ON A GÖDEL-TYPE NONSTATIC COSMOLOGICAL SOLUTION FOR MATTER IN AN ELECTROMAGNETIC FIELD

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Summary

Some solutions of the Einstein–Maxwell equations for gravitational and electromagnetic fields against the background of a rotating and either a stationary or an expanding cosmological model have been obtained. The details of one of these solutions have been given. The solution describes a cosmological model with rotation and shear. The model is initially stationary and then expanding. It is filled with anisotropic fluid and is pervaded by the electromagnetic field, the net charge density being zero.

I. Introduction

In context with the present-day controversial picture regarding the empirical evidence in favour of Mach's philosophy of inertia of matter and the various cosmological issues, it seems quite worth while to look into the possibilities of solutions of the Einstein–Maxwell equations that exhibit universal rotation along the line of approach initiated by Gödel (1949). The better a cosmological solution fits with the physical aspects of the Universe the more will it be accepted as physically reliable. In the actual Universe we observe phenomena like Doppler shift, intergalactic magnetic fields, and, to a certain extent, anisotropy also. Hence in this paper an attempt is made to obtain cosmological solutions for a homogeneous and expanding universe that exhibits time-dependent rotation as well as shear and is filled with anisotropic fluid and pervaded by an electromagnetic field. It is found that such a solution does exist, the net proper charge density in the universe being zero although a nonvanishing electromagnetic field is present. Moreover, there follows a stationary solution for which the proper charge density is non-zero and the background model is pervaded by the electromagnetic field and filled with the perfect fluid. This has been considered earlier by Raval and Vaidya (1967).

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Consequently in what follows we investigate the metric form

$$ds^2 = (dx^0)^2+2e^{x^1}(dx^0)(dx^2)+ae^{2x^1}(dx^2)^2+b(dx^1)^2+c(dx^3)^2, \quad (1)$$

where a, b, and c are supposed to be the functions of time x^0 that are to be determined under the influence of the Einstein–Maxwell equations.

II. The Field Equations

We consider the universe as a Riemannian fourfold system described by the coordinates x^i with the metric g_{il} defining the line element (1). The signature of the g_{il} is given by $(+ - - -)$, while a, b, and c are, for the present, unknown functions of time.

The metric g_{il} is related to the energy-impulse tensor T_{il} through the equations of field

$$G_i^l = R_i^l-\tfrac{1}{2}Rg_i^l+\lambda g_i^l = -8\pi T_i^l. \quad (2)$$

Following the line of approach used by Lichnerowicz (1955), we make the following choice for T_i^l

$$T_i^l = M_i^l+E_i^l = (\rho+p)v_i v^l-pg_i^l+(q-p)V_i V^l-F^{al}F_{ai}+\tfrac{1}{4}\delta_i^l F^{ab}F_{ab}, \quad (3)$$

where ρ is the proper mass density, p is the pressure associated with the x^1 and x^2 axes, q is the pressure associated with the x^3 axis, v^i is the flow vector and is hence timelike, while V^i is a spacelike vector. Also, because v^i and V^i are both unit vectors we require

$$v_i v^i = 1, \qquad V_i V^i = -1. \quad (4)$$

F_{il} is the electromagnetic field tensor satisfying Maxwell's equations

$$F^{il}{}_{;l} = J^i, \qquad F_{il} = k_{i,l}-k_{l,i}, \quad (5)$$

where J^i is the 4-current density and k_i is the 4-potential.

Next we make use of the comoving frame of reference and therefore require that

$$v^i = \delta_0^i. \quad (6)$$

Moreover, we choose V^i with the components

$$V^i = \delta_3^i V^3. \quad (7)$$

For the space–time described by the metric form (1), the nonvanishing components of the mixed Einstein tensor are

$$\left.\begin{aligned}
G_0^0 &= -\frac{a'b'}{4bu}-\frac{a'c'}{4cu}-\frac{ab'c'}{4bcu}-\frac{4a-1}{4bu}+\lambda\,,\\
G_0^1 &= \frac{1}{4bu}\left(\frac{(2a-3)a'}{u}-\frac{(2a-1)b'}{b}+\frac{c'}{c}\right),\\
G_0^2 &= \frac{\mathrm{e}^{-x^1}}{2u}\left(-\frac{b''}{b}-\frac{c''}{c}+\frac{(b')^2}{2b^2}+\frac{(c')^2}{2c^2}+\frac{a'b'}{2bu}+\frac{a'c'}{2cu}\right),\\
G_1^1 &= -\frac{a''}{2u}-\frac{ac''}{2cu}+\frac{(a')^2}{4u^2}+\frac{a(c')^2}{4c^2u}-\frac{(a-2)a'c'}{4cu^2}+\frac{1}{4bu}+\lambda\,,\\
G_1^2 &= \frac{\mathrm{e}^{-x^1}}{4u}\left(-\frac{a'}{u}+\frac{b'}{b}+\frac{c'}{c}\right),\\
G_2^2 &= -\frac{ab''}{2bu}-\frac{ac''}{2cu}+\frac{a(b')^2}{4b^2u}+\frac{a(c')^2}{4c^2u}+\frac{a'b'}{4bu^2}+\frac{a'c'}{4cu^2}-\frac{ab'c'}{4bcu}+\frac{1}{4bu}+\lambda\,,\\
G_3^3 &= -\frac{a''}{2u}-\frac{ab''}{2bu}+\frac{(a')^2}{4u^2}+\frac{a(b')^2}{4b^2u}-\frac{(a-2)a'b'}{4bu^2}-\frac{4a-3}{4bu}+\lambda\,.
\end{aligned}\right\} \quad (8)$$

Here and in what follows the primes denote differentiation with respect to time and u is equivalent to $(a-1)$.

We observe that G_α^3 vanishes identically for $\alpha = 0, 1, 2$. From this it follows that $F^{a3}F_{a\alpha}$ should vanish for $\alpha = 0, 1, 2$. These conditions lead to the following two independent relations for the F_{il}

$$\left.\begin{aligned}
aF_{13}F_{10} - \mathrm{e}^{-x^1}F_{03}F_{20} + \mathrm{e}^{-2x^1}F_{23}F_{20} &= 0\,,\\
aF_{03}F_{10} - \mathrm{e}^{-x^1}F_{23}F_{10} - \mathrm{e}^{-x^1}F_{03}F_{12} + \mathrm{e}^{-2x^1}F_{23}F_{12} &= 0\,.
\end{aligned}\right\} \quad (9)$$

We now choose the 4-potential k_l of the electromagnetic field as

$$k_l = (0, \phi_1, \phi_2\,\mathrm{e}^{x^1}, \phi_3)\,, \quad (10)$$

where ϕ_1, ϕ_2, and ϕ_3 are functions of x^0. This choice is quite general, keeping in view the cosmological nature of our problem. As a result of (10), it follows that

$$F_{13} = 0\,, \qquad\qquad F_{23} = 0\,. \quad (11)$$

Again from (9), we find that

$$F_{03}F_{02} = 0\,, \qquad F_{03}(aF_{01} - \mathrm{e}^{-x^1}F_{21}) = 0\,. \quad (12)$$

This leads to the following two distinct conditions:

$$\left.\begin{array}{lll}\text{Case 1} & F_{02}=0\,, & aF_{01}-\mathrm{e}^{-x^1}F_{21}=0\,; \\ \text{Case 2} & F_{03}=0\,. & \end{array}\right\} \tag{13}$$

III. Solution for Case 1

Here we have

$$F_{02}=0\,,\qquad aF_{01}-\mathrm{e}^{-x^1}F_{21}=0\,. \tag{14}$$

Now $F_{il}=k_{i,l}-k_{l,i}$, with k_i given by (10), would in the present case require that

$$\phi_2=N\,,\qquad \phi_1'=-N/a\,,\qquad \phi_3=\phi_3(x^0)\,, \tag{15}$$

where N is a constant.

Taking into account the second set of Maxwell's equations, that is, $F^{il}_{\ ;l}=J^i$, we obtain

$$J^0=0\,,\qquad \phi_3'=M(-cu/b)^{\frac{1}{2}}\,, \tag{16}$$

where J^0 is the proper charge density and M is a constant.

Finally the gravitational field equations in (2) imply that

$$\left.\begin{aligned}
G_0^0 &= -8\pi\left(\rho+\frac{N^2}{2ab}-\frac{a(\phi_3')^2}{2cu}\right),\\
G_0^1 &= 0\,,\\
G_0^2 &= -8\pi\frac{\mathrm{e}^{-x^1}}{a}\left(-\frac{N^2}{ab}+\frac{a(\phi_3')^2}{cu}\right),\\
G_1^1 &= -8\pi\left(-p-\frac{N^2}{2ab}+\frac{a(\phi_3')^2}{2cu}\right),\\
G_1^2 &= 0\,,\\
G_2^2 &= -8\pi\left(-p-\frac{N^2}{2ab}+\frac{a(\phi_3')^2}{2cu}\right),\\
G_3^3 &= -8\pi\left(-q+\frac{N^2}{2ab}-\frac{a(\phi_3')^2}{2cu}\right).
\end{aligned}\right\} \tag{17}$$

From the equations (17), on elimination of the physical quantities we obtain the following four equations for the three unknown functions a, b, and c

$$G_0^1 = 0\,, \tag{18a}$$

$$ba^2 e^{x^1} G_0^2 - 8\pi(N^2 + a^2 M^2) = 0\,, \tag{18b}$$

$$G_1^2 = 0\,, \tag{18c}$$

$$G_1^1 - G_2^2 = 0\,. \tag{18d}$$

Equations (18a) and (18c) lead to the following solution

$$b = \alpha a, \qquad c = \beta(a-1)/a\,, \tag{19}$$

where α and β are the constants of integration. Because of (19) equation (18d) is found to be identically satisfied by the function a. Finally, equation (18b) leads to the following differential equation for the function a

$$a'' - \frac{a^2+a-1}{a^2(a-1)}(a')^2 + \frac{16\pi(N^2+a^2M^2)}{\alpha a^3}(a-1)^2 = 0\,. \tag{20}$$

IV. Solution for Case 2

Here we have

$$F_{03} = 0\,. \tag{21}$$

Now $F_{il} = k_{i,l} - k_{l,i}$, with k_i given by (10), would in the present case require that

$$\phi_3 = \text{constant}\,. \tag{22}$$

Taking into account the second set of Maxwell's equations, that is, $F^{il}{}_{;l} = J^i$, we obtain the following three equations which should be satisfied by the ϕ's

$$\left.\begin{aligned} J^0 &= -(a\phi_1' + \phi_2)/b(a-1)\,, \\ \phi_1'' + \left(\frac{a'}{a} - \frac{a'}{2u} - \frac{b'}{2b} + \frac{c'}{2c}\right)\phi_1' + \frac{\phi_2'}{a} + \frac{1}{2a}\left(\frac{a'}{u} + \frac{b'}{b} - \frac{c'}{c}\right)\phi_2 &= 0\,, \\ \phi_2'' - \frac{1}{2}\left(\frac{a'}{u} - \frac{b'}{b} - \frac{c'}{c}\right)\phi_2' &= 0\,, \end{aligned}\right\} \tag{23}$$

where J^0 denotes the proper charge density.

Now the gravitational field equations in (2) imply that the following relations should be satisfied

$$\left.\begin{aligned}
G_0^0 &= -8\pi\left(\rho-\frac{a(\phi_1')^2}{2bu}-\frac{(\phi_2')^2}{2u}+\frac{\phi_2^2}{2bu}\right),\\
G_0^1 &= -8\pi\left(-\frac{\phi_2}{bu}\right)\left(\phi_2+\phi_1'\right),\\
G_0^2 &= -8\pi\left(\frac{\phi_1'\,e^{-x^1}}{bu}\right)\left(\phi_2+\phi_1'\right),\\
G_1^1 &= -8\pi\left(-p-\frac{a(\phi_1')^2}{2bu}-\frac{\phi_2\phi_1'}{bu}+\frac{(\phi_2')^2}{2u}-\frac{\phi_2^2}{2bu}\right),\\
G_1^2 &= -8\pi\left(-\frac{\phi_1'\phi_2'\,e^{-x^1}}{u}\right),\\
G_2^2 &= -8\pi\left(-p+\frac{a(\phi_1')^2}{2bu}-\frac{(\phi_2')^2}{2u}-\frac{\phi_2^2}{2bu}\right),\\
G_3^3 &= -8\pi\left(-q+\frac{a(\phi_1')^2}{2bu}+\frac{\phi_2\phi_1'}{bu}+\frac{(\phi_2')^2}{2u}+\frac{\phi_2^2}{2bu}\right).
\end{aligned}\right\} \tag{24}$$

The equations (23) and (24), after elimination of the physical quantities p, q, ρ, and J^0, imply the following six differential equations for the functions a, b, c, ϕ_1', and ϕ_2

$$\phi_1''+\left(\frac{a'}{a}-\frac{a'}{2u}-\frac{b'}{2b}+\frac{c'}{2c}\right)\phi_1'+\frac{\phi_2'}{a}+\frac{1}{2a}\left(\frac{a'}{u}+\frac{b'}{b}-\frac{c'}{c}\right)\phi_2=0\,, \tag{25a}$$

$$\phi_2''-\frac{1}{2}\left(\frac{a'}{u}-\frac{b'}{b}-\frac{c'}{c}\right)\phi_2'=0\,, \tag{25b}$$

$$8\pi(\phi_2')^2G_0^2+uG_1^2G_0^1=0\,, \tag{25c}$$

$$8\pi\{u(\phi_1')^2-b(\phi_2')^2\}-bu(G_1^1-G_2^2)-bue^{x^1}G_0^2=0\,, \tag{25d}$$

$$8\pi(\phi_1')^2G_0^1+ue^{2x^1}G_1^2G_0^2=0\,, \tag{25e}$$

$$8\pi\phi_2^2G_0^1G_1^2+uG_0^2(bG_0^1-e^{x^1}G_1^2)^2=0\,. \tag{25f}$$

With six differential equations for the five unknown functions a, b, c, ϕ_1', and ϕ_2, this system of equations (25) is, in general, inconsistent. However, in the case when ϕ_2 is a constant, say N, equation (25b) is identically satisfied and (25c), (25e), and (25f) lead to the following two independent equations

$$G_0^1 = 0\,, \tag{26a}$$

$$G_1^2 = 0\,. \tag{26b}$$

We are thus left with the four equations (25a), (25d), (26a), and (26b) for the four functions a, b, c, and ϕ_1', which can be solved very easily and lead to the following two solutions for the system of equations (25).

The first solution requires that

$$\left.\begin{array}{ccc} \phi_1' = \text{constant}, & \phi_2 = \text{constant}, & \\ a = \text{constant}, & b = \text{constant}, & c = \text{constant}. \end{array}\right\} \tag{27}$$

The second solution requires that

$$\left.\begin{array}{cc} \phi_1' = -N/a\,, & \phi_2 = N\,, \\ b = \alpha a\,, & c = \beta(a-1)/a\,, \end{array}\right\} \tag{28}$$

where N, α, and β are the constants of integration, while the function a satisfies the differential equation

$$a'' - \frac{a^2+a-1}{a^2(a-1)}(a')^2 + \frac{16\pi N^2(a-1)^2}{\alpha a^3} = 0\,. \tag{29}$$

The solution (27) has already been discussed by Raval and Vaidya (1967), and describes the electromagnetic field due to the constant rotation of a stationary universe when there is a charge distribution present. This charge distribution may be supposed to be due to the excess of the magnitude of the positive charge on the proton over that of the negative charge on the electron. The solution (28) is new. When N is put equal to zero in equation (29), the electromagnetic field disappears and (29) reduces to the equation that was investigated earlier by Raval and Vaidya (1966).

A comparison of equations (29) and (20) shows that when M is put equal to zero in (20) it reduces to (29). Hence the solution (28) follows as a particular case of the solution described in Section III. We shall therefore only discuss the details of this latter solution.

V. Discussion of Solution for Case 1

The geometrical component of the solution for case 1 has been described in Section III. That is, the functions a, b, and c in the metric form

$$ds^2 = (dx^0)^2 + 2e^{x^1}(dx^0)(dx^2) + ae^{2x^1}(dx^2)^2 + b(dx^1)^2 + c(dx^3)^2 \tag{30}$$

are expressed by the relations

$$\left.\begin{aligned} b &= \alpha a\,, \qquad\qquad c = \beta(a-1)/a, \\ a'' - \frac{a^2+a-1}{a^2(a-1)}(a')^2 &+ \frac{16\pi(N^2+a^2M^2)}{\alpha a^3}(a-1)^2 = 0\,. \end{aligned}\right\} \tag{31}$$

The first integral of the differential equation in (31) is given by

$$(a')^2 = \frac{\mu(a-1)^2}{e^{2/a}}\left(\frac{\nu}{\mu} + \frac{a-2}{a}e^{2/a} + \frac{4M^2}{N^2}f(a)\right), \tag{32}$$

where

$$f(a) = \int_\varepsilon^a a^{-1}e^{2/a}\,da \qquad (\epsilon = \text{constant}),$$

$\mu = -8\pi N^2/\alpha$, and ν is an arbitrary constant of integration. We may write $(a')^2$ in the form

$$(a')^2 = \mu y(a-1)^2 e^{-2/a}, \tag{33}$$

where
$$y \equiv \frac{\nu}{\mu} + \frac{a-2}{a}e^{2/a} + \frac{4M^2}{N^2}f(a)\,.$$

It is required that $(a')^2$ be greater than zero and the resulting conditions must be imposed upon μ and y.

The expressions for nonvanishing F_{il} are

$$F_{12} = -Ne^{x^1}, \qquad F_{01} = N/a, \qquad F_{03} = -M(-cu/b)^{\frac{1}{2}}\,. \tag{34}$$

The expressions for p, q, ρ, and the spur T of T_i^l are

$$\begin{aligned} -8\pi p = &\left\{\left(\frac{a+1}{e^{2/a}} + \frac{a-1}{1}\frac{\mu}{\nu}\right)\nu - \frac{1}{\alpha(a-1)}\right\}\left(\frac{1}{4a}\right) - \lambda \\ &- \frac{8\pi M^2}{\alpha}\left\{\frac{1}{2} + \left(\frac{a+1}{a}\right)\frac{f(a)}{e^{2/a}}\right\}, \end{aligned} \tag{35a}$$

$$\begin{aligned} -8\pi q = &\left\{\left(\frac{3a^2+3a-4}{ae^{2/a}} + \frac{3(a-1)}{1}\frac{\mu}{\nu}\right)\nu + \frac{4a-3}{\alpha(a-1)}\right\}\left(\frac{1}{4a}\right) - \lambda \\ &- \frac{8\pi M^2}{\alpha}\left\{\frac{1}{2} + \frac{2(a-1)}{a} + \left(\frac{3a^2+3a-4}{a^2}\right)\frac{f(a)}{e^{2/a}}\right\}, \end{aligned} \tag{35b}$$

$$\begin{aligned} -8\pi\rho = &\left\{\left(\frac{a+1}{e^{2/a}} + \frac{a-1}{1}\frac{\mu}{\nu}\right)\nu + \frac{4a-1}{\alpha(a-1)}\right\}\left(-\frac{1}{4a}\right) + \lambda \\ &+ \frac{8\pi M^2}{\alpha}\left\{\frac{1}{2} + \left(\frac{a+1}{a}\right)\frac{f(a)}{e^{2/a}}\right\}, \end{aligned} \tag{35c}$$

$$-8\pi T = \left\{\left(\frac{3a^2+3a-2}{ae^{2/a}}+\frac{3(a-1)}{1}\frac{\mu}{\nu}\right)\nu+\frac{4a-3}{\alpha(a-1)}\right\}\left(-\frac{1}{2a}\right)+4\lambda$$
$$+\frac{16\pi M^2}{\alpha}\left\{\frac{2a-1}{a}+\left(\frac{3a^2+3a-2}{a^2}\right)\frac{f(a)}{e^{2/a}}\right\}. \tag{35d}$$

We require that p, q, and T should be non-negative while ρ should be positive.

In the present paper we have used -2 for the signature of the metric (30). Because of this, the determinant g of the metric tensor g_{ij} should be negative. This implies that $bc(a-1)e^{2x^1}$ should also be negative. Therefore, using the values for b and c in terms of a from (31), we require that $\alpha\beta$ should be negative. Consequently α and β should have opposite signs and hence also a and $(a-1)/a$. We are thus at liberty to consider only the two following ranges for the function a

$$-\infty < a < 0, \qquad 0 < a < 1. \tag{36}$$

However, the expressions for p and ρ (equations (35a) and (35c)) imply that

$$8\pi(\rho+p) = 1/\alpha(a-1). \tag{37}$$

It is essential that $(\rho+p)$ be greater than zero. However, since the x^1 coordinate is spacelike, b must be negative and consequently, in the range $-\infty < a < 0$, α must be chosen to be positive. This would indicate that if a is in the range $-\infty < a < 0$, $(\rho+p)$ is less than zero. As a result, we consider only the range $0 < a < 1$ for the values of a.

Since b should be negative in the interval $0 < a < 1$, we require $\alpha < 0$ and hence $\beta > 0$. Consequently μ should be greater than zero. So, in the interval $0 < a < 1$, we require

$$\alpha < 0, \qquad \beta > 0, \qquad \mu > 0. \tag{38}$$

Again, because $(a')^2$ should be always positive, μ and y must possess the same sign and consequently y must also be positive for $0 < a < 1$. It will be seen that y is a monotonically increasing function of a in this range and that, if

$$-e^2+\frac{\nu}{\mu}+\frac{4M^2}{N^2}\Big(f(1)-f(0)\Big) > 0, \tag{39}$$

y will vanish for a value of $a = a_0$ in the range $0 < a < 1$. The requirement that y be positive will be satisfied if the constants μ (or N), ν, and M satisfy (39) and if, further, the interval of a is restricted to $a_0 < a < 1$.

In the solution up to this stage, there are several constants that remain undetermined. The further physical requirements that ρ be positive and p, q, and T be non-negative can be satisfied by suitably choosing these constants, in exactly the same manner as was done in an earlier paper (Raval and Vaidya 1966).

Consider now a local observer using a Minkowskian reference frame in his neighbourhood. The electromagnetic field as observed by him can be visualized by choosing the following tetrads at a point.

$$\left.\begin{aligned} \lambda^{i}_{(0)} &= (0,0,a^{-\frac{1}{2}}\mathrm{e}^{-x^1},0), \\ \lambda^{i}_{(1)} &= (0,(-b)^{-\frac{1}{2}},0,0), \\ \lambda^{i}_{(2)} &= (a^{\frac{1}{2}}(1-a)^{-\frac{1}{2}},0,-a^{-\frac{1}{2}}(1-a)^{-\frac{1}{2}}\mathrm{e}^{-x^1},0), \\ \lambda^{i}_{(3)} &= (0,0,0,(-c)^{\frac{1}{2}}). \end{aligned}\right\} \tag{40}$$

Using (40), we form the tetrad components $F_{(ab)}$ of F_{il} as

$$F_{(ab)} = \lambda^{i}_{(a)}\,\lambda^{l}_{(b)}\,F_{il} = -F_{(ba)}\,. \tag{41}$$

It is found that the only nonvanishing components of $F_{(ab)}$ are

$$\left.\begin{aligned} F_{(23)} &= H_1 = -M(-\alpha)^{-\frac{1}{2}}, \\ F_{(10)} &= E_1 = -N(-\alpha)^{-\frac{1}{2}}a^{-1}, \end{aligned}\right\} \tag{42}$$

where H_α and E_α ($\alpha = 1,\ 2,\ 3$) are the components of the magnetic and electric fields respectively, in the local Minkowskian frame. Thus an observer finds in his neighbourhood an electric field or a magnetic field in the direction $\lambda^{i}_{(1)}$.

The scalar of expansion θ, in the present case, is found to be

$$\theta = \frac{-(\mu)^{\frac{1}{2}}}{\mathrm{e}^{1/a}}\left(\frac{\nu}{\mu}+\frac{a-2}{a}\mathrm{e}^{2/a}+\frac{4M^2}{N^2}f(a)\right)^{\frac{1}{2}}. \tag{43}$$

With this value of θ, the components of the angular-velocity vector ω^i and shear tensor q_{il} take the same expressions as given earlier (Raval and Vaidya 1966). Thus, from geometrical considerations, the universe described here possesses properties similar to the one described in the earlier paper.

VI. Conclusions

As β is positive and not appearing in any g_{il} except g_{33}, we can choose the x^3 coordinate in such a way as to make $\beta = 1$. Thus the metric form (30) can be written as

$$\mathrm{d}s^2 = (\mathrm{d}x^0)^2+2\mathrm{e}^{x^1}(\mathrm{d}x^0)(\mathrm{d}x^2)+a\mathrm{e}^{2x^1}(\mathrm{d}x^2)^2+\alpha a(\mathrm{d}x^1)^2-a^{-1}(1-a)(\mathrm{d}x^3)^2, \tag{44}$$

where

$$a' = \pm\frac{\mu^{\frac{1}{2}}(1-a)}{\mathrm{e}^{1/a}}\left(\frac{\nu}{\mu}+\frac{a-2}{a}\mathrm{e}^{2/a}+\frac{4M^2}{N^2}f(a)\right)^{\frac{1}{2}}. \tag{45}$$

The metric form (44) has four disposable constants α, ν, μ (or N), and M. These could be chosen by stipulating certain initial conditions and the particular models could be evaluated. Next, for an expanding model, the scalar of expansion θ, that is, $a'/(a-1)$, must be positive. Hence, since $a < 1$, a' must be negative and so a must decrease. Now equation (45) shows that by choosing the appropriate constants we can start with an initial situation with $a' = 0$ and then, as a decreases, a' can be maintained negative. Hence for this solution, we can start with a stationary model and then pass over to an expanding model.

In the present analysis we have thus formulated a cosmological model with rotation and shear which is initially stationary and then expanding. It is filled with anisotropic fluid and is pervaded by the electromagnetic field, the net proper charge density being zero. By choosing the proper order for the rate of expansion a', the model could be made to have a required lifetime, after which the density in the model would become negative and would thus lose physical significance. The geometry of the model is very similar to that of Gödel's model.

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