TRANSPORT AND IONIZATION PROPERTIES OF MOLECULAR GASES IN A TRANSVERSE MAGNETIC FIELD

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Summary

Measurements are reported of pre-breakdown ionization in $E \times B$ fields under conditions that permit unrestricted motion of the electrons in the $E \times B$ direction. These measurements have been carried out in a coaxial cylindrical ionization chamber, details of which are given. An alternative method of examining the development of ionization as a function of pressure has been used, and a comparison is made of the results of such investigations with earlier spatial measurements using plane geometry. The use of an electromagnet capable of producing magnetic fields up to 10 000 G substantially uniform over the required constant annular gap spacing has permitted an extension of earlier work to the "strong" magnetic field limit, i.e. values of $B/p \simeq 2500$ G/torr(0°C). Details of measurements made in the molecular gases H₂ and N₂ are given.

I. INTRODUCTION

Considerable progress has been made recently in understanding the complexities of the transport and ionizing properties of electrons in partially ionized gases. Phelps and his colleagues (Frost and Phelps 1962; Engelhardt and Phelps 1963; Engelhardt, Phelps, and Risk 1964) have, by means of a numerical solution of the Boltzmann equation, derived elastic and inelastic cross sections for electrons in a number of gases and, by comparing experimental and theoretical values of transport coefficients, have investigated in some detail the cross sections for momentum transfer, rotational and vibrational excitation, electronic excitation, and ionization. Their analysis has been confined to low values of the parameter E/N, where E is the applied electric field strength and N is the neutral molecule number density, corresponding to a region where the Lorentz approximation is valid. This is in essence equivalent to assuming that the distribution function is substantially spherically symmetric, and for hydrogen this sets an upper limit to E/N of approximately 1.5×10^{-15} V cm² or $E/p \sim 50$ V cm⁻¹ torr⁻¹ at room temperature. Although the results of their analysis are not based on a unique solution of the Boltzmann equation, nevertheless the elastic and inelastic collision cross sections so derived form a consistent and realistic set which can now be subjected to further experimental tests.

In the particular case of molecular hydrogen, a check on the derived cross sections has been possible by an examination of the corresponding transport coefficients measured in alternating electric fields and in crossed electric and magnetic fields. This has revealed a number of inconsistencies that are as yet unexplained. One of these concerns an "effective field" plot of measured values of primary ionization coefficients in $E \times B$ fields (Engelhardt and Phelps 1963). In this plot, Bernstein's

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measured values of α/p (Bernstein 1962) have been converted into an ionization frequency ν_i/N and plotted against a parameter E_{eff}/N , where

$$E_{\rm eff} = E\{1 + (\omega^2/\nu^2)\}^{-\frac{1}{2}},\tag{1}$$

with $\omega = eB/m =$ electron cyclotron frequency, $\nu =$ some effective collision frequency.

In this way, a comparison can be made not only of the crossed field data but also of steady field and corresponding alternating field data with the theoretically predicted values. As Engelhardt and Phelps (1963) point out, however, "his [Bernstein's] results for non-zero magnetic fields fall distressingly far from the [predicted] curve". This has raised doubts about the interpretation and reliability of the measurements themselves, particularly since the techniques and analytical methods adopted by Bernstein did not permit great accuracy in the determination of the primary ionization coefficients.

Precise determinations of these same primary ionization coefficients are also of special importance in assessing the validity of an equivalent pressure approach to the problem of the influence of a transverse magnetic field on the ionizing properties of a swarm of electrons. It has been shown previously that the application of crossed electric (E) and magnetic (B) fields to a partially ionized gas has the same effect on some of the transport properties of the electrons in that gas as would an increase in the gas pressure p. Several theoretical attempts have been made to explain pre-breakdown phenomena in terms of this "equivalent pressure" p'. The simplest treatment (Blevin and Haydon 1958) assumes a Maxwellian distribution of electron velocities and a constant mean free time $\tau = l/u$, where l is the mean free path and u the electron velocity. This leads to the result that, when the transverse magnetic field does not alter the form of the velocity distribution function, the equivalent pressure is given by

$$p' = p\{1 + (\omega^2/\nu^2)\}^{\frac{1}{2}},\tag{2}$$

where again ν has the dimension of a collision frequency. When l/u is not constant, however, then ν can only be regarded as some *effective* collision frequency, its precise interpretation depending on both the relationship l = F(u) and the actual distribution of electron velocities (Haydon 1966). When this is the case, a value for the equivalent pressure p' may be obtained from equation (2.14) of Blevin and Haydon (1958), namely,

$$p'/p = W_{0,E/p'}/W_{B/p,E/p}.$$
(3)

At present, direct experimental values of drift velocities are not available for an extensive range of the parameters, and a detailed knowledge of the collision cross section for momentum transfer is required before equation (3) can be evaluated.

If the assumption that the form of the velocity distribution remains unaltered is correct, then it should be possible to obtain the same equivalent situation corresponding to p' irrespective of the particular values of B/p and E/p actually applied; that is, the value of the effective collision frequency ν of equation (2) should be a function only of E/p' and not of E/p or B/p separately. It seems less likely that the form of the distribution will remain unaltered the lower the mean energies of the electrons, and for this reason there is a great deal of interest in obtaining precise measurements of transport and ionization coefficients in this region.

The purposes of the present paper are therefore threefold. First, to establish the validity of an alternative method for studying the primary ionization in $E \times B$ fields that is capable of considerably greater accuracy than that used by Bernstein; secondly, to report measurements of the ionization coefficients in molecular hydrogen and nitrogen over a greatly extended range of values of E/p and B/p than previously reported; and thirdly, to re-examine, in the light of these measurements, the "equivalent field" or "equivalent pressure" approach to the influence of a transverse magnetic field.

II. APPARATUS AND PROCEDURE

Previous investigations in this laboratory of the pre-breakdown ionization in $E \times B$ fields have been confined to the use of a plane-parallel electrode system in which the interelectrode gap distance was varied whilst maintaining the gas pressure constant. Values of the ratio of magnetic field to gas pressure (B/p) were confined to ≤ 300 G/torr; lateral loss of electrons from the ionization region prevented measurements being made over an extended range of values of B/p and E/p. Bernstein overcame the latter restriction by using coaxial cylindrical electrodes and obtained his variable interelectrode spacings by substituting cathodes of different diameters. The method therefore involved a series of separate experiments in each of which the surface conditions and the uniformity of the applied electric fields were not identical. The initial photoelectric current obtained by irradiating the cathode surface with ultraviolet light from an external source was found to vary by as much as 20% from one cathode surface to another, and in some cases it was not possible to quote values of α/p to better than 60%. Thus, although it was possible by the use of this geometry and large magnetic fields to extend the investigation to cover a very wide range of parameters, there were still many limitations. One particularly important limitation was the neglect of any consideration of the secondary ionization in the analytical procedures adopted for evaluating α/p from the pre-breakdown measurements.

In the present investigations cylindrical geometry has been used, and, in order to avoid the complication associated with varying the interelectrode spacings, the pre-breakdown ionization growth has been examined as a function of the gas pressure with constant gap separation d. Since the important parameter in development of the pre-breakdown ionization is the product pd rather than p or d separately, the procedure should be valid, and this was in fact shown to be the case with planeparallel electrodes (Haydon and Robertson 1961) by measuring a value for α/p by each of the two methods. No significant difference in the values was obtained, and the ionization chamber used in this investigation was designed on the basis of the results of those experiments. The chamber consists of two concentric copper cylinders each split into three sections (Fig. 1). The central concentric cylinders A and B comprised the main electrodes, the outer cylinder B being the anode. Situated on each side of both central electrodes were cylindrical guard rings C, D, E, F, separated from the main electrodes by precision-ground $\frac{1}{8}$ in. sapphire balls G held in grooves on the edges of the electrodes. Ultraviolet radiation incident upon the cathode through forty-nine 0.5 mm holes drilled in the anode released electrons, whose subsequent behaviour and that of their progeny was investigated in the crossed field configuration. The complete electrode assembly was tensioned, by means of springs S, between two nonmagnetic stainless steel end plates H, and the whole chamber was enclosed in a Pyrex glass envelope. This envelope was tensioned between two brass end plates by means of eight tie rods. Indium was used as the vacuum seal. The



Fig. 1.—Ionization chamber for measurement of gas-amplified currents in $E \times B$ fields. A, cathode; B, anode; C, D, E, F, guard rings; G, precisionground $\frac{1}{8}$ in. sapphire balls; H, non-magnetic stainless steel end plates; S, springs.

interelectrode gap separation d was measured as 0.3390 ± 0.0012 cm, with cathode diameter 9.60 cm and anode diameter 10.26 cm. The nonuniformity of the applied electric field in these circumstances was sufficiently small (2% across the gap) to be considered acceptable, and this was confirmed by the subsequent measurements. Any further reduction in the gap spacing introduces distortions of the applied electric field because of the holes in the anode and also gives rise to other nonequilibrium phenomena (Haydon and Robertson 1961). Any further increase in diameter, on the other hand, raises problems of fabrication and with the uniformity of the magnetic fields. Nonmagnetic material was used throughout the ionization chamber and vacuum envelope.

The electrode system was heat-treated before assembly to allow vacuum degassing if necessary up to 150°C. Using a two-stage backing pump in series with an oil diffusion pump and with extensive liquid-air trapping, a base pressure of 10^{-6} torr was obtained with an isolated leak rate of 0.05 mtorr/hr. Cylinder hydrogen was admitted to the experimental system through a palladium osmosis tube; cylinder nitrogen was admitted through a very fine porous plug. These methods were the same as those used in the earlier investigations and were retained at this stage so that direct comparisons could be made. Although ultrahigh vacuum techniques are required for reliable measurements in the inert gases, it should be stressed that no



Fig. 2.—Variation of magnetic field between pole faces (i) radially across the magnet air gap in a plane containing the mid point between the two pole faces (also shown is the location of the annular gap, anode–cathode), and (ii) axially across the magnet air gap along a line passing through the ionization region at 5 cm from the axis of symmetry (also shown are the axial lengths of the inner cathode and outer anode).

evidence has been obtained in these laboratories to indicate that the problem of impurities is a severe one in the molecular gases hydrogen and nitrogen, at least in the region of interest in the present investigations.

Variable potentials from 0 to 1 kV could be applied to the anode of the chamber from a Fluke Model 301C/N power supply which also measured the applied voltage to $\pm 0.1\%$. Ionization currents of the order of 10^{-12} - 10^{-9} A were measured with a Keithley Model 200B electrometer having an input impedance of $10^{14} \Omega$.

Pressures in the range 0–10 torr were measured with a simple U-tube oil manometer containing DC 704 silicone oil. A continuously variable gas leak permitted a fine control of gas input. Gas pressures could be measured to an accuracy of better than $\pm 1\%$ over the range of particular interest.

The magnetic fields were obtained with an air-cooled electromagnet providing horizontal magnetic fields of 10 000 G through an air gap $17 \cdot 15$ cm long between parallel plane pole faces 30 cm in diameter. A continuously variable 37 A, 193 V input to the two magnet coils connected in series permitted the application of any desired magnetic field, which was then monitored with a Model 900 Empire gaussmeter capable of an overall accuracy of $\pm 2\%$.

Figures 2(i) and 2(ii) show the variations of the magnetic field for two cross sections of the ionization chamber, namely, (i) radially across the magnet air gap, in a plane containing the mid point between the two pole faces, and (ii) axially across the magnet air gap, along a line passing through the annular ionization region corresponding to a radial distance of 5 cm from the axis of symmetry. Figure 2(i) also shows the position of the annular gap of the ionization chamber where it intersects the radial line along which measurements were taken. Similarly, Figure 2(ii) shows the axial lengths of inner cathode and outer anode. Over the entire annular region of the gap separation used in this investigation the applied magnetic fields were uniform to better than $\pm 3\%$.

The ionization growth was studied as a function of gas pressure p reduced to 0°C. The measurements were subjected to an analysis which has been described previously (Haydon and Robertson 1961), in which the reciprocals Y_p of the ionization currents, measured at a series of values of pressure p, are plotted against the corresponding reciprocals $Y_{p+\Delta p}$ measured at pressures $p+\Delta p$. It can be shown that the gradient of such a linear plot is given by $\exp(kd \,\Delta p)$, from which $k = \alpha/p$ may be obtained. To overcome the difficulties of setting pressures to high orders of accuracy at specific values of p and $p+\Delta p$, the ionization currents were first measured as a function of pressure and the appropriate values of Y_p and $Y_{p+\Delta p}$ obtained from a smooth curve. The analysis gave values of α/p that were consistent to $\pm 3\%$ independent of any secondary effects.

III. RESULTS

(a) Values for α/p and η

The values of α/p and $\eta(=\alpha/E)$ obtained for the cases B/p = 0 and 100 G/torr in hydrogen agree with the earlier measurements of Haydon and Robertson (1963), whereas the new results for B/p = 200 (see Fig. 3(i)) are rather larger. This is to be expected if, as was suspected in the original measurements with plane-parallel geometry, some undetected electron loss was occurring at the larger values of B/p. Subsequently, detailed investigations were made of the dependence of α/p on E/pfor a number of different, but constant, values of B/p. For hydrogen, measurements were made over the range of values 50 < E/p < 400 V cm⁻¹ torr⁻¹ and 0 < B/p < 2500 G/torr; some typical curves of α/p versus E/p are shown in Figure 3(ii). Also shown in this figure are some values quoted by Bernstein for B/p = 500, where the



Fig. 3.—(i) Comparison of values of η obtained from measurements made with planeparallel (+) and coaxial cylindrical (\bigcirc) geometry in hydrogen for values of B/p = 0and 200 G/torr; (ii) values of α/p versus E/p obtained in the present investigation for B/p = 0, 500, and 1000 G/torr; also shown for comparison are values obtained by Bernstein (\times) corrected to 0°C.

gas pressure p was quoted for a temperature of 20°C. A conversion has been made to a temperature of 0°C for the purposes of plotting these values, but direct comparison with our own measurements is then not possible for the same value of B/p. Nevertheless, it can be seen that at high values of E/p Bernstein's measurements would not agree with the present results. We believe that this is due largely to the fact that his analysis did not take into account the contribution made to the spatial ionization growth by the secondary (e.g. positive ion or photon) effects. At low values of E/p, however, these secondary processes do not dominate the growth of currents to the same extent, so that, despite the very large scatter in his ionization measurements and the inevitable large error in the determination of α/p , Bernstein's values in the low range of E/p are in good agreement with the present results. At very large values of E/p, further complications arise when marked scattering of the electrons occurs in the forward direction, so that a nonequilibrium situation develops in the spatial growth of ionization. These problems will be dealt with elsewhere. We consider here only those measurements where the equivalent value of E/p in the presence of a magnetic field corresponds to the equilibrium region. Some values of $(\alpha/p)_{B/p,E/p}$ for hydrogen and nitrogen obtained for these conditions are given in Tables 1 and 2 respectively, together with the values obtained previously in hydrogen (Haydon and Robertson 1963) and also in nitrogen (Bagnall and Haydon 1965) using planeparallel electrode geometry.

Our agreement with Bernstein's values for hydrogen in the equilibrium region leaves unresolved the discrepancy revealed by Engelhardt and Phelps when they used Bernstein's results to test their solutions of the Boltzmann equation (Engelhardt and Phelps 1963). This raises the problem of whether the equivalent field or equivalent pressure concept is a valid one for comparing microwave, crossed electric and magnetic field, and steady field measurements of transport properties. Before discussing this further, we examine the equivalent pressures obtained for various combinations of electric and magnetic fields in the present investigation.

(b) Values for the Equivalent Pressure p'

Provided that the energy distribution function and the drift velocities of the electrons in the gas are known, and neglecting for the present purpose the effects of diffusion, then the first ionization coefficient is given by

$$\alpha/p = k(W_{0,E/p})^{-1} \left| \int_0^\infty P(\epsilon) \, \epsilon^{\frac{1}{2}} f(\epsilon) \, \mathrm{d}\epsilon \right|_{\epsilon}, \tag{4}$$

where the integral is evaluated for the appropriate mean energy ϵ of the electrons, $k = (e/150 \ m)^{\frac{1}{2}}$, $P(\epsilon) = \text{ionization efficiency of electrons with energy } \epsilon \text{ eV at 1 torr}$ pressure, and $f(\epsilon) = \text{distribution function of electron energies.}$ It can then readily be shown that, provided the assumptions underlying equation (2) are valid, the equivalent pressure p' is given by

$$\frac{p}{p'} = \frac{(\alpha/p)_{0,E/p'}}{(\alpha/p)_{B/p,E/p}}.$$
(5)

Expressing this equation in terms of the coefficient $\eta = \alpha/E$, it also follows that

$$\eta_{B/p,E/p} = \eta_{0,E/p'} \,. \tag{6}$$

By measuring η at specific values of E/p and comparing this value with measured values of η as a function of E/p at B/p = 0, the value of E/p' and hence p' may be

		$lpha/p~({ m cm}^-)$	t^{-1} torr t^{-1})	$\eta imes 10^2 \; (\mathrm{V^{-1}})$	
E/p (V cm ⁻¹ torr ⁻¹)	B/p (G/torr)	Parallel Plates	Coaxial Cylinders	Parallel Plates	Coaxial Cylinders
50	0 100 200	0.330	$0.330 \\ 0.235 \\ 0.125$	0.660	$0.660 \\ 0.470 \\ 0.250$
75	0	0.790		$1 \cdot 05$	
100	0 100 200 500	1 • 23	$1 \cdot 27$ $1 \cdot 20$ $0 \cdot 82$ $0 \cdot 28$	1.23	$1 \cdot 27$ $1 \cdot 20$ $0 \cdot 82$ $0 \cdot 28$
120	0 100 200	$1 \cdot 59 \\ 1 \cdot 45 \\ 1 \cdot 17$	1.61	$1 \cdot 32 \\ 1 \cdot 21 \\ 0 \cdot 975$	1.34
140	0 100 200	$1 \cdot 89$ $1 \cdot 90$ $1 \cdot 62$	1.88	$1 \cdot 35 \\ 1 \cdot 36 \\ 1 \cdot 15$	1.34
150	0 100 200 500 1000	$2 \cdot 00$ $2 \cdot 06$ $1 \cdot 80$	$2 \cdot 04 \\ 2 \cdot 07 \\ 1 \cdot 77 \\ 0 \cdot 930 \\ 0 \cdot 218$	1 · 33 1 · 37 1 · 20	$1 \cdot 36$ $1 \cdot 38$ $1 \cdot 18$ $0 \cdot 620$ $0 \cdot 145$
175	100 200	$2 \cdot 36$ $2 \cdot 22$		$1 \cdot 35$ $1 \cdot 27$	
200	0 100 200 500 1000	$2 \cdot 36 \\ 2 \cdot 56 \\ 2 \cdot 52$	$2 \cdot 37$ $2 \cdot 56$ $2 \cdot 68$ $1 \cdot 58$ $0 \cdot 66$	1 · 18 1 · 28 1 · 26	$1 \cdot 185$ $1 \cdot 28$ $1 \cdot 34$ $0 \cdot 79$ $0 \cdot 33$
250	0 100 200 500 1000	$2 \cdot 48$ $2 \cdot 82$ $3 \cdot 00$	$ \begin{array}{c} 2 \cdot 53 \\ 2 \cdot 70 \\ 3 \cdot 35 \\ 2 \cdot 32 \\ 1 \cdot 27 \end{array} $	0.992 1.13 1.20	$ \begin{array}{r} 1 \cdot 01 \\ 1 \cdot 08 \\ 1 \cdot 34 \\ 0 \cdot 930 \\ 0 \cdot 510 \end{array} $
300	0 100 200 1000	2.52	$ \begin{array}{r} 2 \cdot 58 \\ 2 \cdot 97 \\ 3 \cdot 69 \\ 2 \cdot 04 \end{array} $	0.84	$ \begin{array}{c} 0.860 \\ 0.990 \\ 1.23 \\ 0.680 \end{array} $

Table 1 values of α/p and η in crossed fields in ${\rm H_2}$

		$\alpha/p ~(\mathrm{cm}^{-1} \mathrm{torr}^{-1})$		$\eta imes 10^2 (V^{-1})$	
E/p (V cm ⁻¹ torr ⁻¹)	B/p (G/torr)	Parallel Plates	Coaxial Cylinders	Parallel Plates	Coaxial Cylinders
75	0		0.150		0.200
100	0	0.423	0 • 425	0 · 423	0.425
120	0		0.780		0.650
140	0		1.17		0.840
150	0 500	1 · 22	$1 \cdot 25$ $0 \cdot 664$	0.812	$\begin{array}{c} 0\cdot 834 \\ 0\cdot 442 \end{array}$
175	0 500	$1 \cdot 62$	$1 \cdot 63$ $1 \cdot 00$	0.927	$\begin{array}{c} 0\cdot 931 \\ 0\cdot 570 \end{array}$
200	0 250 500 750	2.08	$2 \cdot 17$ 1 · 89 1 · 31 0 · 845	1.045	$1 \cdot 085$ $0 \cdot 945$ $0 \cdot 655$ $0 \cdot 423$
225	0	$2 \cdot 54$		1.13	
250	0	3 • 10	3 ·10	$1 \cdot 24$	1.24
275	0	3 ·54		1 • 29	
300	0	3 ⋅ 95	3 · 90	$1 \cdot 32$	1.30
325	0	4 · 36		1.34	
350	0	$4 \cdot 65$	4 ·83	1.33	1.38
400	0 750 1000 1250 1500 1750	5.32	$5 \cdot 37$ $4 \cdot 52$ $3 \cdot 78$ $2 \cdot 85$ $2 \cdot 43$ $1 \cdot 90$	1.33	$ \begin{array}{r} 1 \cdot 34 \\ 1 \cdot 13 \\ 0 \cdot 945 \\ 0 \cdot 713 \\ 0 \cdot 608 \\ 0 \cdot 475 \end{array} $
500	0 2000	6 • 58	$2 \cdot 72$	$1 \cdot 32$	0.544
600	0 2000 2500	7.53	$7 \cdot 57$ $4 \cdot 23$ $2 \cdot 85$	$1 \cdot 25$	$1 \cdot 26 \\ 0 \cdot 704 \\ 0 \cdot 474$

Table 2 values of α/p and η in crossed fields in $\rm N_2$

found. Values of the ratio p'/p obtained in this way in both H₂ and N₂ are given in Table 3 together with the actual values of E/p, B/p, and the corresponding values for the effective collision frequency v_{eff}/p obtained from equation (2). It should be noted that the effective collision frequency is not constant for all values of the effective ratio of electric field strength to gas pressure.

E/p (V cm ⁻¹ torr ⁻¹)	B/p (G/torr)	p'/p	E/p' (V cm ⁻¹ torr ⁻¹)	$(\nu_{\rm eff}/p) \times 10^{-9}$ $({ m sec}^{-1} { m torr}^{-1})$
Hydrogen				
250	2000	9.09	$27 \cdot 5$	3.90
150	1000	$5 \cdot 30$	$28 \cdot 3$	$3 \cdot 38$
200	1500	$6 \cdot 90$	$29 \cdot 0$	$3 \cdot 87$
250	1500	7.50	33.3	$3 \cdot 55$
250	1500	$7 \cdot 35$	34.0	$3 \cdot 62$
100	500	$2 \cdot 85$	$35 \cdot 0$	$3 \cdot 40$
150	800	$4 \cdot 20$	$35 \cdot 7$	$3 \cdot 45$
200	1000	$5 \cdot 38$	$37 \cdot 1$	3.33
250	1000	5.71	$43 \cdot 8$	$3 \cdot 14$
300	1000	$5 \cdot 87$	$51 \cdot 1$	$3 \cdot 04$
200	600	$3 \cdot 10$	$64 \cdot 5$	$2 \cdot 66$
300	600	$4 \cdot 17$	$72 \cdot 0$	$2 \cdot 61$
250	400	$2 \cdot 97$	84 · 1	$2 \cdot 52$
Nitrogen				
150	500	1.56	98	7.3
200	750	$2 \cdot 00$	100	$7 \cdot 6$
600	2500	5.77	104	7.7
400	1750	$3 \cdot 85$	104	8.3
500	2000	$4 \cdot 46$	112	$8 \cdot 1$
175	500	$1 \cdot 52$	115	7.7
400	1500	$3 \cdot 39$	118	8.1
600	2000	$4 \cdot 61$	130	$7 \cdot 8$
400	1250	$2 \cdot 85$	133	$7 \cdot 8$
400	1000	$2 \cdot 31$	173	8.4
400	750	$1 \cdot 85$	216	8.4

TABLE 3 VALUES OF p'/p and $u_{
m eff}/p$ for ${
m H_2}$ and ${
m N_2}$

(c) The Ionization Frequency v_i/N in H_2

For the purposes of testing their solution of the Boltzmann equation, Engelhardt and Phelps compare computed and experimental values of the ionization frequency ν_i/N as a function of E/N, where N is the number of gas molecules per cm³. The quantity ν_i/N is given by

$$\frac{\nu_{\mathbf{i}}}{N} = \frac{\alpha}{N} W, \qquad (7)$$

where W is the appropriate drift velocity in the electric field direction.

The results of this test are shown in the points A and in curve B of Figure 4. The points A are based on the experimental values of α/p obtained by Bernstein, and curve B represents the values of ν_i/N computed by Engelhardt and Phelps and plotted against $(E/N)_{\text{eff}}$ on the basis of a constant effective collision frequency $\nu_{\text{eff}}/N = 1.68 \times 10^{-7} \text{ cm}^3 \text{ sec}^{-1}$. Since our measurements of α/p in crossed fields have shown



Fig. 4.—Ionization frequency ν_i/N as a function of effective field $(E/N)_{\text{eff}}$ for hydrogen. A, values obtained from Bernstein's measurements of α/p in crossed fields with constant effective collision frequency $\nu_{\text{eff}}/N = 1 \cdot 68 \times 10^{-7} \text{ cm}^3 \text{ sec}^{-1}$; curve B, values computed by Engelhardt and Phelps and plotted against $(E/N)_{\text{eff}}$ on the assumption that $\nu_{\text{eff}}/N = 1 \cdot 68 \times 10^{-7} \text{ cm}^3 \text{ sec}^{-1}$; curve C, values obtained using measurements of W by Schlumbohm and experimental values of α/p (× present authors, \Box Rose (1956)); curve D, values obtained from the present authors' measurements of α/p in crossed fields and using equation (8); curve E, variation of ν_{eff}/N with $(E/N)_{\text{eff}}$.

that ν_{eff}/N cannot be assumed to be constant but must vary, as shown in curve E of Figure 4, a re-assessment of the situation is made in curves C and D. For this purpose, curve C has been derived from equation (7) using the experimental values for α_i/N obtained by Rose (1956) and the drift velocities W measured by Schlumbohm (1965). Curve D has been plotted using the values of α_i/N obtained in the present

investigation with crossed fields and using the values of W given by the following expression for the drift velocity $W_{\rm T}$ in the direction of the electric field:

$$W_{\rm T} = \frac{E}{B} \left(\frac{\omega/\nu_{\rm eff}}{1 + \omega^2/\nu_{\rm eff}^2} \right). \tag{8}$$

The following comments are necessary.

- (1) The discrepancy between curves B and C arises because Schlumbohm's values for the drift velocity are larger than those computed by Engelhardt and Phelps in their determination of v_i/N .
- (2) If we accept the available experimental information as reliable and accurate, then the remaining discrepancy between curves C and D must arise because of the inadequacy of equation (8) to represent the drift velocity in the electric field direction in the presence of a transverse magnetic field. In order to establish the magnitude of the discrepancy between curves C and D, further measurements of the drift velocity are obviously needed. These measurements should provide an independent check on the values of W obtained by Schlumbohm and should also, if possible, provide direct determinations of the appropriate drift velocity W_T in crossed fields.

IV. DISCUSSION

In view of the results of Section III, any discussion of the concept of an equivalent field or equivalent pressure must centre around the precise meaning of the "effective" collision frequency ν_{eff} . It has already been stressed (Blevin and Haydon 1958) that ν_{eff} is simply defined only when the mean free time between collisions of electrons with neutral molecules is constant. For any other situation one must evaluate the equivalent pressure p' from equation (3), which, because of the relations

$$W_{\mathbf{0},E/p'} = rac{Ee}{3m} \left| \overline{u^{-2} rac{\mathrm{d}}{\mathrm{d}u} (lu^2)} \right|_{ar{\epsilon}}$$

and

$$W_{B/p,E/p} = rac{Ee}{3m}\left|\overline{u^{-2}rac{\mathrm{d}}{\mathrm{d}u}\!\!\left(\!rac{lu^2}{1\!+\!l^2\omega^2}\!
ight)}
ight|_{arepsilon},$$

requires knowledge of both f(u) and l = F(u). In these circumstances, any comparison of p'/p with the values predicted in equation (2) can only yield values of some effective collision frequency v_{eff} , and it is the variation of this quantity that has been revealed in the present investigations in both H_2 and N_2 . An understanding of this variation in terms of the fundamental collision cross sections and velocity distribution functions is an extremely complex problem. Some preliminary studies of the problem have been reported (Haydon 1966) in which Maxwellian and Druyvesteyn velocity distributions have been assumed in an attempt to assess the influence of the form of the relationship l = F(u) on the calculated values of v_{eff}/p . For a given variation of l with u, the effect of the distribution function is small. More important is the actual variation of l with u, and, in order to obtain any satisfactory agreement with experiment, it was found necessary to assume collision frequencies for momentum transfer very different from those previously accepted. In the case of hydrogen, the appropriate momentum transfer collision frequencies had to be considerably reduced throughout the electron energy range. At higher energies, these lower values for collision frequency can be explained on the basis of marked forward scattering processes, and this explanation is confirmed by the limited data available on angular scattering for this gas. In fact, a similar variation of momentum transfer collision frequency with energy was required by Heylen (1960) in his calculations of electron mobility in hydrogen. All this appears to confirm that the momentum transfer cross sections $Q_{\rm m}$ at large electron energies are much lower than those assumed by Engelhardt and Phelps. These authors have, of course, already drawn attention to this problem in discussing the discrepancies between their computed and experimental values of drift velocity and characteristic energy ϵ_k at large values of E/N. Although they were reluctant, in the absence of reliable and adequate ϵ_k data, to lower the cross sections for momentum transfer, there is now increasing evidence that this would in fact be more realistic.

Nevertheless, since the effect of the distribution function is not large, it is not clearly understood from the very limited data at present available why the momentum transfer collision frequencies need to be so much lower than the total collision frequencies at the lower electron energies. Much more information about the fundamental collision data appears to be necessary.

It seems especially desirable to measure the quantities $W_{0,E/p}$ and $W_{B/p,E/p}$ directly over as wide a range as possible beyond $E/p \simeq 20$. This would provide further information about the equivalent pressure p' using equation (3) and would also provide an essential check, through equations (7) and (8), of any assumptions made about l = F(u) and of the form of the velocity distribution function f(u).

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VI. References

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