THE CLUSTERING OF RADIO SOURCES

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[*Manuscript received January* 9, 1967]

Summary

Two methods of analysis have been used in an investigation of the possible anisotropic distribution of radio sources listed in the Parkes catalogues. No significant evidence was found for clustering of sources on scales of 15′ arc to 40°.

I. Introduction

A number of radio source surveys have been examined statistically for any evidence of non-randomness of source positions. These analyses, apart from the recent 4C survey, were restricted by the limited resolution and sensitivity of the instruments to angular separations of greater than a few degrees.

Shakeshaft (1955), in an investigation based on the 2C survey at 81·5 MHz, found little evidence for clustering. Edge (1958), in an analysis of the 3C survey, concluded that for different ranges of flux densities (limiting flux 8×10^{-26} W m^{-2} Hz^{-1}) the source positions were randomly distributed in sample areas of 42·5 and 900 sq deg. Mills, Slee, and Hill (1960) concluded that for sources having $S \geqslant 6\times10^{-26}$ W m^{-2} Hz^{-1}, for $-90° \leqslant \delta \leqslant +10°$, there is no significant departure from a random distribution, although small-scale clustering may occur in regions of the order of $\frac{1}{2}°$. Results obtained with the 178 MHz Cambridge interferometer, operating as both an interferometer and a total power system, were used by Leslie (1961) in two methods of analysis, and these gave no evidence for clustering on a scale down to 2′·5 arc. Leslie also derived values for the maximum percentage of double or multiple systems that could occur without producing a detectable effect. Holden (1966), in an analysis based on the 4C survey, found the distribution of source positions for areas from 25 to 3600 sq deg to be uniform within the limits of statistical fluctuations. Further, Holden found no evidence of clustering for angular separations of $\frac{1}{2}°$ to 4°, although there was some evidence to suggest associations of sources having angular separations in the range 15′ to 30′.

The Parkes 210 ft telescope has recently been used to complete two surveys in the declination zones 0° to +20° (Day *et al.* 1966) and 0° to −20° (Shimmins *et al.* 1966) at frequencies of 408, 1410, and 2650 MHz. The present investigation is based on these surveys at 1410 MHz. The sensitivity of the instrument at this frequency is $0\cdot3\times10^{-26}$ W m^{-2} Hz^{-1}, while the beamwidth is 13′·9 arc. Source surveys have also been made in the regions −20° to −60° (Bolton, Gardner, and Mackey 1964) and −60° to −90° (Price and Milne 1965) with an instrument sensitivity of approximately 0·8 flux units† at 1410 MHz. These surveys were restricted to sources with flux densities greater than 4 f.u. at 408 MHz that did not noticeably broaden the 48′ beam of the telescope.

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† 1 flux unit (f.u.) = 10^{-26} W m^{-2} Hz^{-1}.

II. The Probable Scale of Clustering

(1) Several authors (Mills 1960; van den Bergh 1961; Wills 1966) have shown that there is a significant correlation between the positions of radio sources and rich clusters of galaxies. The probability of collisions between galaxies will be greatly increased, and we might expect radio source associations with a scale of a few degrees. It can easily be shown, however, that even in clusters of galaxies the probability of encounters between galaxies is small, and the contribution of this process to the degree of clustering should be negligible.

(2) It is now well established that a radio source is generally larger than the associated optical galaxy and is often double, with the components situated on either side of the visible object. The most extensive investigations of the brightness distributions of radio sources are the long-baseline interferometer observations made at the California Institute of Technology (Moffet and Maltby 1962), using spacings out to 1557 wavelengths, and at Jodrell Bank (Allen *et al.* 1962), where by means of radio links spacings out to 61100 wavelengths were obtained. It was found that about 70% of the sources are apparently double with an angular separation of about 1′ arc. On the basis of these observations we should expect association of extragalactic radio sources with a scale of a few minutes of arc.

Table 1

Results for 5° by 5° zones

Number of sources in zone	0	1	2	3	4	5	6	7	8	9
Number of times observed	56	98	122	100	70	30	10	4	1	0
Number of times expected	45·5	108·2	128·7	102·1	60·7	28·9	11·4	3·9	1·16	0·31

III. Large-scale Clustering

The distribution of source positions was investigated by dividing the sky into square areas and counting the number of squares containing 0, 1, 2, 3, etc. sources. The scales of clustering considered were for areas of 25, 100, 400, and 1600 sq deg. The sky was divided into regions 5°, 10°, 20°, and 40° wide in δ and the appropriate extent in α to produce the required areas. For areas of 400 and 1600 sq deg, where the number of zones was too small for a statistical investigation, the analysis was extended to $\delta = -80°$.

(a) 25 Sq Deg

There is no evidence for a non-random distribution of sources having $S \geqslant 0\cdot3$. The results are shown in Table 1.

The mean of the distribution is 2·38, while the variance is 2·44. The observed results were compared with the theoretical Poisson distribution predicted for a random sky. A chi-square test gave a probability of 0·65 that the distribution is Poisson. The distribution of sources per zone as a function of α was also investigated, and a plot of the number of sources per zone against α showed a general tendency for the number of sources to decrease between 0800 and 1700 hr. It was found that the number of sources in the equal areas between 0800 and 1700 hr, and between 2000 and 0500 hr, was 485 and 587 respectively, i.e. an increase of approximately 25% in the latter region.

(*b*) *100 Sq Deg*

In this case there is a greater number of sources per zone, and three different flux ranges, $S \geqslant 0 \cdot 3$, $S \geqslant 1 \cdot 0$, and $S \geqslant 2 \cdot 0$, were considered. The results are summarized in Table 2, which gives the calculated mean and variance and the probability P that the distribution is Poisson. It can be seen that there is no significant evidence for clustering on this scale.

TABLE 2

RESULTS FOR 10° BY 10° ZONES

	Flux Range		
	$S \geqslant 0 \cdot 3$	$S \geqslant 1 \cdot 0$	$S \geqslant 2 \cdot 0$
Mean	9·3	4·5	1·5
Variance	11·6	5·6	1·4
P	0·35	0·72	0·3

(*c*) *400 Sq Deg*

The analysis for sources having $S \geqslant 0 \cdot 3$ was restricted to declinations $+20° \geqslant \delta \geqslant -20°$, while for sources having $S \geqslant 1 \cdot 0$ and $S \geqslant 2 \cdot 0$ the analysis was extended to $\delta = -80°$. The results of the analysis are shown in Table 3.

TABLE 3

RESULTS FOR 20° BY 20° ZONES

	Flux Range		
	$S \geqslant 0 \cdot 3$	$S \geqslant 1 \cdot 0$	$S \geqslant 2 \cdot 0$
Mean	37·5	15·2	6·3
Variance	67·6	25·1	9·0
P	0·35	0·01	0·25

It can be seen from this table that for zone areas of 400 sq deg there is no evidence for a non-random distribution of sources having $S \geqslant 0 \cdot 3$ and $S \geqslant 2 \cdot 0$. There is, however, significant evidence for clustering of sources having $S \geqslant 1 \cdot 0$ in the region $+20° \geqslant \delta \geqslant -80°$. An analysis was made of sources having $S \geqslant 1 \cdot 0$ in the region $+20° \geqslant \delta \geqslant -20°$, and the distribution was found to have a mean and variance of 17·8 and 30·7 respectively, while the probability that the distribution is Poisson was 0·60. In the region $-20° \geqslant \delta \geqslant -80°$ the distribution of sources was found to have a mean of 15·7, a variance of 37·2, and a probability of 0·20 that the distribution is random. The surveys in the declination zones $-20° \geqslant \delta \geqslant -60°$ and $-60° \geqslant \delta \geqslant -90°$ did not include sources that broadened the 48′ beam of the telescope, whereas the surveys between +20° and −20° included a large number of extended sources, so that the apparent clustering on this scale can be explained in terms of source selection effects in the separate surveys.

(*d*) *1600 Sq Deg*

For sources having $S \geqslant 1 \cdot 0$ and $S \geqslant 2 \cdot 0$ the analysis has been extended to $\delta = -80°$. The number of zones of this size was insufficient for a complete analysis to be made, but to allow for possible clustering on the boundaries of a zone each zone was shifted half a zone length in α. In both cases there were no obvious trends in the numbers of sources per zone to suggest any departure from a non-random distribution.

Table 4

NUMBERS OF PAIRS OF SOURCES IN REAL AND RANDOM SKIES

Flux Range	Type of Sky	Angular Separation 15′–30′	30′–1°	1°–2°	2°–3°	3°–4°
$S \geqslant 0 \cdot 3$	Real	71	130	654	1103	1440
	Random	66·7	126	636	1077	1445
$S \geqslant 0 \cdot 6$	Real	42	75	402	726	892
	Random	39·4	69·1	388	734	872
$S \geqslant 1 \cdot 0$	Real	18	32	163	290	342
	Random	16·3	29·1	149	291	336
$S \geqslant 1 \cdot 5$	Real	6	11	47	74	101
	Random	3·1	8·4	42·2	70·9	98·2
$S \geqslant 2 \cdot 0$	Real	5	6	22	29	43
	Random	2·3	5·1	18·0	29·3	38·8

IV. Small-scale Clustering

An investigation of small-scale clustering was made for sources having flux densities greater than 0·3, 0·6, 1·0, 1·5, and 2·0 f.u., the total number of sources in each range being 1184, 949, 576, 305, and 191 respectively. The analysis did not include the regions of the galactic plane that were not covered in the Parkes surveys. The half-power beamwidth of the Parkes telescope at 1410 MHz is 13′·9 arc, so that the investigation of small-scale clustering is limited to a scale of 15′ arc. The analysis of small-scale clustering is similar to that described by Holden (1966). The analysis uses a method of comparison between the distribution of its neighbours for each source in the real sky and the distribution obtained from a series of random skies. The positions of sources having $S \geqslant 0 \cdot 3$ were compiled in an Elliott 503 computer, and for each source the number of sources falling within a specified angular separation, in α and δ, was calculated. Each pair of sources was included only once in the analysis, and the ranges of angular separation considered were 15′ to 30′, 30′ to 1°, 1° to 2°, 2° to 3°, and 3° to 4° in α and δ. The analysis was then repeated for sources having $S \geqslant 0 \cdot 6$, $S \geqslant 1 \cdot 0$, $S \geqslant 1 \cdot 5$, and $S \geqslant 2 \cdot 0$.

A total of 60 random skies was compiled in the same computer. The number of sources within each flux range was made identical to that of the real sky. Any other parameters, such as those required for the exclusion of the galactic plane, simulated those of the real sky. The effects of the finite resolution of the telescope were accounted for in the generation of the random skies. The results are summarized in Table 4. It can be seen from the values listed that there is, overall, no evidence for clustering of sources with angular separations 15′ to 4°. Except for sources having $S \geqslant 1\cdot5$ and $S \geqslant 2\cdot0$ with an angular separation of 15′ to 30′, all departures from the random values can be accounted for by statistical fluctuations.

The minimum number of double sources occurring at the 10% significance level was calculated from each random sky distribution and subtracted from the observed number of double sources in the real sky. This number was then expressed as a percentage of the total number of sources, giving the maximum percentage of clustering that could be present but undetected at the 10% significance level. The maximum percentage of clustering for each flux range and for each angular separation is shown in Table 5.

TABLE 5

MAXIMUM PERCENTAGE OF DOUBLE SOURCES THAT COULD OCCUR AT THE 10% SIGNIFICANCE LEVEL

Flux Range	Angular Separation				
	15′–30′	30′–1°	1°–2°	2°–3°	3°–4°
$S \geqslant 0\cdot3$	3%	3%	4%	6%	8%
$S \geqslant 0\cdot6$	2	6	5	2	9
$S \geqslant 1\cdot0$	9	4	12	7	10
$S \geqslant 1\cdot5$	4	4	8	7	8
$S \geqslant 2\cdot0$	5	2	3	2	11

The results given in Tables 4 and 5 show that there is no evidence for clustering of sources with angular separations 15′ to 4°. Although there is a number of small-scale associations of intense sources, they represent only a small percentage of the total number of sources with the appropriate flux density. Thus, it is found that $4\cdot5\%$ of sources having $S \geqslant 1\cdot5$, and $5\cdot5\%$ of sources having $S \geqslant 2\cdot0$, lie within 15′ to 30′ of another such source.

V. Conclusions

The analysis of large-scale clustering has shown the source positions listed in the Parkes catalogues to be randomly distributed in areas of 25 to 1600 sq deg. It was found that approximately 50% of the sources in the declination zone $+20° \geqslant \delta \geqslant -20°$ occur between 2000 and 0500 hr.

The distribution of sources having angular separations of 15′ to 4° was found to be isotropic within the limits of statistical fluctuations.

VI. Acknowledgments

I should like to thank Mr. P. A. Hamilton of the Department of Physics, University of Tasmania, for his assistance in the preparation of the programmes and Mr. G. A. Day for making available material prior to publication. This work was carried out during the tenure of a Commonwealth Post-graduate Scholarship.

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