations (Abashian et al. 1964; Christenson et al. 1964). $\dagger$

Failure of CP invariance as a local effect due to dissymmetry of the environment was also suggested by Bell and Perring (1964).

[^1]
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## References

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## Appendix

Transverse beam function $\Psi(R)$, corresponding to a uniform momentum distribution, is

$$
\begin{aligned}
\Psi(R) & =\frac{1}{2 \pi} \int_{0}^{K_{0}}\{\exp (\mathrm{i} K R \cos \theta)\} K \mathrm{~d} K \mathrm{~d} \theta \\
& =\int_{0}^{K_{0}} K \mathrm{~d} K \mathrm{~J}_{0}(K R) \\
& =\left(K_{0} / R\right) \mathrm{J}_{1}\left(K_{0} R\right) .
\end{aligned}
$$

Thus

$$
\Psi(R)=C\left(R_{0} / R\right) \mathrm{J}_{1}\left(3 \cdot 83 R / R_{0}\right),
$$

where $R_{0}=$ beam radius, and $3 \cdot 83=1$ st zero of $\mathrm{J}_{1}$, the Bessel function of order one. The constant $C$ is fixed by the condition

$$
2 \pi \int_{0}^{R_{0}} \frac{R \mathrm{~d} R}{R_{0}^{2}} \Psi^{2}(R)=1
$$

Hence, after numerical integration, we get

$$
C=0 \cdot 64,
$$

and accordingly

$$
\Psi(R)=0 \cdot 64\left(R / R_{0}\right)^{-1} \mathrm{~J}_{1}\left(3 \cdot 83 R / R_{0}\right) .
$$


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[^1]:    * According to the Appendix this corresponds to a magnetic field at a distance of order $R=0.97 R_{0}$ from a beam of radius $R_{0}$.
    $\dagger$ The results of Christenson et al. have been repeated independently at CERN and in England. In every case the decay fraction of $\left(\mathrm{K}_{2}^{0} \rightarrow 2 \pi\right) / \mathrm{K}_{2}^{0}$ was $\epsilon=2 \times 10^{-3}$, although the experimental set-ups must have differed greatly. All electromagnetic regeneration effects vary as $\left(p_{\kappa} / M_{0}\right)^{2} \rightarrow \gamma^{2}=1 /\left(1-\beta^{2}\right)$. Christenson et al. had already made preliminary measurements showing the absence of any $\gamma^{2}$ effect.

