

# THE UPPER LIMITING PRIMARY RIGIDITY OF THE COSMIC RAY SOLAR ANISOTROPY

By R. M. JACKLYN\* and J. E. HUMBLE†

[Manuscript received June 8, 1965]

## Summary

A method of determining the upper limiting rigidity of the solar diurnal variation of the cosmic ray primaries in free space is described. It involves a comparison of the response to the anisotropy of neutron monitors at sea level and of meson telescopes underground. Making use of the model for the free-space first harmonic proposed by Rao, McCracken, and Venkatesan, the annual average value for the upper limiting rigidity ( $R_u$ ) in 1958 is estimated to have been 95 GV with an error of estimate of about 10–20 GV. Changes in the observed annual mean daily variation between 1958 and 1962 indicate that  $R_u$  may have decreased by about 20–40 GV over this period, but a more refined analysis is needed to confirm this.

Changes in  $R_u$  could also influence a sidereal daily variation as observed at the Earth. Evidence is presented for such an effect, from the observations underground at Hobart.

It is shown that the annual average pressure-corrected solar diurnal variation observed at a depth of 40 m.w.e. must be largely under the control of the solar anisotropy. The most significant evidence presented comes from observations in three different directions underground at Hobart during 1961 and 1962. The first harmonics of the solar daily variations observed in these directions have been found to be broadly compatible with the model for the anisotropy proposed by Rao, McCracken, and Venkatesan.

We also present the results of an investigation of the response of an underground telescope to generalized free-space first and second harmonics, for different approximations to the geomagnetic field and for small and large sets of arrival directions.

## I. INTRODUCTION

Recent theories of modulation of galactic cosmic rays within the interplanetary medium (Ahluwalia and Dessler 1962; Parker 1964; Axford 1965) have indicated that charged particles approaching the Earth's orbit with gyro-radii greater than about 1 a.u. would be too energetic to take part in the cosmic ray solar anisotropy, which appears to be responsible for most of the observed pressure-corrected solar daily variation at the Earth. Consequently it has been predicted that the upper limiting rigidity  $R_u$  (the rigidity above which the solar daily variation in free space vanishes) would have been approximately in the vicinity of  $10^2$  GV at the recent maximum of solar activity when the average interplanetary magnetic field strength at the Earth's orbit was a few gammas. At solar minimum  $R_u$  should be somewhat smaller, depending on the nature of the minimum. This solar-cycle dependence should be reflected in changes in amplitude and phase of the solar daily variation.

\* Antarctic Division, Department of External Affairs, Melbourne, and Physics Department, University of Tasmania, Hobart.

† Department of Physics, University of Tasmania, Hobart.

It is also reasonable to suppose that  $R_u$  might be the threshold rigidity for observations at the Earth's orbit of any sidereal anisotropy of the charged primaries, and there could be a significant relationship between the solar and sidereal daily variations because of this common dependence on  $R_u$ . In short, there are a number of reasons why experimental determinations of  $R_u$  are needed at various stages of the solar cycle, in addition to the more fundamental one, that it is one of the constants that characterize the region of modulation. It is proposed to describe here a method of determining annual average values of  $R_u$  within reasonably narrow limits and to report a provisional value for 1958, the year of minimum cosmic ray intensity in the present cycle. This follows an earlier estimate of 200 GV, to within a factor of two, given by Rao, McCracken, and Venkatesan (1963), based on the Hobart underground observations in 1958. The present method depends on the fact that changes in the value of  $R_u$  would have a much greater effect on the solar daily variation observed underground than on the daily variation observed with neutron monitors at high latitude sea level stations.

Towards the end of 1960 two inclined telescopes began operating at the Hobart underground site, in addition to the two vertical semi-cubes. Their axial directions were  $30^\circ$  north of the zenith and  $45^\circ$  south of the zenith respectively, in the plane of the geographic meridian. The whole arrangement would permit a simultaneous scan of the southern sky along three broadly overlapping asymptotic latitude strips. Some results from the three directions during 1961–62 are presented here, partly to indicate how  $R_u$  may have changed since 1958, and also to demonstrate that the first harmonics observed underground at about 40 m.w.e. must have been produced in the greater part by an extraterrestrial anisotropy.

It should be emphasized that the following treatment concerns only the first harmonic of the solar anisotropy. However, it does appear, notably from the work of Sarabhai and his associates (Sarabhai and Subramanian 1963*a*; Subramanian 1963), that a second harmonic must be considered when attempting to describe some of the characteristics of the anisotropy as a whole. There is also evidence that the second harmonic is greater at higher rigidities. We wish to make the point here that, although the free-space daily variation may not be completely described by a first harmonic as Rao, McCracken, and Venkatesan had tentatively concluded, they have shown that the first harmonic has certain definite characteristics that enable an upper limiting rigidity to be worked out for it quite simply. In this instance what is true for the first harmonic might reasonably be expected to apply to the free-space daily variation as a whole.

It is also important to note the argument (Sarabhai and Subramanian 1963*b*) that annual-averaged daily variations fail to provide a realistic picture of the solar anisotropy because the daily variations observed on individual days tend to fall into distinct groups that are attributed to solar anisotropies of differing character. Thus, although on the annual average the index  $\beta$  of variation spectrum may be almost zero, it seems that it is not characteristically zero on individual days. On the other hand, annual averages of upper and lower limiting rigidities and of the amplitude constant of the anisotropy are not open to misinterpretation in the same way. It appears that the two methods of approach are complementary and should lead to

conclusions that are compatible, provided the limitations of each method are understood. Perhaps the greatest single advantage of using an annual average of the solar daily variation is that the influence of a sidereal anisotropy is thereby completely averaged out.

## II. OUTLINE OF A METHOD OF DETERMINING $R_u$

The first harmonic of the pressure-corrected daily variation observed underground in 1958 can now be related to the solar anisotropy of the primaries with some degree of confidence. In the first instance, Rao, McCracken, and Venkatesan, making careful allowance for the asymptotic cones of acceptance of particle detectors, were able to delineate the free-space first harmonic of the anisotropy in 1958, at least so far as to account for the manner in which the first harmonic of the observed annual mean daily variation of neutron intensity depended on the geographic location of the detector. Since adequate coupling coefficients that could be applied to underground detectors of charged particles were not available then, the authors could make only a rough estimate of  $R_u$ . However, recent calculations have indicated that underground telescopes have a mean energy of response that is lower than had been previously thought. Specifically, Fenton (1963) has worked out the yield functions for the pions that are thought to be responsible for the  $\mu$ -meson intensity at the underground depth of 40 m.w.e. Thereby he has obtained the differential coupling coefficients, which give the fraction of the observed counting rate underground due to primaries of any given energy  $E$ . He has found that the coupling coefficients are similar to those derived by Mathews (1963) from an empirical response curve extrapolated from latitude-intensity data. Fenton calculates the mean energy of response to be about 200 GeV, and that less than 10% of the counting rate is due to primaries of energy  $< 50$  GeV. From such information the cone of acceptance technique enables the first harmonic underground to be estimated as a function of  $R_u$ , in relation to the given free-space first harmonic.

Rao, McCracken, and Venkatesan could find no convincing evidence from the neutron data that a semi-diurnal component of extraterrestrial origin existed. Accordingly they specified the anisotropic component of intensity in free space for primaries of rigidity  $R$ , and for a very narrow angle detector, as being simply a first harmonic  $\alpha_1 I_0(R) \cos \lambda \cos(\psi - \psi_0)$ , where  $\alpha_1$  is the amplitude constant,  $I_0(R)$  is the omnidirectional differential intensity of primaries,  $\lambda$  is the ecliptic latitude of the detector,  $\psi$  is the direction of viewing in the plane of the ecliptic, measured eastward of the Earth-Sun line, and  $\psi_0$  is the direction of maximum intensity. Initially the expression given by them also contained the factor  $R^\beta$ , denoting the dependence of amplitude on rigidity, but one of their most important findings was that the annual average amplitude was independent of rigidity in 1958, so that  $\beta = 0$ . On transforming to rotating terrestrial coordinates it can be shown (Jacklyn 1963a) that the corresponding free-space solar diurnal variation is, to a sufficient degree of approximation,

$$\Delta I(R)/I_0(R) = \alpha_1 \cos \delta \cos(\phi - \phi_0), \quad (1)$$

where  $\delta$  is the geographic latitude of viewing,  $\phi$  is the time of day relative to noon, in angular measure (or longitude east of the observer's meridian at noon), and

$\phi_0$  ( $\simeq \psi_0$ ) is the time of maximum intensity. In this context, free space, or the asymptotic region, is the region of modulation situated beyond the effective limits of the Earth's magnetic field.

Now, the fraction of the observed first harmonic underground that is due to anisotropic primaries of rigidity  $R$  will differ considerably from  $\Delta I(R)/I_0(R)$ , partly because at any instant the primaries involved do not approach the Earth's magnetic field from a single direction ( $\delta, \phi$ ) but from a cone of directions that will allow them to arrive at the meson production level within the solid angle of viewing of the recorder, after penetrating the field. The cone is the asymptotic cone of acceptance of which the concept and general method of application to an anisotropy have been fully outlined by Rao, McCracken, and Venkatesan. The present treatment differs somewhat in detail from theirs and is perhaps only appropriate when dealing with high energy primaries. The cone of acceptance is divided up into  $N$  segments specified by intervals of asymptotic latitude. These intervals can be conveniently defined so that each segment of the cone contributes about the same amount to the fraction of the counting rate that is due to primaries of the given rigidity. The contribution  $v(R)$  to the first harmonic of the daily variations underground from primaries of rigidity  $R$  is then approximated by the summation

$$v(R) = \alpha_1 \frac{Y_R}{N} \sum_{n=1}^N A_{Rn} \cos \delta_{Rn} \cos(\phi - \phi_0 + \phi_{Rn}). \quad (2)$$

In this expression  $Y_R$  is the differential coupling coefficient and  $A_{Rn}$  is an amplitude reduction factor determined by the manner in which the contributions to  $Y_R$  from the  $n$ th segment of the cone are distributed in asymptotic longitude. An individual contribution will be defined more precisely below as the "differential radiation sensitivity"  $I(R, \omega)$ . The distribution also determines an effective longitude of viewing  $\phi_{Rn}$  (east of the observer's meridian) with respect to a first harmonic of the anisotropy. The mean asymptotic latitude  $\delta_{Rn}$  is obtained from distribution in asymptotic latitude of the contributions to  $Y_{Rn}$  from the  $n$ th segment of the cone.

The total first harmonic  $v$  is estimated as

$$\begin{aligned} v &= \sum_{R=R_c}^{R_u} v(R) \\ &= \alpha_1 \sum_{R=R_c}^{R_u} \sum_{n=1}^N \frac{Y_R}{N} A_{Rn} \cos \delta_{Rn} \cos(\phi - \phi_0 + \phi_{Rn}), \end{aligned} \quad (3)$$

where  $R_c$  is the lowest primary rigidity to which the recorder can respond, provided it is greater than  $R_L$ , the lower limiting rigidity of the anisotropy. In practice the vector form of  $v(R)$  is more useful, being denoted by

$$\mathbf{v}_R = |v_R| \mathbf{p}_R = \frac{\alpha_1 Y_R}{N} \sum_{n=1}^N A_{Rn} \cos \delta_{Rn} \mathbf{p}_{Rn}, \quad (4)$$

where  $\rho_{Rn}$  is the phase angle on the harmonic dial. In turn, the total first harmonic

vector is

$$\mathbf{v} = |\mathbf{v}|\mathbf{p} = \sum_{R=R_c}^{R_u} |v_R|\mathbf{p}_R. \quad (5)$$

Considerable use is also made of the relative harmonics  $\mathbf{v}_R/\alpha_1$  and  $\mathbf{v}/\alpha_1$ , whose amplitudes are denoted by  $B_{1R}$  and  $B_1$  respectively.

If one substitutes for  $v$  with a reliable observed first harmonic (notably one that is as free as possible from contributions of atmospheric origin), the free-space amplitude  $\alpha_1$  may be estimated from equation (3) for a given value of  $R_u$ . It will be shown that, in relation to the underground observations,  $\alpha_1$  is a steeply varying function of  $R_u$  when  $R_u$  is in the vicinity of 100 GV. On the other hand, when the form of equation (3) that applies to a high latitude neutron monitor is worked out and  $v$  is replaced by the first harmonic of an observed daily variation of neutron intensity, one finds that the variation of  $\alpha_1$  with  $R_u$  flattens off as  $R_u$  increases above 100 GV. Therefore, if  $R_u$  is thought to be about 200 GV, the neutron monitor observations can be used to determine the free-space amplitude  $\alpha_1$  regardless of the exact value of  $R_u$ . This is what Rao, McCracken, and Venkatesan have done. However, the result is less precise if  $R_u$  is closer to 100 GV. In general it is more profitable to find the intersection of the curve specifying  $\alpha_1$  as a function of  $R_u$ , relating to observations of high latitude neutron intensity (curve B in Fig. 3), with the corresponding curve relating to simultaneous observations of meson intensity underground (curve A in Fig. 3). The sharp intersection gives the values of both  $\alpha_1$  and  $R_u$  that apply to the observations. This is the method that shall be used in estimating  $R_u$  for 1958.

### III. METHOD OF DETERMINING $\mathbf{v}/\alpha_1$ AS A FUNCTION OF $R_u$

To calculate the response underground to a first harmonic of the anisotropy we determine sets of values of  $A_{Rn}$ ,  $\delta_{Rn}$ , and  $\phi_{Rn}$  for selected rigidities so that by curve-fitting they may be read off as functions of  $R$ . It should be noted that  $A_{Rn}$ ,  $\delta_{Rn}$ , and  $\phi_{Rn}$  are constants of the detector at the given location and may be used to determine the response to any free-space first harmonic, either solar or sidereal. Corresponding quantities  $A_{2Rn}$ ,  $\delta_{2Rn}$ , and  $\phi_{2Rn}$  relating to a second harmonic may be obtained by the same procedures, and so on for higher harmonics.

Consider the  $\mu$ -mesons, due to isotropic primaries of rigidity  $R$ , which arrive at the detector within a small solid angle  $\omega_r$  specified by zenith angle  $Z_r$  and azimuth  $a_r$ . They constitute a fraction of the total counting rate that is known as the differential radiation sensitivity  $I(R, \omega_r)$  given by

$$I(R, \omega_r) = Y_R F(\omega_r) \cos^n Z_r.$$

The factor  $F(\omega_r)$ , the geometric sensitivity (see Parsons 1957), is the fraction of the total counting rate apportioned to  $\omega_r$  by virtue of the geometry of the telescope, while  $\cos^n Z_r$  expresses the well-known zenith angle dependence of intensity. The primaries of rigidity  $R$  that are responsible for  $I(R, \omega_r)$  will have approached the Earth's magnetic field from directions within some asymptotic volume element  $(\Omega_r)_R$  specified by latitude  $(\delta_r)_R$  and longitude  $(\phi_r)_R$ . Such asymptotic coordinates are usually found either by computing the trajectories of the primaries outward

from the direction  $(Z_r, a_r)$  at the geomagnetic location of the recorder (see McCracken, Rao, and Shea 1962), or else from observations of the deflections of charged particles in a physical simulation of the geomagnetic field (see the terrella experiments of Brunberg and Dattner 1953). When  $I(R, \omega_r)$  and the associated asymptotic coordinates  $(\delta_r)_R$  and  $(\phi_r)_R$  have been calculated for each of the volume elements  $\omega_r$  that make up the solid angle of the recorder, the distribution of the fractional  $\mu$ -meson intensity due to primaries of rigidity  $R$  can be determined with respect to both asymptotic latitude and asymptotic longitude of the primaries. The distribution with latitude can be simply divided up amongst  $N$  latitude intervals giving equal contributions to the differential counting rate, thereby defining the  $N$  segments of the total asymptotic cone. The mean latitudes  $\delta_{Rn}$  are obtained from the latitude distributions within the individual segments. The distribution with respect to longitude within each segment, when applied to a first harmonic  $\cos \phi$ , provides the amplitude reduction factor  $A_{Rn}$  and the longitude displacement angle  $\phi_{Rn}$ . However, it should be noted that the distributions themselves are derived on the assumption of isotropic primaries and an average cosmic ray spectrum.

When the constants  $A_{Rn}$ ,  $\delta_{Rn}$ , and  $\phi_{Rn}$  are applied to a particular model for the anisotropy, the important simplification in the present treatment is that the mean value  $\delta_{Rn}$  replaces a latitude distribution. This suggests that the latitude intervals specifying the segments of the asymptotic cones should be small. Several factors are involved in the choice of intervals, one of them being the manner of variation of the longitude distribution with respect to asymptotic latitude. As far as the underground vertical semi-cube is concerned, the final result does not seem to depend at all markedly on the number of latitude intervals used.

#### IV. APPLICATION TO THE COSMIC RAY DETECTORS

##### (a) *The Vertical Semi-cube Underground*

The solid angle of the semi-cube was divided up into 576 elements ( $\omega_r$ ) of dimensions  $5^\circ$  in azimuth and  $7.5^\circ$  in zenith angle. In calculating the values of  $I(R, \omega_r)$ , Fenton's (1963) coupling coefficients and the geometric sensitivity characteristics calculated by Parsons (1957) were used. Both  $n = 2.0$  and  $n = 2.2$  were tried for the  $\cos^n Z$  zenith angle dependence of intensity, but for all practical purposes they each led to the same final asymptotic distribution.

Since about 90% of the counting rate underground appears to be due to primaries of energy  $> 50$  GeV, it was considered that a centred dipole was a sufficient approximation to the real geomagnetic field for calculations of the asymptotic coordinates  $(\delta_r)_R$  and  $(\phi_r)_R$ . (Calculations using a more complex representation of the field tended to confirm this view and will be mentioned again below.) Consequently, the particle deflections were estimated from the diagrams of Brunberg and Dattner (1953) for the geomagnetic southern latitude of  $50^\circ$ . Considerable interpolation was necessary and for this reason it was found more convenient to express the asymptotic latitude and longitude data in the form of deflections in zenith angle and azimuth.

At the higher rigidities the differential cones of acceptance change rather slowly with rigidity, and therefore it was decided to work them out only for the three most

important rigidities, namely, 50 GV (a practical lower limiting rigidity of response), 150 GV (near the mean rigidity of response), and infinite rigidity (providing an asymptote for the response constants).

Each cone of acceptance was initially divided up into three segments defined by intervals of asymptotic geographic latitude. For example, it was found that, of the 50 GV primaries contributing to the counting rate, one-third came from the latitude range  $90^\circ\text{S.}$  to  $46^\circ\text{S.}$ , one-third from the range  $45^\circ\text{S.}$  to  $28^\circ\text{S.}$ , and the remainder from the range  $27^\circ\text{S.}$  to  $20^\circ\text{N.}$  These high, middle, and low latitude intervals were found to be almost exactly the same at the higher rigidities.

The mean latitude  $\delta_{Rn}$  was worked out for each segment from the distribution  $I(R, \omega_r)$  versus  $(\delta_r)_R$ , after the data had been grouped at  $3^\circ$  intervals of latitude. The distribution  $I(R, \omega_r)$  versus  $(\phi_r)_R$  was based on longitude intervals of  $20^\circ$  and provided the amplitude constant  $A_{Rn}$  and phase constant  $\phi_{Rn}$  when a first harmonic  $\cos \phi$  was impressed on it. In addition, constants  $A_R$ ,  $\delta_R$ , and  $\phi_R$  were obtained for

TABLE I  
ASYMPTOTIC CONSTANTS\* FOR THE VERTICAL SEMI-CUBE UNDERGROUND AT  $50^\circ\text{S.}$  GEOMAGNETIC LATITUDE

Primary Rigidity (GV):	50			150			$\infty$		
	$A_{Rn}$	$\delta_{Rn}$	$\phi_{Rn}$	$A_{Rn}$	$\delta_{Rn}$	$\phi_{Rn}$	$A_{Rn}$	$\delta_{Rn}$	$\phi_{Rn}$
High latitude range (H)	0.84	$-56^\circ$	$25^\circ$	0.75	$-60^\circ$	$12^\circ$	0.74	$-62^\circ$	$0^\circ$
Mid latitude range (M)	0.86	$-36$	35	0.86	$-38$	10	0.86	$-41$	0
Low latitude range (L)	0.91	$-13$	33	0.90	$-15$	12	0.89	$-18$	0
Total cone	0.87	$-34$	30	0.83	$-39$	11	0.83	$-39$	0

\* The constants specify the differential response to a free-space first harmonic at the given primary rigidities.

the total (undivided) cone. Values of the asymptotic constants for individual segments and for the total cone are given in Table I for each of the three rigidities. It can be seen that (a) above 150 GV the cone of acceptance does not change with increasing rigidity except for a gradual displacement towards the observer's meridian, and that (b) the distribution  $I(R, \omega_r)$  versus  $(\phi_r)_R$  changes only slightly with latitude, as would be expected.

Curves of fit, from which  $A_{Rn}$ ,  $\delta_{Rn}$ , and  $\phi_{Rn}$  could be read off against rigidity  $R$ , were drawn using the results from Table I. The vectors  $\mathbf{v}_R/\alpha_1$ , relating to the free-space model of Rao, McCracken, and Venkatesan, were then estimated in accordance with equation (4) as the mean sums of the individual vectors  $(\mathbf{v}_R)H/\alpha_1$ ,  $(\mathbf{v}_R)M/\alpha_1$ , and  $(\mathbf{v}_R)L/\alpha_1$ . Finally, the total first harmonic vector  $\mathbf{v}/\alpha_1$  was estimated as the sum of differential vectors  $\mathbf{v}_R/\alpha_1$  for different values of  $R_u$ .

Now, it was found that if the total vectors  $\mathbf{v}/\alpha_1$  were derived from undivided cones of acceptance, characterized by the constants given in the last line of Table I, they were practically indistinguishable from those calculated from cones that had been divided up into three segments. This greatly simplified the procedure for estimating the response of the vertical semi-cube to free-space harmonics. However,

as we shall see below, such a drastic simplification would not be justifiable in the case of the south-pointing cube underground.

Curves of fit were drawn through the total-cone values  $A_R$ ,  $\delta_R$ , and  $\phi_R$  given in Table 1 and are shown in Figure 1. These curves were used in all subsequent calculations requiring values of the asymptotic constants as functions of  $R$ .

It is of interest at this stage to enquire whether the asymptotic constants would have been very different (*a*) if approximations to the geomagnetic field other than a centred dipole had been used, and (*b*) if the solid angle of the semi-cubical telescope had comprised fewer and larger volume elements. We also note how the

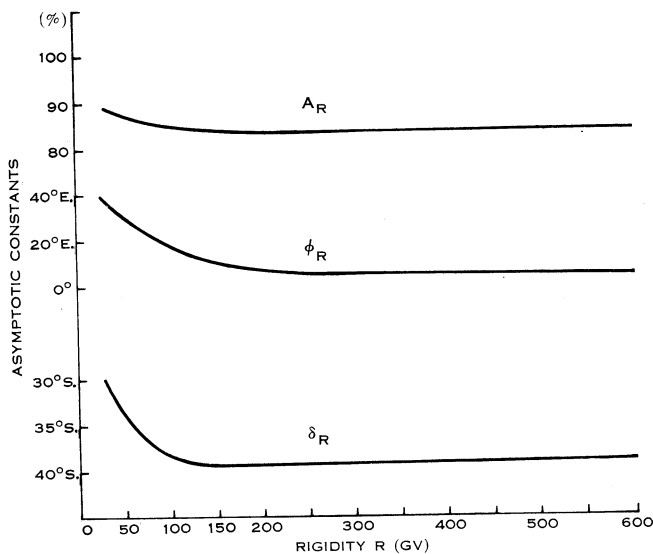


Fig. 1.—The differential asymptotic constants of response to a diurnal variation of the primaries in free space for a vertical semi-cubical telescope at a depth of 40 m.w.e. at  $\lambda = 50^\circ \text{S}$ .

response to a free-space second harmonic would be affected. It was decided to compare three representations of the field: the centred dipole; the Finch and Leaton sixth-order field (Finch and Leaton 1957), as used by Rao, McCracken, and Venkatesan; and the same field ignoring second-order terms and above (approximating an off-centred dipole). In the last two cases the facilities of an Elliott 503 computer were used in the determination of the asymptotic coordinates. For each representation of the field we tried volume elements  $\omega_r$  of small size (about  $10^{-2}$  sr, employed throughout the present paper) and of large size (about  $10^{-1}$  sr, centred on the azimuth  $0^\circ, 45^\circ, 90^\circ, \dots, 315^\circ$  and zenith angles  $5^\circ, 10^\circ, 20^\circ, 30^\circ, 40^\circ, 50^\circ$ , and  $60^\circ$ , 56 elements making up the solid angle of the telescope). Consequently, with three representations of the geomagnetic field and a small and a large number of arrival directions available we were able to compare six sets of the constants  $\delta_R$ ,  $A_R$ ,  $\phi_R$ ,  $A_{2R}$ , and  $\phi_{2R}$ , the last two constants characterizing the differential response to a generalized second harmonic of an anisotropy. Attention was confined to a primary rigidity of 50 GV,



the lowest effective rigidity of response, where the greatest discrepancies would be expected. The first result was that the estimates of  $\delta_R$ ,  $\phi_R$ , and  $\phi_{2R}$  were essentially unaffected. On the other hand, it can be seen from Table 2 that the estimated values of the amplitude factors  $A_R$  and  $A_{2R}$  tended to be smaller when a large number of arrival directions was used, and larger when more complex representations of the Earth's magnetic field were used. The differences relating to the volume elements could depend on the particular arrival directions that are chosen, as well as on the number. We conclude that it is generally advisable to use a large number of arrival directions, particularly for the estimation of the second harmonic constant  $A_{2R}$ . It also appears that the centred dipole field, and specifically the results of the terrella experiments of Brunberg and Dattner, should suffice for present calculations of the response underground to a free-space first harmonic, introducing uncertainties of not more than 2 or 3% in estimates of  $A_R$ .

TABLE 2

ESTIMATES OF RELATIVE AMPLITUDES FOR DIFFERENT GEOMAGNETIC FIELD REPRESENTATIONS AND FOR SMALL AND LARGE NUMBERS ( $N$ ) OF ARRIVAL DIRECTIONS

Values are estimated for the response of a vertical semi-cube at Hobart to first and second harmonics of an anisotropy in the intensity of 50 GV primaries

Geomagnetic Field Representation	First Harmonic $A_R$		Second Harmonic $A_{2R}$	
	$N = 56$	$N = 576$	$N = 56$	$N = 576$
Brunberg and Dattner (1953), centred dipole	0.893	0.874	0.633	0.589
Finch and Leaton (1957), first-order terms only	0.915	0.899	0.676	0.635
Finch and Leaton (1957), sixth order	0.915	0.915	0.704	0.663

In curve 1 of Figures 2(a) and 2(b) the relative amplitude  $B_1$  and the phase angle  $\rho_1$  of the estimated relative first harmonic  $v/\alpha_1$  are shown as functions of the upper limiting rigidity  $R_u$ . The phase angle  $\rho_1$  is measured east of the direction of maximum intensity of the free-space first harmonic, so that negative values denote earlier times of maximum. The curves 1 demonstrate that changes in  $R_u$  have a much greater effect on the estimated amplitude of the first harmonic than on the phase. For example, if  $R_u$  decreases from 200 to 100 GV the amplitude decreases by 50% while there is a phase shift to an earlier time of maximum of only about  $\frac{1}{2}$  hr.

#### (b) *The Inclined Telescopes Underground*

There is an average of about 35 m.w.e. of material absorber within the viewing cone of the north-pointing telescope. This has been estimated from the difference in counting rate between the inclined cube and the vertical semi-cube, having regard to their respective radiation sensitivity patterns. The axial direction of the telescope was set  $30^\circ$  north of the zenith, the equivalent latitude being  $13^\circ$  S. In the axial direction, the telescope views upwards along the magnetic field at Hobart and at the

same time this represents a low latitude of viewing. It is for this reason that the asymptotic acceptance cones are relatively compact, centred only slightly east of the observer's meridian, and are not strongly dependent on rigidity, while the distribution of  $I(R, \omega_r)$  with asymptotic longitude is virtually independent of asymptotic latitude. Consequently the constants  $A_R$ ,  $\delta_R$ , and  $\phi_R$  can be computed from the undivided cones of acceptance with rather less loss of accuracy than in the case of the vertical semi-cube. Following the same procedures as in Section IV(a) above, we determine  $B_{1N}$  and  $\rho_{1N}$  as functions of  $R_u$ . The result is shown in curves 2 of Figure 2.

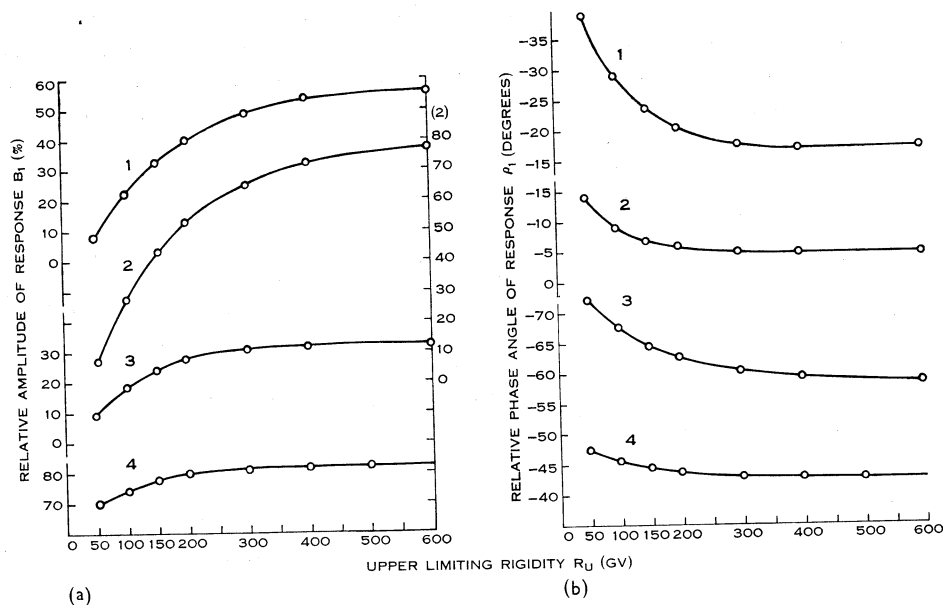


Fig. 2.—(a) The estimated amplitude of the solar diurnal variation, relative to the amplitude of the average free-space first harmonic, and (b) the estimated direction of the solar diurnal variation, measured east of the direction of maximum intensity of the free-space first harmonic, versus upper limiting rigidity for

- 1, a vertical semi-cube at a depth of about 40 m.w.e. at Hobart;
- 2, a cube inclined  $30^\circ$  north of the zenith at a depth of about 40 m.w.e. at Hobart;
- 3, a cube inclined  $45^\circ$  south of the zenith at a depth of about 40 m.w.e. at Hobart; and
- 4, a high latitude neutron monitor at sea level.

Open circles indicate calculated values. The model for the free-space first harmonic is that proposed by Rao, McCracken, and Venkatesan.

It suffices to note here that, in comparison with the response of the vertical telescope, the amplitude of the first harmonic in the north-pointing direction depends more steeply on  $R_u$ , while the phase is rather less dependent on  $R_u$ .

The south-pointing cube underground has an average of about 42 m.w.e. of earth and shale absorber within its field of view. The axial direction of the telescope is  $45^\circ$  south of the zenith and this corresponds to a geographic latitude of  $88^\circ$  S. The situation is the reverse of that in the north-pointing direction. Directions of arrival tend to be transverse to the magnetic field and at higher zenith angles are accessible from all  $360^\circ$  of asymptotic longitude. The cones of acceptance are centred far to the

east of the observer's meridian and change very markedly with primary rigidity. The cones were divided into high, middle, and low latitude segments from which the constants  $A_{Rn}$ ,  $\delta_{Rn}$ , and  $\phi_{Rn}$  were derived, leading finally to the evaluation of  $B_{1S}$  and  $\rho_{1S}$  versus  $R_u$ , shown in curves 3 of Figure 2. It is clear that the transition from low to high axial latitudes of viewing with a cubical telescope is accompanied by the following changes in response to a first harmonic of the anisotropy:

(1) The amplitude increases less steeply with increasing  $R_u$ . This is reflected in the potentially useful relationship  $B_{1S}/B_{1N}$  versus  $R_u$ , which would enable  $R_u$  to be determined independently of  $\alpha_1$ , given a simultaneous pair of observations in the north and south directions. Unfortunately the statistical inaccuracies in the observed ratios are far too great for this method to be of any use at present.

(2) There is a pronounced shift to later times of maximum and a greater dependence of the time of maximum on  $R_u$ .

### (c) *A High Latitude Neutron Monitor*

Rao, McCracken, and Venkatesan have calculated the diurnal relative amplitudes  $B_1$  and the times of maximum for the neutron monitors (see their Table 1) over a rigidity range from near cut-off to 500 GV. In effect then, they have put  $R_u$  equal to 500 GV. The actual value was of little consequence in their work, provided it was not much less than about 200 GV. In what follows we show approximately how  $B_1$  and  $\rho_1$  are affected as  $R_u$  decreases from 500 to 50 GV.

For rigidities  $> 50$  GV the difference of response between a high latitude neutron monitor and the underground semi-cube is due essentially to the very great differences in the coupling coefficients. The differences in the cones of acceptance are relatively minor. Therefore, since a refined treatment for a neutron monitor is unnecessary at this stage, it suffices to apply the same values of  $A_R$ ,  $\delta_R$ , and  $\phi_R$  at the higher rigidities as have been worked out for the semi-cube. Accordingly, the differential first harmonic vectors  $\mathbf{v}_R/\alpha_1$  were estimated from equation (4) for the rigidities 75, 125, 175, 250, 350, and 450 GV as they were for the semi-cube except that the values of  $Y_R$  were those calculated by Dorman (1957) for a high latitude monitor. The amplitude and the time of maximum of the estimated relative first harmonic  $v/\alpha_1$  for Mt Wellington are given by Rao, McCracken, and Venkatesan in their Table 1 and are the values for  $R_u = 500$  GV. Estimates of  $v/\alpha_1$  for lower values of  $R_u$  were therefore obtained by successive subtraction of the differential vectors  $\mathbf{v}_R/\alpha_1$  from the total vector specified in their table. Thereby the values of  $B_1$  and  $\rho_1$  versus  $R_u$  that are shown in curves 4 of Figure 2 were obtained. It can be seen that the estimated relative amplitude begins to fall off rather significantly as  $R_u$  decreases below about 150 GV, but of course much less steeply than the relative amplitude underground. A comparison is given in Table 3.

## V. 1958 EVALUATION OF $\alpha_1$ AND $R_u$

### (a) $\alpha_1$ versus $R_u$ from the Underground Observations at Hobart

The relative amplitude  $B_1$  in curve 1, Figure 2(a), is  $|v|/\alpha_1$ , where  $|v|$  is the amplitude of that part of the observed first harmonic that is caused by the primary

anisotropy. Given a value of  $|v|$  for 1958, estimates of the free-space amplitude  $\alpha_1$  for different values of  $R_u$  can be obtained directly from this curve.

The amplitude of the first harmonic of the annual mean pressure-corrected daily variation underground in 1958 was  $(0.095 \pm 0.008 \text{ S.E.})\%$  and the time of maximum was  $(1712 \pm 20 \text{ min})$  local solar time. It appears that if there were contributions to the first harmonic from daily variations of atmospheric temperature they must have been very small. This is discussed in a paper on the sidereal effect (Jacklyn 1965). We also present evidence below that the diurnal variations observed in three different directions underground are compatible with the model for the free-space first harmonic put forward by Rao, McCracken, and Venkatesan. It is concluded that the result in 1958 was at least very largely of extraterrestrial origin. With  $|v| = 0.095\%$ , we obtain curve A in Figure 3, giving the estimate of  $\alpha_1$  versus  $R_u$  from the underground observations in 1958. The dashed lines give the limits of error arising from the S.E. of estimate of  $|v|$ .

TABLE 3  
ESTIMATED CHANGES IN RELATIVE AMPLITUDE  $B_1$  OF THE FIRST HARMONIC AS UPPER  
LIMITING RIGIDITY  $R_u$  DECREASES

$R_u$	$\Delta B_1/B_1$ (%)			
	250-200	200-150	150-100	100-50 (GV)
Neutron intensity at Mt Wellington	-1	-2	-4	-8
Meson vertical intensity underground at Hobart	-10	-18	-32	-68

(b)  $\alpha_1$  versus  $R_u$  from Neutron Monitor Observations

Having virtually assumed  $R_u$  to be 500 GV, Rao, McCracken, and Venkatesan obtained  $0.4\%$  as their estimate of  $\alpha_1$  for 1958, this being the average of individual estimates from the records of 17 neutron monitors distributed globally. Looking at their Figure 12, for  $\beta = 0$ ,  $g(\lambda) = \cos \lambda$ , we see that the end-points of all but three of the individual estimates of the free-space vector are grouped within a circle of radius  $0.067\%$ , this being  $3\sigma$ , where  $\sigma$  is the S.E. of estimate of amplitude of the diurnal variation at a single station. It seems that, if we regard the radius of the circle as being about twice the S.E. of estimate of  $\alpha_1$  from a single station, it would fairly take account of other random errors. Thus the estimate of  $0.4\%$  would have a S.E. of about  $0.008\%$ . Clearly, if we use the averaged estimate of  $\alpha_1$ , in the product  $\alpha_1 B_1$  we have a much more accurate indication of the amplitude of the first harmonic at Mt Wellington to be expected from the model than would be given by the actual observation at the station. From Table 1 of their paper, we see that  $B_1$  for Mt Wellington is  $0.813$ , so that the estimated amplitude at that station is  $(0.325 \pm 0.006)\%$ . Their Figures 9 and 12 show that the observed amplitude at Mt Wellington was in fact very close to this estimate. Proceeding as in Section V(a) and substituting  $0.325\%$  for  $|v|$ , we obtain estimates of  $\alpha_1$  for different values of  $R_u$  from curve 4, Figure 2(a). The result is shown as curve B in Figure 3. It indicates how the estimate

of free-space amplitude at a high latitude station, derived from world-wide observations of neutron intensity in 1958, would depend on the value assigned to  $R_u$ . The errors due to counting rate statistics would be very small and are not shown. The intersection of curves A and B gives values of  $R_u$  and  $\alpha_1$  that would satisfy the observations of the solar diurnal variation both underground at Hobart and at the neutron monitor stations, in relation to the model. The intersection value of  $R_u$  is 95 GV with an error tail due to counting rate statistics of perhaps 5 GV. The free-space amplitude is about 0.43%, slightly higher than the estimate given by Rao, McCracken, and Venkatesan.

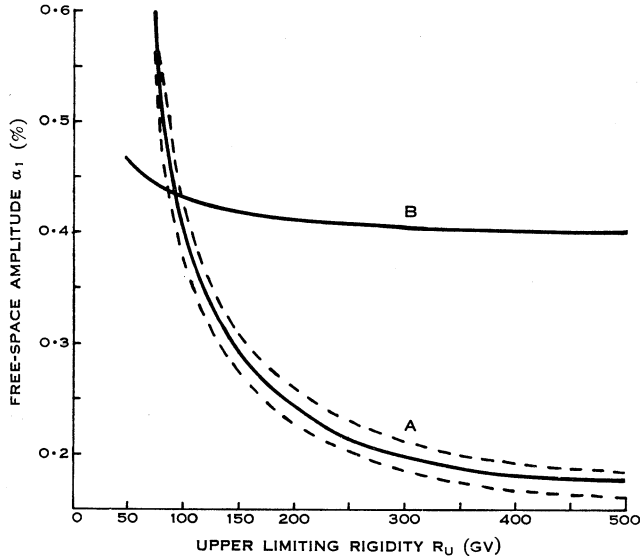


Fig. 3.—Estimates of the amplitude of the average free-space solar diurnal variation of the primaries in 1958 for different values of the upper limiting rigidity based on: A, observations with a vertical semi-cubical telescope at a depth of 40 m.w.e. at Hobart; B, the estimated solar diurnal variation of neutron intensity at a high-latitude sea level station, deduced from world-wide observations of neutron intensity. The average values of  $R_u$  and  $\alpha_1$  for 1958 are given by the intersection of the two curves. All the estimates derive from the model for the free-space first harmonic proposed by Rao, McCracken, and Venkatesan. The dashed lines represent the standard errors of estimate of individual points on curve A.

This result only holds if the average free-space first harmonic is independent of rigidity. Supposing the free-space amplitude to vary as  $R^\beta$ , Rao, McCracken, and Venkatesan showed that  $\beta$  was  $0.00 \pm 0.05$ , averaged over 1958. Therefore we have to find out how our estimates of  $\alpha_1$  and  $R_u$  would be affected if  $|\beta|$  was about 0.05. Accordingly, the curves A and B of Figure 3 were recalculated, starting from equation (3) which now contains the factor  $R^\beta$ , for  $\beta = +0.05$  and for  $\beta = -0.05$ . Essentially the result was that in each case curve A became much steeper for values of  $R_u$  below about 150 GV while curve B was displaced towards lower values of  $\alpha_1$  for  $\beta = 0.05$  and towards higher values for  $\beta = -0.05$ . The intersection values of  $\alpha_1$  and  $R_u$  are shown in Table 4. Note that we now have to refer to  $\alpha_1$  as the free-space amplitude constant since it only becomes the free-space amplitude itself when  $\beta = 0$ .

From these considerations it appears that the error in estimation of  $R_u$  would be about 15 GV. This might even take care of uncertainties in the calculation of curve B that do not apply to curve A. These uncertainties will be mentioned in the Discussion, but it is clear that they would affect the determination of  $\alpha_1$  rather than  $R_u$ .

TABLE 4  
ESTIMATES OF THE UPPER LIMITING RIGIDITY AND THE FREE-SPACE AMPLITUDE  
CONSTANT FOR THREE VALUES OF THE RIGIDITY DEPENDENCE INDEX  
Values are estimated for the first harmonic of the solar anisotropy in 1958

Rigidity Dependence Index $\beta$ :	-0.05	0.00	+0.05
Upper limiting rigidity $R_u$ (GV)	105	95	90
Free-space amplitude constant $\alpha_1$ (%)	0.50	0.43	0.38

## VI. 1961-62. RESULTS FROM THREE DIRECTIONS UNDERGROUND

The four telescopes that have been installed in a disused railway tunnel near Hobart (geographic latitude  $43^\circ$  S.) have already been mentioned in Section I, but it may be useful to summarize here the main features of the arrangement during 1961-62. While the tunnel itself is not quite in the north-south direction, the axes of the inclined telescopes were accurately aligned in the plane of the geographic meridian.

TABLE 5  
CHARACTERISTICS OF THE UNDERGROUND TELESCOPES AT HOBART DURING 1961-62

Telescope Type:	North Single Cube	Vertical Duplex Semi-cube	South Single Cube
Inclination of axis to the zenith	$30^\circ$ N.	$0^\circ$	$45^\circ$ S.
Approximate mean asymptotic latitude of response	$17^\circ$ S.	$39^\circ$ S.	$60^\circ$ S.
Effective absorber excluding atmosphere (m.w.e.)	35	40	42
Particles/hr	16700	70000	8700

Each telescope registered triple coincidences, and individual trays were of 1 square metre sensitive area. Other characteristics of the telescopes are tabulated in Table 5. At the end of 1962 the north-pointing telescope was elongated to give  $20^\circ$  half angle of aperture and the axis was inclined  $70^\circ$  north of the zenith.

The pressure-corrected first harmonic vectors averaged over the two calendar years 1961 and 1962 are shown in Figure 4. The radius of each error circle is  $2\sigma$  so that even in the south-pointing direction the first harmonic is significant. To show that the vector distribution was broadly consistent with an extraterrestrial origin for the diurnal variation underground, it was tested against the model for the anisotropy on the assumption that  $\beta = 0$  and neither  $\alpha_1$  nor  $\psi_0$  had changed appreciably since 1958. Estimates of the first harmonics to be expected in the three directions were

calculated as functions of  $R_u$ . The results are shown as dashed lines in the figure. It can be seen that the phase differences between V, N, and S are compatible with the model although the observed time of maximum for N is later than expected. The amplitudes are also those to be expected from the model but they indicate that the average value of  $R_u$  must have been about 70 GV during 1961–62 if the other constants had not changed. Alternatively, if  $\alpha_1$  alone had changed, it must have decreased by about 40% since 1958. The estimates of  $\alpha_1$  versus  $R_u$  that were derived from each result demonstrate the interrelationships more clearly (Fig. 5). It is quite clear that the same estimate is given by each of the three independent experiments, involving detectors of different geometry and greatly different effective latitudes of scan.

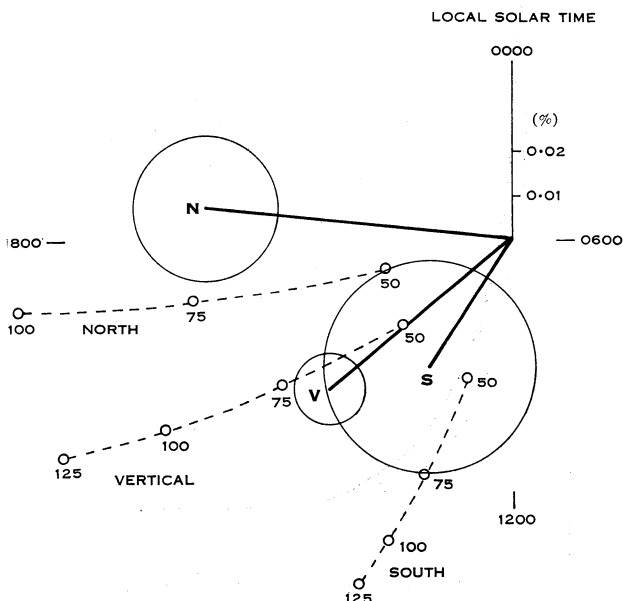


Fig. 4.—The first harmonic vectors of the pressure-corrected daily variations observed in the north (N), vertical (V) and south-pointing (S) directions underground at Hobart, averaged over the years 1961 and 1962. The radius of each error circle is  $2\sigma$ . The dashed lines give estimates of the first harmonics to be expected in each of the three directions, for different values of upper limiting rigidity. Some values of interest are shown in GV. The estimates are based on the model for the free-space solar diurnal variation of the primaries proposed by Rao, McCracken, and Venkatesan.

## VII. 1958–62. SOME RESULTS FROM OTHER PLACES

The large decrease of amplitude of the solar diurnal variation that has taken place at the Hobart underground site since 1958 was confirmed by results from the vertical semi-cubical telescopes located at a depth of 40 m.w.e. at Budapest (geographic coordinates  $47.5^\circ\text{N.}$ ,  $18.9^\circ\text{E.}$ , geomagnetic latitude  $46.4^\circ\text{N.}$ ). The only complete years available from Budapest were 1959 and 1961. The comparison with Hobart for these 2 years, given in Table 6, shows unmistakable agreement except for the differences in time of maximum in 1961.

Also of relevance are some interesting results reported by Sarabhai and Subramanian (1963*a*). They have found that the peak-to-peak amplitude of the total solar daily variation (sum of first and second harmonics) of neutron intensity at a low latitude station (Huancayo) had decreased by about 35% between 1958 and 1961–62, while at a high latitude station (Churchill) it had decreased by about only 5%. There were also interesting shifts of the times of maximum and minimum to earlier hours at Huancayo but not at Churchill. The results from a cubical meson telescope at Churchill were between those from the two neutron monitors. Sarabhai

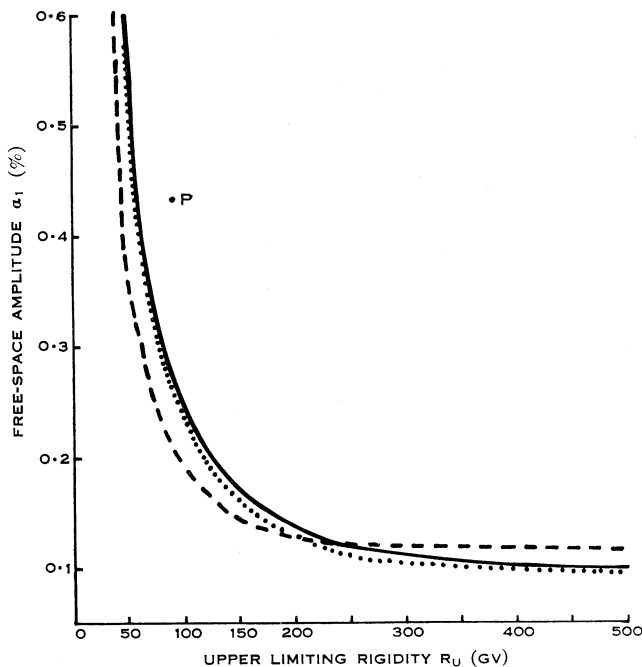


Fig. 5.—Estimates of the amplitude of the free-space solar diurnal variation of the primaries, averaged over 1961 and 1962, for different values of  $R_u$  and for  $\beta = 0$ , based on observations underground at Hobart with (a) the vertical semi-cubes, full line; (b) the north-pointing cube, dotted line; (c) the south-pointing cube, dashed line. The point P represents the intersection of curves A and B for 1958 (Fig. 3).

and Subramanian suggested that the observed changes could have come about if  $\beta$  had become increasingly negative or if  $R_u$  had decreased, but that a decrease in the lower limiting rigidity  $R_L$  could not have been the main cause. Also, their results do not seem to be consistent with a large decrease in the amplitude constant  $\alpha_1$ . This would have produced similar decreases of total amplitude at all stations, bearing in mind that the part played by the second harmonic in their observations is unknown. Finally, changes in the galactic spectrum that might have been expected over the period (see Mathews and Kodama 1964) would clearly not have been responsible for an effect that seems to increase with increasing primary rigidity of response.



# VIII. RELEVANT CHANGES IN THE APPARENT SIDEREAL EFFECT OBSERVED UNDERGROUND

Every year since experiments commenced at the Hobart underground site a significant daily variation in sidereal time has been observed. A general account of the results obtained over the period 1958–62 inclusive has been published recently (Jacklyn 1965). It is submitted in that paper that most, if not all, of the sidereal effect observed at the depth of 40 m.w.e., is genuine. Figure 6 shows the annual running averages of the amplitudes of the total daily variations in solar and sidereal time. As noted in Sections VI and VII, the amplitude in solar time has decreased considerably over the period and it is evident that most of the change took place after December 1960. It seems that the amplitude in sidereal time also decreased somewhat after 1958, but that it began to increase again during 1961 and passed

TABLE 6  
FIRST HARMONICS OF MEAN PRESSURE-CORRECTED SOLAR DAILY VARIATION OF VERTICAL INTENSITY AT BUDAPEST AND HOBART DURING 1959 AND 1961

Values were obtained from semi-cubical telescopes at a depth of 40 m.w.e.

	1959		1961	
	Amplitude (%)	$T_{\max}$ (local solar time)	Amplitude (%)	$T_{\max}$ (local solar time)
Budapest*	0.1114	1640	0.0389	1554
S.E.	$\pm 0.0054$	$\pm 12$ min	$\pm 0.0054$	$\pm 34$ min
Hobart†	0.1062	1638	0.0389	1408
S.E.	$\pm 0.0058$	$\pm 12$ min	$\pm 0.0058$	$\pm 34$ min

\* Geomagnetic latitude  $46^\circ$  N.

† Geomagnetic latitude  $50^\circ$  S.

through quite a conspicuous maximum at the same time as the amplitude of the solar daily variation passed through a minimum, and a few months before the daily intensity reached a maximum value. Over a period of about 6 months centred on the turning points of the two amplitude curves the genuineness of the sidereal effect was strikingly demonstrated; the time of maximum of the monthly mean daily variation in solar time changed from 1500 to 0700 solar time and became earlier month by month at about the rate of 2 hr each month, in the manner of a sidereal daily variation expressed in solar times. In other words, the sidereal component of the daily variation was dominating the solar component over this period. Accounts of this phase anomaly have been given elsewhere (Jacklyn 1963*b*, 1965). Now, the point we wish to make here is that the changes in amplitudes of the solar and sidereal components over this period were probably not negatively correlated by chance but were connected through opposite dependences on changes in  $R_u$ . On the one hand, a decrease in  $R_u$  is a decrease in upper limiting rigidity leading to a decrease of amplitude of the solar component. On the other hand, it is a decrease in threshold rigidity leading to an increase of amplitude of the sidereal component. The effect on the sidereal component (assumed to be observable over a very wide range of rigidities above  $R_u$ ) would not be expected to be very great until  $R_u$  had decreased to values below 100 GV,

where the differential response of the underground detector is greatest. Calculations based on a simple model for the sidereal anisotropy show that the observed increase of amplitude could be explained if  $R_u$  had decreased from, say, 85 to 50 GV over the period of increase, although it might be necessary for the spectrum of variation (for the sidereal effect) to be somewhat negatively dependent on rigidity.

If, alternatively, the changes in amplitude of the solar component were primarily due to  $\beta$  becoming increasingly negative on the average, then there would be no simple explanation for the correlated changes in amplitude of the sidereal component.

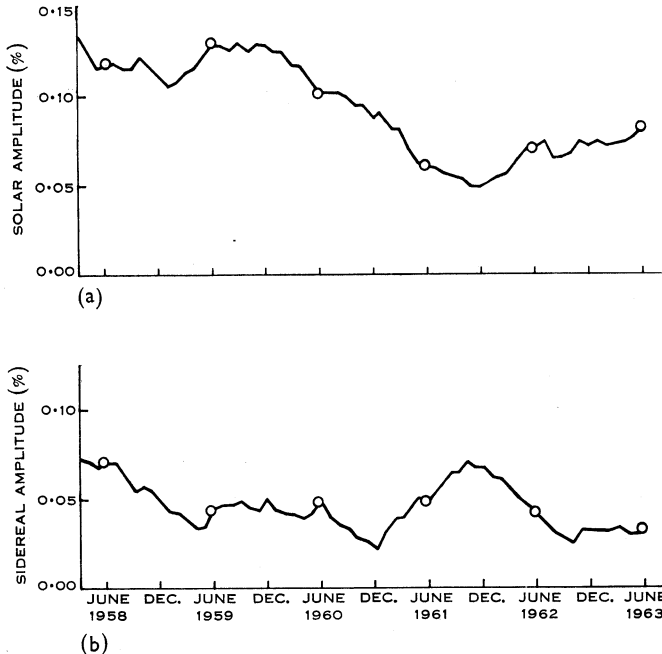


Fig. 6.—Long-term changes observed with the vertical semi-cubical telescopes at the Hobart underground station; month by month annual running averages of (a) amplitude of the solar daily variation, (b) amplitude of the apparent sidereal daily variation. The dates refer to the central months and the open circles indicate the conventional annual mean values.

### IX. DISCUSSION

The fact that the amplitude, but not the phase, of the first harmonic of the solar daily variation underground changed so greatly between 1958 and 1962 is itself evidence that the observed daily variation at this depth must be very largely of extraterrestrial origin. Of greater significance, however, is the evidence that the amplitudes and times of maxima in all three directions underground are compatible with the model for the free-space first harmonic derived from the neutron monitor data. In addition to this there were indications in 1961 that simultaneous changes in the solar and apparent sidereal daily variations were connected. Overall, it seems reasonable to assume that the observed pressure-corrected first harmonic at 40 m.w.e.

is almost entirely due to the anisotropy, a prerequisite for the present method of determining  $R_u$ . We have seen that the method depends essentially on the fact that the differential coupling coefficients ( $Y_R$ ) for the underground semi-cube differ very greatly from the coupling coefficients that apply to a high latitude neutron monitor. This is the main reason for the difference between curves A and B in Figure 3. Moreover, the nature of the difference is such that quite sizeable errors in curve B will not greatly affect the estimation of  $R_u$ .

It would seem that more accurate knowledge of the deflections of high energy particles in the Earth's magnetic field, improved estimates of the radiation sensitivity patterns of the underground telescopes, or a more refined treatment of the asymptotic cones of acceptance are not likely to affect curve A very much. The reliability of the coupling coefficients is a more important factor. It seems unlikely that the coefficients used here would be greatly in error since the values worked out by Fenton (1963) are in reasonable agreement with Mathews's (1963) results obtained by empirical methods. The coefficients are only strictly applicable at vertical incidence where the meson threshold energy at production, for detection at 40 m.w.e. underground, is 15 GeV. However, in the case of the vertical semi-cube they should apply quite well up to the limiting zenith angle of about  $60^\circ$ , where the meson threshold energy has only increased to 18 GeV.

The accuracy of estimation of curve B, relating to the Mt Wellington neutron monitor, should be sufficient for this provisional determination of  $R_u$ . However, in a detailed study of year to year changes in  $R_u$ ,  $\alpha_1$ , and  $\beta$ , other quantities would be important at the low rigidities of response. It is evident from the work of Mathews and Kodama (1964), for example, that the differential coupling coefficients at rigidities below 15 GV must change considerably during the cycle of solar activity. Therefore, at the least, it might be necessary to use different average cosmic ray spectra at solar maximum and solar minimum when analysing the neutron monitor data.

It is conceivable that  $\beta$ , the index of the variational spectrum, changes with increasing rigidity, although *a priori* there is no reason to suppose that this happens. Indeed, Rao, McCracken, and Venkatesan showed that any trend of this kind in 1958 would have been difficult to detect since  $\beta$  was found to be within the limits  $0.0 \pm 0.05$  at least over the rigidity range 1–100 GV. To provide against the possibility, however, it would be an advantage to have additional estimates of  $\alpha_1$  versus  $R_u$  from detectors whose mean primary rigidities of response were between those of neutron monitors and the telescopes at 40 m.w.e. Unfortunately, as the amount of material absorber decreases, the atmospheric negative temperature effect in particular contributes increasingly to the daily variation of meson intensity. At many places, daily variations of atmospheric structure are difficult to estimate and may vary greatly from place to place (see Kane 1963). One way to avoid this problem is to make use of results from the crossed telescopes that operate at Hobart, Mawson, and a number of other places. Even then such data may prove to be of limited value until the differential responses of inclined telescopes at sea level to primaries of rigidity  $> 15$  GV are more reliably known.

So far it has been tacitly assumed that  $R_u$  represents both the upper limiting rigidity of the solar anisotropy and the threshold rigidity for observation of a sidereal

effect. This is not necessarily true of course. In the case of the sidereal anisotropy we are interested to know the rigidity above which primaries have access to the Earth's orbit from some particular direction in space, without having been appreciably scattered. On the other hand, if we accept Parker's recent theory (Parker 1964) that the solar diurnal variation is produced essentially by streaming of cosmic rays with an azimuthal velocity of rigid rotation with the Sun, then, as he points out, two conditions for the upper limiting rigidity must be satisfied.  $R_u$  must be such that (a) the gyro-radius is less than 1 a.u., so that the guiding centre approximation holds for the motion of particles along the spiralling interplanetary field, and (b) the gyro-radius is small enough to allow the particles to be scattered several times beyond the orbit of the Earth as they approach, so that a density gradient of intensity would be largely destroyed. In these circumstances there are clearly several possibilities whereby  $R_u$  could differ from the threshold rigidity for observation of a sidereal effect. Nevertheless, it seems reasonable to assume as a starting point that the two are not very different and that changes in  $R_u$  would be accompanied by similar changes in the sidereal threshold rigidity, but perhaps not vice versa. Moreover, judging from the evidence for a sidereal effect obtained from neutron monitor data by Conforto and Simpson (1957) at the previous sunspot minimum, the average threshold rigidity for observation should usually exceed the lower limiting rigidity of the sidereal anisotropy.

#### X. CONCLUSION

The pressure-corrected solar daily variation at 40 m.w.e. appears to be very largely under the control of the primary anisotropy, as evidenced by the nature of the year to year variations of the first harmonic observed at Hobart and Budapest in the vertical direction and by the characteristics of the first harmonic observed in three different directions underground at Hobart. It would follow that a component of atmospheric origin must be small and this is supported by other evidence, which is discussed in an earlier paper (Jacklyn 1965).

The provisional estimate of the average value of  $R_u$  in 1958 is 95 GV, this being the average primary rigidity above which a first harmonic of the anisotropy would become unimportant. The uncertainty of 10–20 GV in the estimate refers only to the method of measurement and has nothing to do with any inherent variability in  $R_u$ .

It appears that changes in the annual average solar diurnal variation between 1958 and 1962 would have been effected mainly through one or more of the constants  $R_u$ ,  $R_L$ ,  $\alpha_1$ ,  $\beta$ , and  $\psi_0$  of the average free-space first harmonic, and through the average primary spectrum from which the coupling coefficients are derived. All these quantities may have varied to some extent, but it is interesting to note that only a decrease in  $R_u$  (of about 20–40 GV) could by itself have brought about the observed decrease in amplitude of the solar diurnal variation underground, the negatively correlated effects in solar and sidereal time underground in 1961, and the type of change observed in the solar daily variation of neutron intensity at Huancaayo relative to that at Churchill.

A more refined analysis of long-term changes should take into account the variability of each of the parameters specifying the free-space first harmonic. It

would also be necessary to clarify the part played by the second harmonic. Some of the factors (e.g. changes in  $R_L$ ) should only affect the daily variation of neutron intensity, while others (e.g. changes in  $R_u$ ) produce the most noticeable effects underground. Therefore a comprehensive comparative study of year to year changes in the daily variation of neutron intensity and of the underground meson intensity should permit the most important of these factors to be identified and estimated. A project of this type is being undertaken at Hobart. To achieve the desired result many years of continuous and reliable data are needed. Of particular importance are the records from the few neutron monitors at the very high and very low latitudes.

## XI. ACKNOWLEDGMENTS

The authors wish to thank Dr. A. G. Fenton for providing the coupling coefficients for the underground telescopes in advance of publication. We thank the Director of the Cosmic Ray Department of the Central Research Institute of Physics of the Hungarian Academy of Sciences for the underground data from Budapest, made available through World Data Centre WDC 2. One of us (J.E.H.) is grateful to General Motors-Holden's Ltd. for the award of a Research Fellowship during the period of the present work. We are indebted to the Hydro-University Computing Centre at Hobart for the use of the facilities of the Elliott computer.

## XII. REFERENCES

- AHLUWALIA, A. S., and DESSLER, A. J. (1962).—*Planet. Space Sci.* **9**: 195.  
 AXFORD, W. I. (1965).—*Planet. Space Sci.* **13**: 115.  
 BRUNBERG, E. A., and DATNER, A. (1953).—*Tellus* **5**: 269.  
 CONFORTO, A. M., and SIMPSON, J. A. (1957).—*Nuovo Cim.* **6**: 1052.  
 DORMAN, L. I. (1957).—"Cosmic Ray Variations." p. 117. (State Publishing House for Technical and Theoretical Literature, Moscow.) (Translation by Technical Document Liaison Office, U.S. Air Force.)  
 FENTON, A. G. (1963).—Proc. Int. Conf. Cosmic Rays, Jaipur, India. Vol. 2, p. 185.  
 FINCH, H. P., and LEATON, B. R. (1957).—*Mon. Not. R. Astr. Soc., Geophys. Suppl.* **1**: 314.  
 JACKLYN, R. M. (1963a).—*Nature* **200**: 1306.  
 JACKLYN, R. M. (1963b).—Proc. Int. Conf. Cosmic Rays, Jaipur, India. Vol. 2, p. 345.  
 JACKLYN, R. M. (1965).—*Nuovo Cim.* **37**: 1135.  
 KANE, R. P. (1963).—*Ind. J. Phys.* **37**: 151.  
 MCCracken, K. G., RAO, U. R., and SHEA, M. A. (1962).—Tech. Rep. Lab. Nucl. Sci. Engng M.I.T. No. 77 (NYO-2670).  
 MATHEWS, T. (1963).—*Phil. Mag.* **8**: 387.  
 MATHEWS, T., and KODAMA, M. (1964).—*J. Geophys. Res.* **69**: 4429.  
 PARKER, E. N. (1964).—*Planet. Space Sci.* **12**: 735.  
 PARSONS, N. R. (1957).—*Rev. Scient. Instrum.* **28**: 265.  
 RAO, U. R., MCCracken, K. G., and VENKATESAN, D. (1963).—*J. Geophys. Res.* **68**: 345.  
 SARABHAI, V. A., and SUBRAMANIAN, G. (1963a).—Proc. Int. Conf. Cosmic Rays, Jaipur, India. Vol. 2, p. 307.  
 SARABHAI, V. A., and SUBRAMANIAN, G. (1963b).—Proc. Int. Conf. Cosmic Rays, Jaipur, India. Vol. 2, p. 405.  
 SUBRAMANIAN, G. (1963).—Proc. Int. Conf. Cosmic Rays, Jaipur, India. Vol. 2, p. 302.

