R-MATRIX FIT TO ^{9}Be$(\gamma,\mathrm{n})^8$Be CROSS SECTION NEAR THRESHOLD

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[*Manuscript received March* 12, 1968]

Summary

Satisfactory fits to the ^{9}Be$(\gamma,\mathrm{n})^8$Be cross section just above the neutron threshold are obtained with the one-level approximation of R-matrix theory but not with that of complex eigenvalue theory. Restrictions on the R-matrix parameters from other sources are discussed.

I. Introduction

The ^{9}Be$(\gamma,\mathrm{n})^8$Be cross section is observed to have a peak just above the neutron threshold (Lauritsen and Ajzenberg-Selove 1966). Mahaux (1965) has suggested that analysis of the cross section in this region should show significant differences between the R-matrix and complex eigenvalue theories of nuclear reactions.

It is not obvious that these differences exist, as the cross section formulae of the two theories, in the one-level approximation and for s-wave neutron emission, have the same forms of energy dependence in both numerators and denominators. The expressions appear to be equivalent; however, differences can arise due to the restrictions on the parameters of the two theories.

II. R-matrix and Complex Eigenvalue Expressions for Cross Section

In the one-level approximation of R-matrix theory (Lane and Thomas 1958), the contribution to the cross section from a level of spin J is

$$\sigma_R(\gamma,\mathrm{n}) = \frac{(2J+1)\pi\hbar^2c^2}{8}\frac{\Gamma_\gamma\Gamma_\mathrm{n}}{E_\gamma^2}\frac{}{(E_R+\Delta-E)^2+\frac{1}{4}\Gamma^2}. \tag{1}$$

As is usual, we retain in the denominator the contributions to Δ and Γ from the neutron channel only. Then for a $\frac{1}{2}^+$ level formed by E1 γ-radiation and decaying by s-wave neutrons, and for $E>0$ (with E measured from the neutron threshold, so that $E_\gamma = E+E_\mathrm{T}$ with $E_\mathrm{T} = 1\cdot665$ MeV), we have

$$\Gamma_\gamma = \frac{16\pi}{9}\frac{e^2}{\hbar^3c^3}E_\gamma^3 B, \qquad \Gamma_\mathrm{n} = 2(\epsilon_R E)^{\frac{1}{2}}, \qquad \Delta_\mathrm{n} = 0. \tag{2}$$

Here $B \equiv B(\mathrm{E1}, \frac{1}{2}^+ \to \frac{3}{2}^-)$ and $\epsilon_R = 2M_\mathrm{n}a_\mathrm{n}^2\gamma_\mathrm{n}^4/\hbar^2 > 0$, where M_n, a_n, and γ_n^2 are

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the reduced mass, channel radius, and reduced width for the ^{8}Be+n channel, and the boundary condition parameter B_n is taken as zero. Thus (1) becomes

$$\sigma_R(\gamma, \mathrm{n}) = \alpha_R E_\gamma E^{\frac{1}{2}}/(E^2+\beta_R E+\gamma_R), \tag{3}$$

with

$$\alpha_R = \frac{8\pi^2 e^2}{9\ \hbar c}\epsilon_R^{\frac{1}{2}} B, \qquad \beta_R = \epsilon_R - 2E_R, \qquad \gamma_R = E_R^2. \tag{4}$$

Similarly in the complex eigenvalue theory (Mahaux 1965), the one-level approximation can be written

$$\sigma_S(\gamma, \mathrm{n}) = \alpha_S E_\gamma E^{\frac{1}{2}}/(E^2+\beta_S E+\gamma_S), \tag{5}$$

with

$$\alpha_S = \frac{\pi}{4}\frac{\hbar^2 c^2 \bar{G}_{\gamma\mathrm{n}}\bar{G}_{\mathrm{nn}}}{(E_S+E_T)^3\,|\,E_S\,|^{\frac{1}{2}}}, \qquad \beta_S = -2E_S, \qquad \gamma_S = E_S^2+\tfrac{1}{4}\Gamma_S^2, \tag{6}$$

using the notation of Mahaux except that E_S and Γ_S replace E_{nn} and Γ_{n}. Thus the cross sections (3) and (5) are identical provided $\alpha_R = \alpha_S$, $\beta_R = \beta_S$, and $\gamma_R = \gamma_S$, which makes it appear that the two theories can give identical fits to any experimental $\sigma(\gamma, \mathrm{n})$.

There are, however, restrictions on the values of the parameters (4) and (6), i.e. $\gamma_R \geqslant 0$ and $\gamma_S \geqslant \frac{1}{4}\beta_S^2$. These imply that any cross section that is an admissible form of σ_S is also an admissible form of σ_R, with $E_R = +(E_S^2+\frac{1}{4}\Gamma_S^2)^{\frac{1}{2}}$ and $\epsilon_R = 2(E_R - E_S)$, giving $\epsilon_R \leqslant 4E_R$. On the other hand, there are some cross sections that are admissible forms of σ_R with $\epsilon_R > 4E_R$ and these are inadmissible forms of σ_S. In this latter case the same values of α_R, β_R, and γ_R correspond to two sets of values of E_R, ϵ_R, and B, related by

$$E_{R2} = -E_{R1}, \qquad \epsilon_{R2} = \epsilon_{R1} - 4E_{R1}, \qquad B_2 = B_1(\epsilon_{R1}/\epsilon_{R2})^{\frac{1}{2}}. \tag{7}$$

The solution with $E_R < 0$ can be rejected on the following grounds. For reactions of the type ^{9}Be(p, p′)^{9}Be*, the cross section in the one-level R-matrix approximation is (Lane and Thomas 1958; Barker and Treacy 1962)

$$\sigma_R(\mathrm{p}, \mathrm{p}') \propto \tfrac{1}{2}(\Gamma_\gamma+\Gamma_{\mathrm{n}})/\{(E_R+\Delta-E)^2+\tfrac{1}{4}\Gamma^2\}. \tag{8}$$

Whereas $\sigma_R(\gamma, \mathrm{n})$ given by (1) vanishes for $E < 0$ because of the factor Γ_{n} in the numerator, $\sigma_R(\mathrm{p}, \mathrm{p}')$ in the case $E_R < 0$ has a sharp peak at $E = E_{\mathrm{r}} < 0$, where $E_{\mathrm{r}} = E_R + \Delta_{\mathrm{n}}(E_{\mathrm{r}})$ (as was discussed by Barker and Treacy (1962) for a similar case in ^{17}F). If $E_R \geqslant 0$ the cross section $\sigma_R(\mathrm{p}, \mathrm{p}')$ is zero for $E < 0$. Experimentally Spencer, Phillips, and Young (1960) found no peak below the neutron threshold, so fits with $E_R < 0$ (bound states) can be excluded, contrary to the previous conclusion of Corman, Sherwood, and John (1963).

III. Fits to ^{9}Be(γ, n)^{8}Be Cross Section near Neutron Threshold

Mahaux (1965) used the complex eigenvalue formulae (5) and (6) to fit experimental values (Gibbons *et al.* 1959; John and Prosser 1962) of $\sigma(\gamma, \mathrm{n})$ at four energies E between 26 and 185 keV, obtaining $E_S = -85$ keV, $\Gamma_S = 0$, and $\alpha_S = 1 \cdot 87$ mbn keV$^{\frac{1}{2}}$. These give a peak of $\sigma_S(\gamma, \mathrm{n})$ at $E = 29$ keV. Exactly the same fit is obtained with the R-matrix formulae (3) and (4) with $E_R = 85$ keV, $\epsilon_R = 340$ keV, and $B = 1 \cdot 58$ mbn. An earlier R-matrix fit (Barker and Treacy 1962) to the data of Gibbons *et al.* (1959) used a calculated value $\epsilon_R = 820$ keV and obtained $E_R = 110$ keV and $B = 2 \cdot 22$ mbn, giving the peak at $E = 19$ keV.

Recently Berman, Van Hemert, and Bowman (1967) measured $\sigma(\gamma, \mathrm{n})$ from < 1 to about 40 keV above the neutron threshold and found that the peak occurs at $E \simeq 6$ keV, in marked disagreement with the previous fits to $\sigma(\gamma, \mathrm{n})$. Berman, Van Hemert, and Bowman attempted to fit their own and the earlier data using the R-matrix formulae (3) and (4) but did not obtain a satisfactory fit. This appears to be due to the use of unrealistic weights for the data points. Berman, Van Hemert, and Bowman used a linear least squares fit to a quantity Y, which is an energy-dependent function of σ, and gave all the data points equal weights in Y. This corresponds to very unequal weights in σ, e.g. the highest energy point of Berman, Van Hemert, and Bowman had a weight 100 times greater than the lowest energy point, and the highest energy point of the earlier data had a weight 6000 times greater. Consequently the low energy points contributed little to the fit and the peak was obtained at too high an energy.

We measure the goodness of fit by

$$X_R = N^{-1} \sum_{i=1}^{N} |\{\sigma_R(E_i) - \sigma(E_i)\}/\eta(E_i)|^2, \tag{9}$$

where $\sigma(E_i)$ and $\eta(E_i)$ are the measured cross section and error at the energy E_i, $\sigma_R(E_i)$ is calculated from (3) and (4), and $N = 86$ (two points from Gibbons *et al.* 1959, three from John and Prosser 1962, and 81 from Berman, Van Hemert, and Bowman 1967). The assigned experimental errors are used for the earlier data and we take equal errors of $0 \cdot 1$ mbn for all the cross sections of Berman, Van Hemert, and Bowman, on the basis of the scatter of their points. Our results are insensitive to changes in this value of $0 \cdot 1$ mbn.

Berman, Van Hemert, and Bowman normalized their experimental cross sections to the earlier measurements at 26 keV. A somewhat better overall fit may be obtained by increasing their cross sections by about 15%; this seems to be justifiable as their absolute cross sections were subject to an overall uncertainty of about 40%. With this increase of 15%, the minimum values of X_R obtained by varying E_R and ϵ_R for fixed values of B are shown in Figure 1. The minimum X_R stays essentially constant at about $1 \cdot 1$ for $B \gtrsim 3$ mbn. The corresponding values of E_R and ϵ_R are given within about 10% by

$$E_R \simeq 31\,B \quad \mathrm{mbn}^{-1}\,\mathrm{keV}, \qquad \epsilon_R \simeq 200\,B^2 \quad \mathrm{mbn}^{-2}\,\mathrm{keV}. \tag{10}$$

As examples the fits to the experimental $\sigma(\gamma, \mathrm{n})$ are shown in Figure 2 for the two cases

$$B = 2\cdot 0 \text{ mbn}, \qquad E_R = 63 \text{ keV}, \qquad \epsilon_R = 0\cdot 82 \text{ MeV}, \tag{11}$$

and

$$B = 5\cdot 0 \text{ mbn}, \qquad E_R = 151 \text{ keV}, \qquad \epsilon_R = 4\cdot 59 \text{ MeV}. \tag{12}$$

As B increases, the high energy tail of $\sigma_R(\gamma, \mathrm{n})$ increases, since (for $E_R \geqslant 0$)

$$\int_0^\infty E_\gamma^{-1}\, \sigma_R(\gamma, \mathrm{n})\, \mathrm{d}E = (8\pi^3/9)(e^2/\hbar c)B. \tag{13}$$

The increase of X_R as B decreases below 3 mbn is due mainly to poorer fits to the earlier data, and the fits for B much less than 2 mbn are regarded as unacceptable.

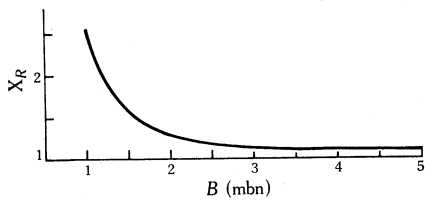

Fig. 1.—Quantity X_R, which measures the goodness of fit of the R-matrix one-level approximation to $\sigma(\gamma, \mathrm{n})$, shown as a function of the transition matrix element B.

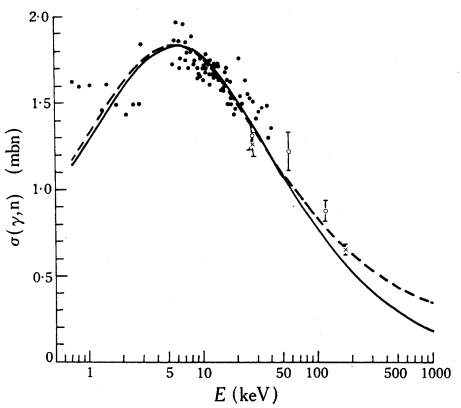

Fig. 2.—Cross section of $^9\mathrm{Be}(\gamma, \mathrm{n})^8\mathrm{Be}$ shown as a function of energy E above neutron threshold. The curves are calculated fits in the R-matrix one-level approximation with the parameter values of (11) (solid curve) and of (12) (dashed curve). The experimental points are from:

x, Gibbons *et al.* (1959)
o, John and Prosser (1962)
●, Berman, Van Hemert, and Bowman (1967)

The fits for $B \gtrsim 5$ mbn appear to be significantly better than those obtained by Berman, Van Hemert, and Bowman (1967).

From the relations (10), it follows that the acceptable R-matrix fits have $\epsilon_R > 4E_R$ and therefore these same fits are not obtainable from the complex eigenvalue formulae (5) and (6). In fact the smallest value of the quantity X_S, corresponding to (9) with σ_S replacing σ_R, is about $5\cdot 0$, and in this case the fit to the earlier data is much poorer than in the best R-matrix fits, the contribution of these points to X_S being about 20 times that to X_R. This is due to the fact that, for a given peak energy, σ_R can have a larger high energy tail than σ_S.

Thus the measured ^{9}Be$(\gamma,\mathrm{n})^8$Be cross section just above the neutron threshold can be fitted satisfactorily by the R-matrix one-level approximation, but the one-level approximation of the complex eigenvalue theory cannot give a satisfactory fit.

IV. Further Restrictions on R-matrix Parameters

In the previous section, satisfactory R-matrix fits to $\sigma(\gamma,\mathrm{n})$ for $E \lesssim 200$ keV are obtained for a range of values of B, E_R, and ϵ_R. Further experimental information may be used to restrict the values of these parameters.

Jakobson (1961) measured $\sigma(\gamma,\mathrm{n})$ at higher energies using bremsstrahlung, and obtained a minimum in σ of about $(0\cdot27\pm0\cdot07)$mbn at $E_\gamma \simeq 2\cdot2$ MeV. A radioactive source measurement at $E_\gamma = 2\cdot185$ MeV gave $\sigma = (0\cdot39\pm0\cdot06)$mbn (Hamermesh and Kimball 1953). Contributions to σ from other levels of ^{9}Be, such as the $\frac{1}{2}^-$ level expected at about $2\cdot4$ MeV excitation energy (Barker 1966), may start to be appreciable at about these energies. Thus we require $\sigma_R(\gamma,\mathrm{n}) \lesssim 0\cdot3$ mbn at $E \simeq 500$ keV, and this implies $B \lesssim 2\cdot0$ mbn (cf. Fig. 2). Acceptable fits to all the $\sigma(\gamma,\mathrm{n})$ data therefore need $B \simeq 2\cdot0$ mbn, and parameter values near those given in (11). These correspond to $\Gamma_\gamma = 0\cdot98$ eV at $E_\gamma = 1\cdot67$ MeV.

From the inelastic scattering of high energy electrons on ^{9}Be, Nguyen Ngoc, Hors, and Perez y Jorba (1963) have given a value $B = (10\cdot4\pm1\cdot2)$mbn. The reliability of this value is doubtful, however, as a more recent similar measurement at Darmstadt has led to a form factor which is 15 times smaller than that of Nguyen Ngoc, Hors, and Perez y Jorba and is compatible with the $\sigma(\gamma,\mathrm{n})$ measurements (Clerc, personal communication). There is the same difficulty in extracting a unique value of B from ^{9}Be$(\mathrm{e},\mathrm{e}')^9$Be* as from ^{9}Be$(\gamma,\mathrm{n})^8$Be, due to possible variation in the high energy tail of the peak.

A calculated value of B, based on a weak coupling model for the $\frac{1}{2}^+$ state of ^{9}Be (Barker 1961), is $B = 3\cdot3$ mbn, but this number depends on some approximations of unknown accuracy.

A given value of ϵ_R implies a relation between a_n and γ_n^2, or alternatively between a_n and the dimensionless reduced width $\theta_\mathrm{n}^2 = \gamma_\mathrm{n}^2(\hbar^2/M_\mathrm{n}a_\mathrm{n}^2)^{-1}$. From the definition of ϵ_R and the relations (10), one has

$$a_\mathrm{n}/\theta_\mathrm{n}^2 \simeq 22\,B^{-1}\ \mathrm{mbn\,fm}\,. \tag{14}$$

The conventional value of the channel radius $1\cdot45(A_1^{\frac{1}{3}}+A_2^{\frac{1}{3}})$fm is in this case $a_\mathrm{n} = 4\cdot35$ fm, and one might reasonably expect $a_\mathrm{n} \gtrsim 3$ fm. Also one expects $\theta_\mathrm{n}^2 \lesssim 1$ (a value estimated from the weak coupling model (Barker 1961) is $0\cdot41$), and these with (14) lead to $B \lesssim 7$ mbn.

Alternatively, another relation between a_n and θ_n^2 may be obtained by fitting the width of the analogous $\frac{1}{2}^+$ level of the mirror nucleus ^{9}B (assuming the same channel radius and reduced width for the ^{8}Be+p channel as for the ^{8}Be+n channel). This first excited state of ^{9}B has apparently been seen at an excitation energy E_x of about $1\cdot5$ MeV and with an observed width Γ^o of order 1 MeV (Islam and Treacy

1965; Treacy, personal communication; Slobodrian *et al.* 1967). For a given value of Γ° and from (14), values of a_{n} and θ_{n}^2 may be obtained for each value of B. These are shown in Figure 3 for $\Gamma^{\circ} = 1\cdot0$, $1\cdot5$, and $2\cdot0$ MeV (at $E_x = 1\cdot5$ MeV). For $B \simeq 2$ mbn, the values of a_{n} and θ_{n}^2 are reasonable (3 fm $\lesssim a_{\mathrm{n}} \lesssim 5$ fm, $0\cdot3 \lesssim \theta_{\mathrm{n}}^2 \lesssim 0\cdot5$) for $1\cdot2$ MeV $\lesssim \Gamma^{\circ} \lesssim 1\cdot5$ MeV. If B were much larger than 2 mbn, the $\frac{1}{2}^+$ level of ^{9}B would probably be too broad to be identified in reactions.

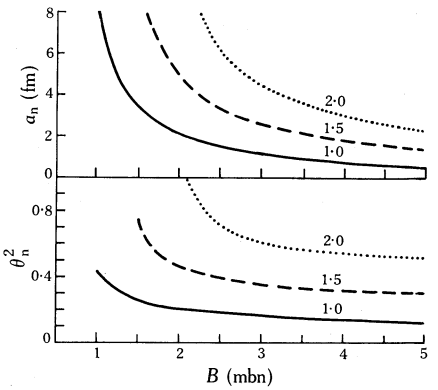

Fig. 3.—Channel radius a_{n} and dimensionless reduced width θ_{n}^2 shown as functions of the transition matrix element B for simultaneous fits to ^{9}Be(γ, n)^{8}Be cross section and to width Γ° (values on curves in MeV) of ^{9}B first excited state.

V. Discussion

The R-matrix one-level approximation with parameters for the $\frac{1}{2}^+$ level of ^{9}Be given in (11) appears to fit satisfactorily the experimental data from ^{9}Be(γ, n)^{8}Be and ^{9}Be(e, e′)^{9}Be* in the region of ^{9}Be excitation energies just above the neutron threshold, and gives an observed width Γ° of about $1\cdot4$ MeV for the corresponding $\frac{1}{2}^+$ level of ^{9}B.

With these parameter values, $\sigma_R(\gamma, \mathrm{n})$ peaks at $E \simeq 6$ keV, in agreement with the data of Berman, Van Hemert, and Bowman (1967). These were obtained with ^{9}Be targets of thickness $0\cdot5$ and $1\cdot0$ in., and it seems likely that neutron energy loss in the targets could be appreciable† as the mean free path is of order $0\cdot5$ in. and the mean energy loss per collision about 20%. Some evidence to support this comes from fits to the data from the $0\cdot5$ in. target and from the $1\cdot0$ in. target separately; the former gives the peak energy as $8\cdot3$ keV, the latter as $4\cdot7$ keV. A rough method of overcoming this discrepancy, assumed to be due to neutron energy loss in the targets, is to increase all the values of E_i obtained with the $0\cdot5$ in. target by a factor $8\cdot3/4\cdot7 = 1\cdot77$, and all those obtained with the $1\cdot0$ in. target by $(1\cdot77)^2 = 3\cdot12$. Acceptable R-matrix fits to these and the earlier data for $E \lesssim 200$ keV are obtained for $B \gtrsim 1\cdot5$ mbn and $X_R \simeq 1$, with the fit to the earlier data much better than before (by a factor of six for $B = 2$ mbn). The best complex eigenvalue fits have $X_S \simeq 2$, with the early data points contributing six times as much to X_S as to X_R. Thus the R-matrix one-level approximation still allows a much better fit to the data than the complex eigenvalue one-level approximation. Consistency of

† We are indebted to Dr. P. B. Treacy for suggesting this possibility.

the R-matrix fit with $\sigma(\gamma,n)$ at higher energies requires $B \lesssim 1\cdot 9$ mbn. For $B \simeq 1\cdot 7$ mbn, reasonable values of a_n and θ_n^2 are obtained for $\Gamma^0 \simeq 1\cdot 2$ MeV. The main difference from the results of Sections III and IV is that $\sigma_R(\gamma,n)$ now peaks at $E \simeq 15$ keV.

Further investigation of the problem of neutron energy loss in $^9Be(\gamma,n)^8Be$ experiments of the type performed by Berman, Van Hemert, and Bowman (1967) seems necessary, and an independent measurement of the peak position in a particle reaction such as $^9Be(p,p')^9Be^*$ is desirable; a previous measurement with this reaction (Bronson, Beckner, and Phillips 1962) indicated that the peak occurs within 14 keV of the neutron threshold.

VI. References

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