THE BEHAVIOUR OF DWBA STRIPPING AMPLITUDES NEAR THE BUTLER POLE

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[*Manuscript received November* 24, 1967]

Summary

Using integral representations of the bound state wave functions it is shown that, near the so-called Butler pole, DWBA amplitudes of (d, p) and certain (d, n) reactions depend only on the asymptotic parts of these wave functions. Furthermore, the zero-range approximation for the n–p interaction in the deuteron is exact at the pole. As well it is found that for (d, n) stripping to loosely bound levels of heavy nuclei the Butler pole disappears in the sense that the amplitude is no longer infinite there.

I. Introduction

The amplitude M of a (d, p) or (d, n) reaction, at fixed incident deuteron energy, may be regarded as a function of the variable Q^2,

$$Q^2 = |\,\boldsymbol{Q}\,|^2,$$

where

$$\boldsymbol{Q} = \boldsymbol{k}_1 - \boldsymbol{k}_2, \qquad \boldsymbol{k}_2 = m_A/m_B\,\boldsymbol{k}_y.$$

$\boldsymbol{k}_1$ and $\boldsymbol{k}_y$ are the momenta, in the centre of mass frame, of the incident deuteron and the outgoing nucleon y, while m_A and m_B are the masses of the target nucleus A and the residual nucleus B, which consists of A plus a captured nucleon x.

For (d, p) and (d, n) reactions the amplitude $M(Q^2)$ has a singularity at the point $Q^2 = -\kappa_x^2$, where

$$\kappa_x^2 = -2m_{xA}B_x/\hbar^2, \qquad m_{xA} = m_x m_A/(m_x+m_A),$$

and B_x is the binding energy of the captured nucleon. If Coulomb interactions are neglected, this singularity is a pole, the Butler pole (Schnitzer 1962), but when Coulomb interactions are included this singularity is a branch point. In the case of (d, p) reactions the cut is in the phase of the amplitude only and there is a simple pole in the modulus. The (d, n) case is somewhat more complicated.

In both cases the amplitude in a neighbourhood of the singularity, the so-called pole term, is of considerable interest. It represents the high partial waves in the partial wave expansion of the amplitude. It can therefore be used to aid the computation of these amplitudes (Bertram and Tassie 1968).

Another reason for current interest in the pole term is that the residue at the pole is proportional to the reduced width of the captured nucleon so that, at least

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in principle, model independent reduced widths may be obtained from the angular distribution of the differential cross section either by extrapolation (Amado 1959) or, in the case of sub-Coulomb stripping, by employing the method of Morinigo (1967).

In the distorted wave Born approximation (DWBA) the amplitude for deuteron stripping reactions is (see, for example, Glendenning 1963)

$$M = \int \psi_{\mathrm{f}}^{(-)*}\{(m_A/m_B)\boldsymbol{r}+\boldsymbol{R}\}\,\phi_x^*(\boldsymbol{r})\,\phi_{\mathrm{d}}(\boldsymbol{R})\,V_{\mathrm{d}}(\boldsymbol{R})\,\psi_{\mathrm{i}}^{(+)}\{\boldsymbol{r}+(m_y/m_{\mathrm{d}})\boldsymbol{R}\}\,\mathrm{d}\boldsymbol{r}\,\mathrm{d}\boldsymbol{R}\,, \tag{1}$$

where $\psi_{\mathrm{i}}^{(+)}$ and $\psi_{\mathrm{f}}^{(-)}$ are elastic scattering optical model wave functions of the incoming deuteron and the outgoing nucleon respectively, ϕ_{d} is the internal wave function of the deuteron, ϕ_x the bound state wave function of the captured nucleon, and V_{d} represents the neutron–proton interaction in the deuteron. The aim of this paper is to show that near the pole the amplitude $M(Q^2)$ depends only on the asymptotic parts of the wave functions and that the zero-range approximation is exact at the pole. We shall neglect the spins of the particles involved and choose our units such that $\hbar = 1$.

II. (d, p) and (d, n) Reactions

If we introduce the Fourier transforms $F_1(\boldsymbol{p}_1)$ and $F_2(\boldsymbol{p}_2)$ of the scattered wave functions by the relations

$$\psi_{\mathrm{i}}^{(+)}(\boldsymbol{r}) = \int F_1(\boldsymbol{p}_1)\exp(\mathrm{i}\boldsymbol{p}_1.\boldsymbol{r})\,\mathrm{d}\boldsymbol{p}_1\,, \tag{2}$$

$$\psi_{\mathrm{f}}^{(-)*}(\boldsymbol{r}) = \int F_2(\boldsymbol{p}_2)\exp(-\mathrm{i}\boldsymbol{p}_2.\boldsymbol{r})\,\mathrm{d}\boldsymbol{p}_2\,, \tag{3}$$

then the amplitude (1) can be written as (Clement 1965)

$$M = -\tfrac{1}{2}m_{\mathrm{np}}^{-1}\int F_1(\boldsymbol{p}_1)\,F_2(\boldsymbol{p}_2)\,(\kappa_{\mathrm{d}}^2+q_2^2)\,G_1(\boldsymbol{q}_1)\,G_2(\boldsymbol{q}_2)\,\mathrm{d}\boldsymbol{p}_1\,\mathrm{d}\boldsymbol{p}_2\,, \tag{4}$$

where

$$\boldsymbol{q}_1 = \boldsymbol{p}_1-(m_A/m_B)\boldsymbol{p}_2\,, \qquad \boldsymbol{q}_2 = (m_y/m_{\mathrm{d}})\boldsymbol{p}_1-\boldsymbol{p}_2\,,$$

$$(m_B/m_A)(q_1^2+\kappa_x^2) = (m_{\mathrm{d}}/m_y)(q_2^2+\kappa_{\mathrm{d}}^2)\,, \tag{5}$$

$$G_1(\boldsymbol{q}_1) = \int \phi_x^*(\boldsymbol{r})\exp(\mathrm{i}\boldsymbol{q}_1.\boldsymbol{r})\,\mathrm{d}\boldsymbol{r}\,, \tag{6}$$

$$G_2(\boldsymbol{q}_2) = \int \phi_{\mathrm{d}}(\boldsymbol{R})\exp(\mathrm{i}\boldsymbol{q}_2.\boldsymbol{R})\,\mathrm{d}\boldsymbol{R}\,. \tag{7}$$

Suppose that the potential V_{d} and the potential V_{xA}, acting between A and the captured nucleon, can be expressed as superpositions of exponential or Yukawa potentials

$$V_{\mathrm{d}}(r) = -\int_{\gamma}^{\infty} C(\alpha)\exp(-\alpha r)\,\mathrm{d}\alpha\,, \tag{8}$$

$$V_{xA} = (2\eta\kappa_x)r^{-1} - \int_{\mu}^{\infty} B(\alpha)\, r^{-1} \exp(-\alpha r)\, \mathrm{d}\alpha\,. \tag{9}$$

The use of exponential potentials in one case and Yukawa potentials in the other is of no great significance as they can be related by partial integration. The first term of V_{xA} represents the Coulomb interaction between x and A with

$$\eta = \begin{array}{ll} Z_A\, e^2 m_{xA}/\kappa_x & \text{if } x \text{ is a proton} \\ 0 & \text{if } x \text{ is a neutron.} \end{array}$$

The wave function for the internal motion of the deuteron, which is assumed to be in a pure s-state, can be expressed as (Martin 1959)

$$\phi_{\mathrm{d}}(R) = (4\pi)^{-\frac{1}{2}} N_{\mathrm{d}} R^{-1} \exp(-\kappa_{\mathrm{d}} R)\left(1 + \int_{\gamma}^{\infty} \rho(\alpha) \exp(-\alpha R)\, \mathrm{d}\alpha\right), \tag{10}$$

where $\rho(\alpha)$ satisfies the equation

$$\alpha(\alpha + 2\kappa_{\mathrm{d}})\rho(\alpha) = C(\alpha) + \int_0^{\alpha-\gamma} C(\alpha-\beta)\, \rho(\beta)\, \mathrm{d}\beta$$

and N_{d} is a normalization constant.

The bound state wave function of the captured particle, which is assumed to be captured into a state of definite angular momentum, may be written as (Andrews, to be published)

$$\phi_x(\boldsymbol{r}) = N_L\, Y_{LM}(\boldsymbol{r})\, r^L \exp(-\kappa_x r) \int_0^{\infty} \sigma(\alpha) \exp(-\alpha r)\, \mathrm{d}\alpha\,. \tag{11}$$

The constant N_L is related to the reduced width (see, for example, Dullemond and Schnitzer 1963) and $\sigma(\alpha)$ satisfies the equation

$$\alpha(\alpha + 2\kappa_x)\sigma'(\alpha) - 2\{L(\alpha + \kappa_x) + \kappa_x \eta\}\sigma(\alpha) = \int_0^{\alpha-\mu} B(\alpha-\beta)\, \sigma(\beta)\, \mathrm{d}\beta\,. \tag{12}$$

The right-hand side does not contribute for $0 \leqslant \alpha \leqslant \mu$, and the solution can easily be seen to be

$$\sigma(\alpha) = \sigma_0(\alpha) \qquad 0 \leqslant \alpha \leqslant \mu\,, \tag{13}$$

where

$$\sigma_0(\alpha) \equiv \alpha^{L+\eta}(\alpha + 2\kappa_x)^{L-\eta}\,.$$

For all greater values of α equation (12) can then be solved recursively.

If we define

$$\sigma_1(\alpha) = \sigma(\alpha) - \sigma_0(\alpha)\,,$$

equation (11) becomes

$$\phi_x(\boldsymbol{r}) = N_L\, Y_{LM}(\boldsymbol{r})\, r^L \exp(-\kappa_x r)\left(\int_0^{\infty} \sigma_0(\alpha) \exp(-\alpha r)\, \mathrm{d}\alpha + \int_{\mu}^{\infty} \sigma_1(\alpha) \exp(-\alpha r)\, \mathrm{d}\alpha\right). \tag{14}$$

Substituting (14) in (6) and (10) in (7), we find

$$G_1(\boldsymbol{q}_1) = 8\pi(2\mathrm{i})^L(L+1)!\,q_1^L\,Y_{LM}^*(\boldsymbol{q}_1)\,N_L\biggl(\int_0^\infty \sigma_0(\alpha)\frac{\alpha+\kappa_x}{\{q_1^2+(\alpha+\kappa_x)^2\}^{L+2}}\,\mathrm{d}\alpha + \int_\mu^\infty \sigma_1(\alpha)\frac{\alpha+\kappa_x}{\{q_1^2+(\alpha+\kappa_x)^2\}^{L+2}}\,\mathrm{d}\alpha\biggr), \tag{15}$$

$$G_2(q^2) = (4\pi)^{\frac{1}{2}}N_\mathrm{d}\biggl(\frac{1}{q_2^2+\kappa_\mathrm{d}^2} + \int_\mu^\infty \frac{\rho(\alpha)}{q_2^2+(\alpha+\kappa_\mathrm{d})^2}\,\mathrm{d}\alpha\biggr). \tag{16}$$

Using (15) and (16) in equation (4), the amplitude M becomes

$$-2m_{\mathrm{np}}M = M_1+M_2+(m_\mathrm{d}\,m_A/m_y\,m_B)(M_3+M_4).$$

Writing

$$R(\boldsymbol{p}_1,\boldsymbol{p}_2) = (4\pi)^{3/2}(2\mathrm{i})^L(L+1)!\,2N_\mathrm{d}N_L q_1^L\,Y_{LM}^*(\boldsymbol{q}_1)F_1(\boldsymbol{p}_1)F_2(\boldsymbol{p}_2),$$

we have

$$M_1 = \int_0^\infty \sigma_0(\alpha)\,\mathrm{d}\alpha \int R(\boldsymbol{p}_1,\boldsymbol{p}_2)\frac{\alpha+\kappa_x}{\{q_1^2+(\alpha+\kappa_x)^2\}^{L+2}}\,\mathrm{d}\boldsymbol{p}_1\,\mathrm{d}\boldsymbol{p}_2, \tag{17a}$$

$$M_2 = \int_\mu^\infty \sigma_1(\alpha)\,\mathrm{d}\alpha \int R(\boldsymbol{p}_1,\boldsymbol{p}_2)\frac{\alpha+\kappa_x}{\{q_1^2+(\alpha+\kappa_x)^2\}^{L+2}}\,\mathrm{d}\boldsymbol{p}_1\,\mathrm{d}\boldsymbol{p}_2, \tag{17b}$$

$$M_3 = \int_0^\infty \sigma_0(\alpha)\,\mathrm{d}\alpha \int_\gamma^\infty \rho(\beta)\,\mathrm{d}\beta \int R(\boldsymbol{p}_1,\boldsymbol{p}_2)\frac{(\alpha+\kappa_x)(q_1^2+\kappa_x^2)}{\{q_1^2+(\alpha+\kappa_x)^2\}^{L+2}\{q_2^2+(\beta+\kappa_\mathrm{d})^2\}}\,\mathrm{d}\boldsymbol{p}_1\,\mathrm{d}\boldsymbol{p}_2, \tag{17c}$$

$$M_4 = \int_\mu^\infty \sigma_1(\alpha)\,\mathrm{d}\alpha \int_\gamma^\infty \rho(\beta)\,\mathrm{d}\beta \int R(\boldsymbol{p}_1,\boldsymbol{p}_2)\frac{(\alpha+\kappa_x)(q_1^2+\kappa_x^2)}{\{q_1^2+(\alpha+\kappa_x)^2\}^{L+2}\{q_2^2+(\beta+\kappa_\mathrm{d})^2\}}\,\mathrm{d}\boldsymbol{p}_1\,\mathrm{d}\boldsymbol{p}_2. \tag{17d}$$

In Section III it is shown that the term M_1 is infinite at the point $Q^2 = -\kappa_x^2$ for $\eta \leqslant 1$ and that this singularity is an end-point singularity from the lower limit of the integration with respect to α. Since $R(\boldsymbol{p}_1,\boldsymbol{p}_2)$ does not explicitly depend on α, κ_d, and κ_x, and since $\sigma_0(\alpha) = 0$ at $\alpha = 0$, the singularity of M_1 can only come from the vanishing of the term $q_1^2+(\alpha+\kappa_x)^2$ at $\alpha = 0$ in the integrand, i.e. the singularity arises from $q_1^2+\kappa_x^2 = 0$ or equivalently from $q_2^2+\kappa_\mathrm{d}^2 = 0$.

Comparing equations (17b) and (17d) with (17a) it follows immediately that M_2 and M_4 in general have no singularities at $Q^2 = -\kappa_x^2$.

The term M_3 may be rewritten as

$$M_3 = \int_\gamma^\infty \rho(\beta)\,\mathrm{d}\beta \int \frac{R(\boldsymbol{p}_1,\boldsymbol{p}_2)}{q_2^2+(\beta+\kappa_\mathrm{d})^2}\,\mathrm{d}\boldsymbol{p}_1\,\mathrm{d}\boldsymbol{p}_2 \int_0^\infty \alpha^{L+\eta}(\alpha+2\kappa_x)^{L-\eta}\frac{\alpha+\kappa_x}{\{q_1^2+(\alpha+\kappa_x)^2\}^{L+1}}\,\mathrm{d}\alpha - \int_\gamma^\infty \rho(\beta)\,\mathrm{d}\beta \int \frac{R(\boldsymbol{p}_1,\boldsymbol{p}_2)}{q_2^2+(\beta+\kappa_\mathrm{d})^2}\,\mathrm{d}\boldsymbol{p}_1\,\mathrm{d}\boldsymbol{p}_2 \int_0^\infty \alpha^{L+\eta+1}(\alpha+2\kappa_x)^{L-\eta+1}\frac{\alpha+\kappa_x}{\{q_1^2+(\alpha+\kappa_x)^2\}^{L+2}}\,\mathrm{d}\alpha. \tag{18}$$

When $q_1^2 = -\kappa_x^2$ the integrands of both terms on the right of (18) behave like $\alpha^{\eta-1}$ as $\alpha \to 0$. Therefore, for $\eta > 0$, the lower limit of the integration with respect to α does not cause any singularities in M_3 at $Q^2 = -\kappa_x^2$.

In the case of $\eta = 0$, the integration with respect to α in (17c) can be carried out explicitly to yield

$$M_3 = \int_\gamma^\infty \rho(\beta)\,\mathrm{d}\beta \int R(\boldsymbol{p}_1,\boldsymbol{p}_2)\{q_2^2+(\beta+\kappa_\mathrm{d})^2\}^{-1}\,\mathrm{d}\boldsymbol{p}_1\,\mathrm{d}\boldsymbol{p}_2\,.$$

The integrand is finite at the point $q_2^2 = -\kappa_\mathrm{d}^2$, and M_3 cannot have a singularity at $Q^2 = -\kappa_x^2$. Therefore, as $Q^2 \to \kappa_x^2$, $M(Q^2) \to M_1(Q^2)$, except when the Coulomb parameter η of the captured particle is greater than unity.

The amplitude M_1 when expressed in terms of wave functions is

$$M_1 = \int \psi_\mathrm{f}^{(-)*}\{(m_A/m_B)\boldsymbol{r}\}\,\phi_0^*(\boldsymbol{r})\,\psi_\mathrm{i}^{(+)}(\boldsymbol{r})\,\mathrm{d}\boldsymbol{r}\,, \tag{19}$$

with

$$\phi_0(\boldsymbol{r}) = N_L\,Y_{LM}(\boldsymbol{r})\exp(-\kappa_x r)\,r^L \int_0^\infty \alpha^{L+\eta}(\alpha+2\kappa_x)^{L-\eta}\exp(-\alpha r)\,\mathrm{d}\alpha\,. \tag{20}$$

Thus M_1 is the DWBA amplitude for the stripping reaction when the zero-range approximation for the deuteron is made and with the bound state wave function of the captured nucleon replaced by its asymptotic part.

After carrying out the integration in (20) we find that for (d, p) reactions ($\eta = 0$) the correct asymptotic form of the bound state wave function is

$$\phi_0(\boldsymbol{r}) = N_L\,\mathrm{h}_L^{(1)}\,(\mathrm{i}\kappa_x r)\,Y_{LM}(\boldsymbol{r})\,,$$

where $\mathrm{h}_L^{(1)}(z)$ is the spherical Hankel function of the first kind, and for (d, n) reactions ($\eta > 0$)

$$\phi_0(\boldsymbol{r}) = N_L\,r^{-1}\,\mathrm{W}_{L,\eta}(\kappa_x r)\,Y_{LM}(\boldsymbol{r})\,,$$

where $\mathrm{W}_{L,\eta}(z)$ is the Whittaker function.

III. The Term M_1

In this section we shall show that the term M_1 in the amplitude is infinite at $Q^2 = -\kappa_x^2$, the singularity arising from the lower limit of the integration with respect to α in equation (17a).

The scattering wave functions $\psi_\mathrm{i}^{(+)}$ and $\psi_\mathrm{f}^{(-)}$ in (19) can be expanded as Born series; the leading terms in these expansions are given as (Mott and Massey 1965)

$$\psi_\mathrm{i}^{(+)}(\boldsymbol{r}) = \exp(-\tfrac{1}{2}\pi\eta_1)\,\Gamma(1+\mathrm{i}\eta_1)\exp(\mathrm{i}\boldsymbol{k}_1\cdot\boldsymbol{r})\,F\{-\mathrm{i}\eta_1;1;\mathrm{i}(k_1 r-\boldsymbol{k}_1\cdot\boldsymbol{r})\}\,, \tag{21}$$

$$\psi_\mathrm{f}^{(-)*}(\boldsymbol{r}) = \exp(-\tfrac{1}{2}\pi\eta_2)\,\Gamma(1+\eta_2)\exp(-\mathrm{i}\boldsymbol{k}_2\cdot\boldsymbol{r})\,F\{-\mathrm{i}\eta_2;1;\mathrm{i}(k_2 r+\boldsymbol{k}_2\cdot\boldsymbol{r})\}\,, \tag{22}$$

where

$$\eta_1 = (Z_A\,e^2 m_{A\mathrm{d}})/k_1$$

and

$$\eta_2 = \begin{cases} Z_B\,e^2 m_{By}/k_y & \text{for (d, p) reactions} \\ 0 & \text{for (d, n) reactions}\,. \end{cases}$$

The amplitude M_1 is then

$$M_1 = K\int_0^\infty \sigma_0(\alpha)\,\mathrm{d}\alpha \int \exp(\mathrm{i}\boldsymbol{Q}\,.\,\boldsymbol{r})\, r^L\, Y_{LM}^*(\boldsymbol{r}) \exp\{-(\alpha+\kappa_x)r\}\, F\{-\mathrm{i}\eta_1;1;\mathrm{i}(k_1 r-\boldsymbol{k}_1\,.\,\boldsymbol{r})\}$$
$$\times F\{-\mathrm{i}\eta_2;1;\mathrm{i}(k_2 r+\boldsymbol{k}_2\,.\,\boldsymbol{r})\}\,\mathrm{d}\boldsymbol{r}\,. \tag{23}$$

All the constant factors that occur in M_1 have been absorbed in the quantity K. In order to keep the expressions in the remainder of this section as concise as possible, any other constant factors that may appear as the result of our manipulations shall be automatically included in K.

If we assume that η_1 and η_2 contain a small imaginary part $\mathrm{i}\epsilon$ ($\epsilon > 0$), which we can put equal to zero after the calculations, we may use the integral representation of the hypergeometric functions (Erdélyi 1953) and write

$$M_1 = K\int_0^1\int_0^1 s^{-1-\eta_1}(1-s)^{\mathrm{i}\eta_1} t^{-1-\mathrm{i}\eta_2}(1-t)^{\mathrm{i}\eta_2}\,\mathrm{d}s\,\mathrm{d}t \int_0^\infty \sigma_0(\alpha)\,\mathrm{d}\alpha$$
$$\times \int r^L \exp\{-(\alpha+a)r\}\, Y_{LM}^*(r) \exp(\mathrm{i}\boldsymbol{q}\,.\,\boldsymbol{r})\,\mathrm{d}r\,, \tag{24}$$

where

$$a = k_x - \mathrm{i}(k_1 s + k_2 t)\,, \tag{25}$$

$$\boldsymbol{q} = \boldsymbol{Q} - \boldsymbol{k}_1 s + \boldsymbol{k}_2 t\,. \tag{26}$$

Integration with respect to $\boldsymbol{r}$ yields

$$M_1 = K\int_0^1\int_0^1 s^{-1-\mathrm{i}\eta_1}(1-s)^{\mathrm{i}\eta_1} t^{-1-\mathrm{i}\eta_2}(1-t)^{\mathrm{i}\eta_2}\,\mathrm{d}s\,\mathrm{d}t$$
$$\times q^L\, Y_{LM}(\boldsymbol{q}) \int_0^\infty \sigma_0(\alpha)\,(\alpha+a)\{q^2+(\alpha+a)^2\}^{-(L+2)}\,\mathrm{d}\alpha\,. \tag{27}$$

For large α the integrand goes as α^{-4} and thus the integral converges at the upper limit of the α-integration. Hence, if M_1 is to have a singularity at $Q^2 = -\kappa_x^2$, the only way this can happen is when the term $\{q^2+(\alpha+a)^2\}$ in the integrand vanishes at $Q^2 = -\kappa_x^2$. From equations (25) and (26) we find that this occurs only when $\alpha = s = t = 0$. Hence we have shown that if M_1 has a singularity at $Q^2 = -\kappa_x^2$ it must arise from the lower limit of the integration with respect to α.

In order to show that M_1 is in fact infinite at $Q^2 = -\kappa_x^2$ we make use of a result due to Cejpek (1966)

$$A(Q^2,\beta) \equiv \int_0^1 s^{-1-\mathrm{i}\eta_1}(1-s)^{\mathrm{i}\eta_1}\,\mathrm{d}s \int_0^1 t^{-1-\mathrm{i}\eta_2}(1-t)^{\mathrm{i}\eta_2}\,\mathrm{d}t$$
$$\times \int r^{L-1}\, Y_{LM}^*(\boldsymbol{r}) \exp\{\mathrm{i}(\boldsymbol{Q}-\boldsymbol{k}_1 s+\boldsymbol{k}_2 t)\,.\,\boldsymbol{r}\} \exp[-\{\beta-\mathrm{i}(k_1 s+k_2 t)\}r]\,\mathrm{d}\boldsymbol{r}$$
$$= K\frac{(1+C)^{\mathrm{i}\eta_1}(1+B)^{\mathrm{i}\eta_2}}{(Q^2+\beta^2)^{L+1}} \sum_{l=0}^{L}\sum_{m=-l}^{l}\left(\frac{4\pi}{2l+1}\right)^{\frac{1}{2}}\binom{2L+1}{2l}^{\frac{1}{2}}\Big(L-l, M-m, l, m \mid LM\Big)\times$$

$$\times k_1^{L-l}(-k_2)^l\, Y^*_{L-l,M-m}(\boldsymbol{k}_1)\, Y^*_{lm}(\boldsymbol{k}_2)\,\Gamma(L-l+1+\mathrm{i}\eta_1)\,\Gamma(l+1)$$

$$\times \sum_{s=0}^{l} \frac{\Gamma(-\mathrm{i}\eta_1+s)}{\Gamma(-\mathrm{i}\eta_1)} \frac{1}{(L-l+s+1)\,\Gamma(l-s+1)\,\Gamma(s+1)} \frac{(-1)^s C^s}{(1+C)^s}$$

$$\times \sum_{j=0}^{L-l} (1+B)^{-j} \frac{\Gamma(-\mathrm{i}\eta_2+j)\,\Gamma(L-j+1+\mathrm{i}\eta_2)}{\Gamma(-\mathrm{i}\eta_2)\,\Gamma(L-l-j+1)\,\Gamma(j+1)}$$

$$\times {}_2F_1(-\mathrm{i}\eta_2+j;\, -\mathrm{i}\eta_1+s;\, L-l+s+1;\, H)\,, \tag{28}$$

with

$$B = \frac{-2\mathrm{i}k_2\beta + k_1^2 - k_2^2 - Q^2}{Q^2+\beta^2}, \qquad C = \frac{-2\mathrm{i}k_1\beta - k_1^2 + k_2^2 - Q^2}{Q^2+\beta^2}, \qquad H = 1 - \frac{Q^2+\beta^2}{\beta^2+(k_1-k_2)^2}.$$

Comparing this formula with equation (24) it follows immediately that, with $\beta = \alpha + \kappa_x$,

$$M_1 = K \int_0^\infty \sigma_0(\alpha) \frac{\mathrm{d}}{\mathrm{d}\beta}\Big(A(Q^2,\beta)\Big)\, \mathrm{d}\alpha\,. \tag{29}$$

When $Q^2 = -\kappa_x^2$, a typical term in the integrand of (29) behaves like $\alpha^{\eta+j-2}$ as $\alpha \to 0$. Therefore, due to the terms with $j = 0$ in (28), M_1 is infinite at $Q^2 = -\kappa_x^2$ only if $\eta \leqslant 1$.

IV. Conclusions

We have shown that the DWBA amplitude $M(Q^2)$ for stripping reactions can be expressed as a sum of four terms, the first of which $M_1(Q^2)$ corresponds to DWBA when the zero-range approximation is used for the deuteron, and with the bound state wave function of the captured nucleon replaced by its asymptotic part. The term M_1 is infinite at $Q^2 = -\kappa_x^2$, this singularity being known as the Butler pole. Since the other terms are finite at the Butler pole, $M(Q^2) \to M_1(Q^2)$ as $Q^2 \to -\kappa_x^2$.

There is, however, one exception. In the case of (d, n) reactions with $\eta > 1$, that is, when the proton is captured into a loosely bound level of a highly charged nucleus, M_1 is no longer infinite at the point $Q^2 = -\kappa_x^2$ and therefore in this case $M(Q^2)$ does not approach $M_1(Q^2)$ as $Q^2 \to -\kappa_x^2$.

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