

THE SEMICLASSICAL MODEL OF DIRECT REACTIONS*

By I. ROBERTSON†

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Summary

The semiclassical model of direct reactions is reviewed and its predictions are presented for a reaction that exhibits the phenomenon of backward peaking, following the work of Pearson. The curve $\sigma(180^\circ)$ against energy is fitted quite closely and the shape of the curve has a simple explanation in terms of the bound state radial wave functions.

To check the validity of the model the corresponding distorted wave Born approximation (DWBA) calculations are carried out; little agreement is found. The semiclassical model severely overestimates the spatial localization in the exact matrix element, and therefore gives cross sections that have a much faster energy variation than the DWBA cross sections. The DWBA calculations cannot explain the experimental results on backward peaking and it is concluded that the agreement between the semiclassical model and experiment is fortuitous.

I. INTRODUCTION

The semiclassical model was first used by Butler (1957) and Butler, Austern, and Pearson (1958) to investigate the physical reasons for the well-known "Bessel function" shape of angular distributions in direct reactions. The model considers the incident particle to be described by a ray (or WKB wave function) that is, in general, refracted and absorbed by the nuclear potential. The reaction occurs at some well-defined *point* in the nucleus and the emitted particle is described by another ray having different direction, wave number, and absorption coefficient from the first. Simple models of the bound state wave functions involved in the reaction usually impose strong restrictions on the angular momentum transfer L (often L is limited to a single value) and in the semiclassical model this limitation is assumed to apply locally, at the point of scattering. For a particular scattering angle, the angular momentum condition defines the regions of the nucleus that can contribute to the reaction.

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† Australian Atomic Energy Commission Research Establishment, Lucas Heights, N.S.W.; present address: Department of Theoretical Physics, Institute of Advanced Studies, Australian National University, Canberra.

In the original papers, refraction and absorption of the rays are neglected so that the initial and final rays follow straight lines (i.e. the wave functions are plane waves $\exp(i\mathbf{k}_i \cdot \mathbf{r})$ and $\exp(i\mathbf{k}_f \cdot \mathbf{r})$) and the angular momentum condition is

$$|\mathbf{Q} \times \mathbf{r}| = L, \quad (1)$$

where $\mathbf{Q} = \mathbf{k}_i - \mathbf{k}_f$ is the momentum transfer. Equation (1) shows that the reaction is confined to the surface of a cylinder, radius L/Q , with axis parallel to \mathbf{Q} , and the oscillatory shape of the angular distribution may be explained in terms of interference between rays arising from different parts of this cylinder. Small cross sections at forward angles occur because the cylinder for these angles often does not intersect the nucleus and there are therefore no regions in which the reaction can occur with the correct angular momentum transfer.

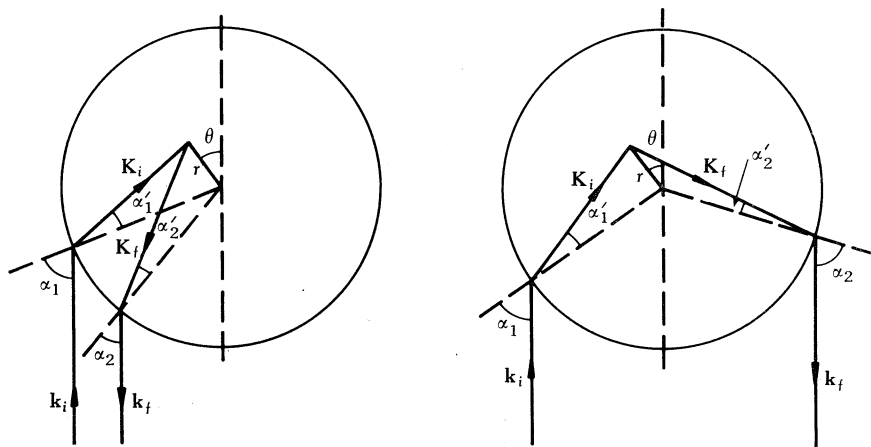


Fig. 1.—Semiclassical scattering processes.

Butler gave the semiclassical model some justification by showing that the angular integral in the (plane wave) matrix element,

$$S_{fi} = \int \exp(i\mathbf{Q} \cdot \mathbf{r}) F_L(r) P_L(\cos \theta) d\mathbf{r}, \quad (2)$$

may be plausibly evaluated by the method of stationary phase. The region of stationary phase is found to coincide with the cylinder defined by equation (1), so that it seems reasonable to evaluate the matrix element by integrating over that cylinder. The model may thus be viewed as an approximate way of evaluating the Born approximation matrix element by integrating over the surface defined by the angular momentum condition, rather than over all space.

II. MODEL FOR DISTORTED WAVES

Pearson (1962, 1963) has extended the model to cover distorted wave calculations and has applied it to certain reactions that show the phenomenon of backward peaking. Although angular distributions have not been calculated, the model is able to fit the rapid energy variation of the backward cross section, which is an important feature of these reactions.

When refraction and absorption are included the ray diagrams are as shown in Figure 1 (we assume a square well for the nuclear potential and consider only scattering in the backward direction), and the angular momentum condition is

$$|\mathbf{Q}^* \times \mathbf{r}| = L, \quad (3)$$

with $\mathbf{Q}^* = \mathbf{K}_i - \mathbf{K}_f$. It is found that equation (3) confines the reaction to an area on the surface of the nucleus facing the incident beam, and to a small interior region on the far side of the nucleus which coincides with the optical model focus noted by McCarthy (1959). In the evaluation of the semiclassical matrix element the contributions from these two regions are separated, and we shall refer to them as the "surface term" and "focal term" respectively.

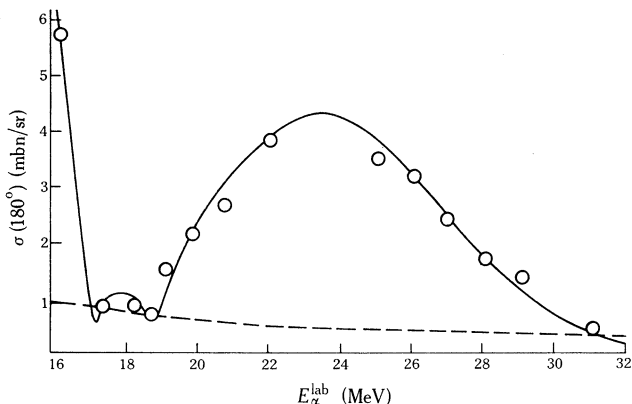


Fig. 2.—Backward cross section $\sigma(180^\circ)$ against α -particle energy (laboratory system) for the reaction $^{12}\text{C}(\alpha, p)^{15}\text{N}$: \circ experimental values; — semiclassical model; --- DWBA calculations.

If the optical model wave functions $\Psi_{\mathbf{k}_i}^{(+)}(\mathbf{r})$, $\Psi_{\mathbf{k}_f}^{(-)}(\mathbf{r})$ appearing in the distorted wave matrix element are replaced by semiclassical wave functions of the form

$$\Psi_{\mathbf{k}}(\mathbf{r}) = \sum_j A_j(\mathbf{r}) \exp\{iS_j(\mathbf{r})\}, \quad (4)$$

it is again possible to justify the model by the stationary phase argument and so maintain the view that the semiclassical model is simply an approximation to the distorted wave Born approximation (DWBA) matrix element. (The sum in equation (4) is a sum over rays passing through the point \mathbf{r} , and the amplitude and phase are

$$A_j(\mathbf{r}) \propto \exp\left(-\int_{\mathbf{r}_0}^{\mathbf{r}} \Gamma ds_j\right),$$

$$S_j(\mathbf{r}) = S_j(\mathbf{r}_0) + \int_{\mathbf{r}_0}^{\mathbf{r}} \hbar^{-1} \{2m(E - V)\}^{\frac{1}{2}} ds_j,$$

where V is the real part of the optical potential and Γ is simply related to the imaginary part.)

Figure 2 shows the predictions of the semiclassical model for the curve $\sigma(180^\circ)$ against energy in the reaction $^{12}\text{C}(\alpha, p)^{15}\text{N}$ (Yamazaki, Kondo, and Yamabe 1963),

which is assumed to proceed by the α -particle knocking out a proton from a $1p$ state and being itself captured into a (cluster model) $3s$ state. The theory is fitted to experiment at 17.3 MeV and the model is able to reproduce the experimental results quite closely. Similar fits were obtained for the two other reactions considered by Pearson ($^{19}\text{F}(p, \alpha)^{16}\text{O}$ and $^{13}\text{C}(p, n)^{13}\text{N}$).

The features of Figure 2 may be explained by examining Figure 3, which shows the bound state radial wave functions for the α -particle and the proton, together with the positions and widths of the semiclassical focal spot at various energies. At 16 MeV the focus is at a point where the bound state wave functions have a large overlap, so that the focal term is large and the cross section has a maximum. At 18 MeV the cross section has a minimum because the focus falls on a zero of the α -particle wave function, severely reducing the focal contribution. Above 20 MeV the focus is again in a region of large overlap, and a broad maximum is seen as a result. At energies where the focal term is small the surface term will be dominant and the experimental angular distributions at these energies often have the oscillatory "surface reaction" shape.

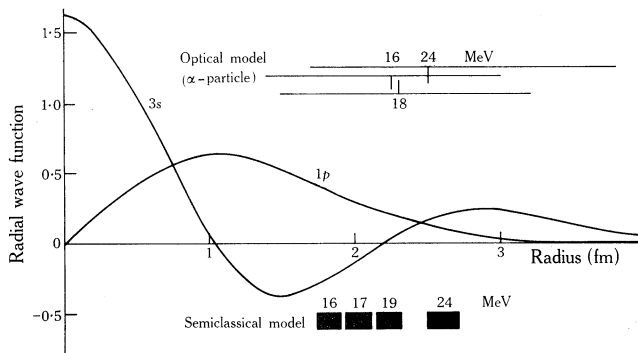


Fig. 3.—Bound state wave functions and focal positions for the reaction $^{12}\text{C}(\alpha, p)^{15}\text{N}$.

The explanation of the energy variation of the backward cross section in terms of the bound state wave functions and a small moving focal spot leads to the interesting idea that careful measurements of $\sigma(180^\circ)$ against energy might be used to obtain information about bound state wave functions when these are not well known. In particular the positions of zeros could be determined accurately and from these the depth of the bound state potential could be found.

III. THE DWBA CALCULATIONS

To check the validity of the semiclassical model, a general purpose DWBA code was written for an IBM 7040 computer. The DWBA calculations were carried out using exactly the same parameters as those used in the semiclassical model (Table 1); the results for $^{12}\text{C}(\alpha, p)^{15}\text{N}$, which are typical of those obtained for the three reactions, are shown in Figure 2. There is no agreement whatsoever between the semiclassical and DWBA curves.

The positions and widths of the focus in the full optical model α -particle wave function are plotted in Figure 3 (the proton focus is almost stationary at 3.7 fm and may be disregarded) and it is easily seen why the two calculations disagree. Although the peak positions of the semiclassical and quantum mechanical foci are similar, the quantum mechanical focus is about an order of magnitude wider than that obtained in the semiclassical model. Instead of a small "bright spot" tracing out the bound state wave functions, there is a very large bright region, and so much averaging occurs within the region that all details of the bound state wave functions are lost. The $\sigma(180^\circ)$ -energy curve is quite devoid of structure.

A series of calculations was carried out to see if the DWBA theory was able in any way to reproduce the experimental results for backward peaking. (The code calculates angular distributions, permitting a more detailed comparison with experiment than is possible with the semiclassical model.) Angular distributions as a function of energy were calculated both with Pearson's parameters and with the best parameter values currently available, namely, those of Rosen *et al.* (1965) for nucleons and those of McFadden and Satchler (1966) for α -particles. A peculiar

TABLE 1
OPTICAL POTENTIALS USED IN SEMICLASSICAL AND DWBA CALCULATIONS
All potential wells square and of radius 3.6 fm

Reaction	V	Parameter Values (MeV)	
		W	$E_p^{c.m.} (= 0.75 E_\alpha^{lab} - 4.96)$
α - ^{12}C	45	12	
p - ^{15}N	49- n	6+0.5 n	7+ n
	45	8.5	> 11

feature of the measured angular distributions is the rapid change that can occur (within 0.2 MeV in one case) from forward to backward peaking, or vice versa. In contrast to this the theoretical distributions changed slowly with energy and it was always possible to recognize gradual variation of the one basic shape throughout the whole energy range. The DWBA calculation could not reproduce the features of the angular distributions even in small energy regions, except for $^{13}\text{C}(p,n)^{13}\text{N}$ between 5 and 6 MeV where the theoretical distributions did peak in the backward direction, giving qualitative agreement with experiment. One must conclude from these calculations that the DWBA theory can sometimes give backward peaked angular distributions, as suggested by Kromminga and McCarthy (1961), but it is quite incapable of giving a detailed fit to experiment, particularly in the matter of rapid energy variation.

Since the semiclassical model is nothing more than an approximation to the DWBA matrix element, the results of the DWBA calculations must take precedence over those obtained with the simplified model, and any agreement between the latter and experiment must be regarded as fortuitous. The error of the semiclassical model lies in considering the incident or emitted particle to be described by a *single* ray and thinking of the reaction as occurring at a *point*. The particles are allowed, in effect, to have well-defined positions and momenta, in complete disregard for the require-

ments of the uncertainty principle, and the exaggerated spatial localization to a focal region about 0.1 fm wide follows automatically.

If semiclassical wave functions (equation (4)) are to be used correctly to evaluate the probability of finding a particle in a region, the probability must be summed over the bundle of rays within the region and one must not try to be specific about which particular ray the particle is travelling along. This point has been made by Eisberg, McCarthy, and Spurrier (1959) in connection with semiclassical calculations of transmission coefficients. Thus the evaluation of the focal contribution to the DWBA matrix element using semiclassical wave functions would require two separate sums or integrations over rays: over those entering the focus, to find the probability of a particle reaching the area, and over those leaving the focus, to find the probability that a particle starting from there will emerge in the backward direction. This is just a complicated way of evaluating $\Psi_{\mathbf{k}_i}^{(+)}(\mathbf{r})$ and $\Psi_{\mathbf{k}_f}^{(-)}(\mathbf{r})$ at the focus. The total wave functions so obtained would then be multiplied by the bound state wave functions and interaction potential (the $F_L(r)P_L(\cos\theta)$ in equation (2)) and the result integrated over the focal region to obtain the focal term. If the calculation were made in this way the focal region would be 3 or 4 fm wide (see the ray diagrams in McCarthy 1959) and the structure of the bound state wave function would be quite lost in the final result. There is, of course, no advantage in following such a procedure, and it might well turn out to be more difficult than exact evaluation of the quantum mechanical matrix element.

The semiclassical model has proved useful in qualitative discussions of angular distribution shapes. The results presented in this paper, however, show that it should not be expected to give detailed quantitative agreement either with the exact quantum mechanical calculation or (therefore) with experiment.

IV. ACKNOWLEDGMENTS

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