2^+ STATES OF ^{8}Be

By F. C. Barker*

[*Manuscript received November* 14, 1968]

Summary

Analysis by many-level R-matrix theory of the d-wave α–α scattering phase shift suggests the existence of broad 2^+ excited states of ^{8}Be, but their properties depend sensitively on the assumed channel radius a_2. A simultaneous fit to the ^{9}Be(p, d)^{8}Be deuteron spectrum near the $2\cdot 9$ MeV peak requires $a_2 \approx 7\cdot 1$ fm, while a simultaneous fit to the α-particle spectra following ^{8}Li and ^{8}B β-decays requires $a_2 \approx 6\cdot 7$ fm. For the best overall fit with $a_2 = 6\cdot 75$ fm, the first 2^+ excited state is at $2\cdot 84$ MeV excitation energy with a width at half maximum of $1\cdot 30$ MeV. It is shown that data from other reactions which appeared to give much larger widths for this level can be fitted using the same R-matrix parameters. A second 2^+ excited state is obtained at about 9 MeV with a width of about 10 MeV. Properties of the narrow 2^+ states at $16\cdot 6$ and $16\cdot 9$ MeV are also discussed.

I. Introduction

In a previous paper on the 0^+ states of ^{8}Be (Barker, Hay, and Treacy 1968; hereinafter referred to as BHT), the s-wave α–α scattering phase shift and data from the ^{9}Be(p, d)^{8}Be reaction were analysed using the three-level approximation of R-matrix theory. A consistent fit required an α–α channel radius a_0 of order 7 fm, implying a second 0^+ level of ^{8}Be at about 6 MeV excitation. Such a level had not been identified previously, but its existence is consistent with the systematics of neighbouring nuclei.

In this paper the same methods of analysis are used for a study of the 2^+ states of ^{8}Be for excitation energies below about 17 MeV. Many of the arguments and formulae that would be the same as those in BHT are not repeated here. The formulae in Section III of BHT, obtained from the many-level one-channel approximation of R-matrix theory, are used firstly to fit the d-wave α–α scattering phase shift δ_2 neglecting contributions of known narrow levels. These fits are given in Section II for a wide range of values of the channel radius a_2 and of the level parameters. In order to restrict the acceptable range of parameters, the formulae are then used to obtain simultaneous fits to experimental data from various reactions proceeding through 2^+ states of ^{8}Be.

Reactions of two types are suitable for this purpose. The first includes reactions, such as ^{9}Be(p, d)^{8}Be(α)^{4}He, where reasonable assumptions may be made about the feeding of the higher 2^+ levels; the second includes reactions, such as ^{8}Li(β^-)^{8}Be(α)^{4}He, where the order in which the particles are emitted is reasonably certain. In Section III, parameters are obtained which fit both δ_2 and the experimental data from

* Research School of Physical Sciences, Australian National University, Canberra, A.C.T. 2600.

^{9}Be(p, d) for ^{8}Be excitation energies E_x up to about 4 MeV. In order to fit the ^{8}Li β-decay data, which are available for $E_x \lesssim 14$ MeV, it is necessary to include contributions from the known narrow 2^+ levels of ^{8}Be at 16·6 and 16·9 MeV, so data from reactions involving these levels are investigated in Section IV. Simultaneous fits to δ_2 and to the ^{8}Li β-decay data are then given in Section V.

Other reactions are discussed in Section VI, particularly insofar as they led to different widths of the 2·9 MeV level, and the results of the paper are summarized in Section VII. The one-channel approximation, used throughout this paper, was justified in Appendix I of BHT, but some further discussion is necessary here owing to the inclusion of the narrow 16·6 and 16·9 MeV levels; this is given in Appendix I. Appendix II gives the relations between level parameters that are required in order that fits to data involving the 16·6 and 16·9 MeV levels should be independent of the boundary condition parameter B_2 and of a_2. Appendix III contains formulae used in the analysis of the β-decay of ^{8}Li and ^{8}B.

II. R-matrix Parameters from α–α Scattering Data

The experimental values δ_2^{exp} and errors ϵ_2 used for the d-wave α–α scattering phase shift are those of Heydenberg and Temmer (1956) for channel energies $E = 1\cdot0$–$1\cdot5$ MeV (with the errors increased to 1°), Tombrello and Senhouse (1963) for $E = 1\cdot92$–$5\cdot94$ MeV, and Nilson *et al.* (1958) for $E = 6\cdot15$–$11\cdot45$ MeV. For $E = 11\cdot55$–$17\cdot1$ MeV, we use values extracted from the data of Bredin *et al.* (1959) by Berztiss (1965), with the errors increased slightly to 4° (but 8° for $E = 12\cdot75$ MeV).

Table 1

Parameter values for best fits to δ_2^{exp} in the three-level approximation for various channel radii

$B_2 = 0$

a_2 (fm)	E_{max} (MeV)	N	E_1 (MeV)	γ_1^2 (MeV)	E_2 (MeV)	γ_2^2 (MeV)	E_3 (MeV)	γ_3^2 (MeV)	X_2
5·5	17·1	30	2·629	0·603	13·45	1·202	185·0	14·9	0·52
6·0	17·1	30	2·667	0·471	10·76	0·943	185·0	17·4	0·49
6·5	17·1	30	2·650	0·390	8·87	0·809	45·2	3·83	0·48
6·75	17·1	30	2·627	0·364	8·13	0·747	36·0	2·95	0·47
7·0	17·1	30	2·599	0·339	7·43	0·686	27·5	1·98	0·44
7·5	17·1	30	2·526	0·306	6·34	0·560	20·2	1·20	0·39
8·0	15·15	28	2·438	0·286	5·56	0·455	16·3	0·83	0·53
9·0	11·55	24	2·244	0·272	4·51	0·289	12·4	0·61	0·47

Previous fits to δ_2^{exp} in the R-matrix one-level approximation (Barker and Treacy 1962; Tombrello and Senhouse 1963) were over a limited energy range near the well-known 2·9 MeV level and required a channel radius $a_2 \approx 3\cdot5$ fm. We now use a three-level one-channel approximation to try to fit δ_2^{exp} for $E \lesssim 17$ MeV; for the larger channel radii the energy range over which the fits are made is restricted by requiring $\delta_2^{\text{exp}}+\phi_2 \lesssim 440°$, where $-\phi_2$ is the hard sphere phase shift. Contributions to δ_2 from the known narrow 2^+ levels of ^{8}Be at $E_x \approx 16\cdot6$ and $16\cdot9$ MeV are neglected here. Such contributions could well be appreciable in the measured value of δ_2^{exp} at $E = 17 \cdot 1$ MeV ($E_x \approx 17 \cdot 0$ MeV), but are difficult to calculate owing to the experimental uncertainty in E of $\pm 0 \cdot 15$ MeV and spread of $\pm 0 \cdot 2$ MeV (Bredin *et al.* 1959). Exclusion of the 17·1 MeV measurement from the fits does not appreciably change the parameter values.

TABLE 2

PARAMETER VALUES FOR FITS TO δ_2^{exp} IN THE THREE-LEVEL APPROXIMATION FOR $a_2 = 7 \cdot 0$ fm AND VARIOUS FIXED VALUES OF E_1

$E_{max} = 17 \cdot 1$ MeV, $B_2 = 0$

E_1 (MeV)	γ_1^2 (MeV)	E_2 (MeV)	γ_2^2 (MeV)	E_3 (MeV)	γ_3^2 (MeV)	X_2
2·525	0·360	7·36	0·672	26·3	1·75	0·98
2·55	0·352	7·37	0·675	26·3	1·76	0·68
2·575	0·346	7·40	0·681	27·2	1·91	0·50
2·6	0·339	7·43	0·686	27·5	1·99	0·44
2·625	0·334	7·48	0·693	29·1	2·25	0·51
2·65	0·327	7·50	0·698	29·4	2·31	0·68
2·675	0·322	7·54	0·705	31·0	2·58	0·97

The parameter values that give best fits to δ_2^{exp} for various channel radii are given in Table 1. These values are obtained by taking $B_2 = 0$ and varying the parameters E_λ and γ_λ^2 ($\lambda = 1, 2, 3$) to minimize X_2, defined in equation (3) of BHT. For convenience the suffix l on $E_{\lambda l}$ and $\gamma_{\lambda l}^2$ is omitted. Identical fits can be obtained for any other value of B_2 by using the relations in Appendix II of BHT. Variations of the parameter values about those given in Table 1 can still lead to acceptable fits to δ_2^{exp}, defined as fits with $X_2 \lesssim 1$, which is about twice the minimum value of X_2; thus for $a_2 = 7 \cdot 0$ fm, Table 2 gives parameter values for fits with $X_2 \lesssim 1$ obtained by taking a set of fixed values of E_1 and varying only the remaining level parameters.

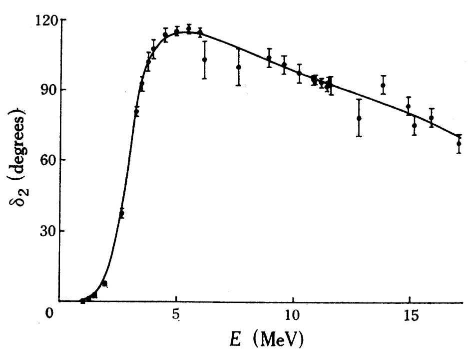

Fig. 1.—The α–α scattering d-wave phase shift δ_2 as a function of the ^{8}Be channel energy E. The points are experimental values and the curve is the R-matrix three-level fit for the channel radius $a_2 = 6 \cdot 75$ fm and other parameters as in Section VII.

In Tables 1 and 2, no allowance has been made for uncertainty in the α-particle energy in the scattering experiments. This is most significant in the region $E \approx 3$ MeV where δ_2 is changing most rapidly with energy; here Tombrello and Senhouse (1963) have an uncertainty in E of ± 18 keV. When allowance is made for this, the parameter values giving best fits are not changed greatly, but for fits like those given in Table 2, X_2 increases less rapidly as E_1 is changed from its optimum value.

Figure 1 shows the calculated fit to δ_2^{exp} for $a_2 = 6 \cdot 75$ fm and the other parameter values given in Section VII; these provide the best overall fit to the data discussed in this paper.

From Table 1, it is seen that acceptable fits to δ_2^{exp} can be obtained for a wide range of channel radii, including at least $5 \cdot 5$ to $9 \cdot 0$ fm.

III. Restriction of R-matrix Parameters from ^{9}Be(p, d)^{8}Be Reaction

Restrictions on the acceptable values of the channel radius and the level parameters may be obtained by requiring them to give simultaneous fits to the α–α scattering data and to other data obtained from some reaction that proceeds through an intermediate stage involving 2^+ states of ^{8}Be. One such reaction is ^{9}Be(p, d)^{8}Be(α)^{4}He, which is suitable to the extent that it proceeds as a direct transition and therefore populates preferentially the lowest 2^+ state rather than the higher broad 2^+ states. As was the case for the 0^+ states discussed in BHT, these higher 2^+ states probably contain little of the lowest shell-model configuration.

Of the data available on the ^{9}Be(p, d)^{8}Be reaction, the most accurate and useful for the present purpose are those of Hay (personal communication) obtained at a proton energy of 8 MeV and at the peak of the angular distribution at 20°. The deuteron spectrum was obtained for ^{8}Be excitation energies $E_x \leqslant 4 \cdot 4$ MeV ($E \leqslant 4 \cdot 5$ MeV), as the competing mode ^{9}Be(p, α)^{6}Li(d)^{4}He proceeding through the $2 \cdot 18$ MeV state of ^{6}Li contributes strongly for $E > 4 \cdot 4$ MeV (see Fig. 3). Transitions through other ^{6}Li states and also through the 0^+ states of ^{8}Be provide non-interfering backgrounds.

After a neutron penetration factor P_{n} is extracted, as in BHT, in order to provide a spectral density, the background contribution from 0^+ states of ^{8}Be is assumed to have the form (10) of BHT, with all the parameters except the normalization taken from the best fit obtained there, while the background contribution due to transitions through ^{6}Li states is assumed for simplicity to be a linear function of energy, vanishing at $E = 0$. The contribution from 2^+ levels of ^{8}Be is obtained from equation (9) of BHT, by neglecting the $16 \cdot 6$ and $16 \cdot 9$ MeV levels altogether, by extracting the neutron penetration factor (using $G_{\lambda x} = g_{\lambda x}^2 P_{\mathrm{n}}$), and by assuming that the levels $\lambda = 2$ and 3 are weakly fed. Then contributions from different x-values can be lumped together, giving

$$g_1^2 = \sum_x g_{1x}^2, \qquad g_\lambda = \sum_x g_{1x} g_{\lambda x}/g_1, \qquad \lambda = 2, 3, \tag{1}$$

so that the 2^+ contribution is similar to BHT, equation (10). Also, as in BHT, the restrictions

$$|g_2/g_1| \lesssim 0 \cdot 3, \qquad |g_3/g_1| \ll 0 \cdot 3, \tag{2}$$

are imposed on the feeding amplitudes of the 2^+ levels. This in principle makes the range of acceptable fits dependent on the choice of B_2, but in practice the dependence is slight for a reasonable range of B_2 values.

The initial fitting is done with $B_2 = 0$ and $g_3 = 0$, and the relations of Appendix II of BHT may then be used to obtain parameter values that give the same fit for other values of B_2. For a given a_2 and for a set of level parameters that gives an acceptable fit to $\delta_2^{\exp}$, such as those given in Table 2 for $a_2 = 7 \cdot 0$ fm, the value of g_2/g_1 for the 2^+ levels and the normalizations of the two background contributions and of the 2^+ contribution are varied to minimize Y_2, defined by

$$Y_2 = N_2^{-1} \sum_{i=1}^{N_2} |\{\rho^{\exp}(E_i) - \rho(E_i)\}/\eta(E_i)|^2, \tag{3}$$

where $\rho(E_i)$, $\rho^{\exp}(E_i)$, and $\eta(E_i)$ are respectively the calculated and measured spectral densities and the error at the channel energy E_i. The data are fitted over the energy range $E = 0 \cdot 7$–$4 \cdot 2$ MeV ($N_2 = 135$).

TABLE 3

PARAMETER VALUES FOR FITS TO ^{9}Be(p, d)^{8}Be DATA IN THE THREE-LEVEL APPROXIMATION FOR $a_2 = 7 \cdot 0$ fm AND VARIOUS SETS OF PARAMETER VALUES GIVING ACCEPTABLE FITS TO $\delta_2^{\exp}$

E_1 (MeV)	X_2	Y_2	$B_2 = 0$		$B_2 = -0 \cdot 5$	
			g_2/g_1	g_3/g_1	g_2/g_1	g_3/g_1
2·525	0·98	4·06	−0·18	0·0	−0·13	0·01
2·55	0·68	3·25	−0·09	0·0	−0·04	0·01
2·575	0·50	3·29	0·01	0·0	0·06	0·02
2·6	0·44	4·11	0·12	0·0	0·17	0·02
2·625	0·51	6·05	0·23	0·0	0·28	0·02

TABLE 4

PARAMETER VALUES FOR BEST FITS TO ^{9}Be(p, d)^{8}Be DATA IN THE THREE-LEVEL APPROXIMATION FOR VARIOUS CHANNEL RADII AND FOR PARAMETER VALUES GIVING ACCEPTABLE FITS TO $\delta_2^{\exp}$

a_2 (fm)	E_1 (MeV)	X_2	Y_2	$B_2 = 0$		$B_2 = -0 \cdot 5$	
				g_2/g_1	g_3/g_1	g_2/g_1	g_3/g_1
6·0	2·605	0·74	3·19	0·70	0·0	0·77	0·02
6·5	2·597	0·72	3·12	0·26	0·0	0·31	0·02
6·75	2·579	0·70	3·18	0·10	0·0	0·15	0·02
7·0	2·561	0·58	3·16	−0·05	0·0	0·00	0·01
7·5	2·516	0·40	3·20	−0·24	0·0	−0·19	0·01
8·0	2·448	0·54	3·40	−0·41	0·0	−0·35	0·01

The smallest Y_2 obtained in this way is about $3 \cdot 1$ and fits with $Y_2 \lesssim 5$ are, rather arbitrarily, taken as acceptable, provided that $X_2 \lesssim 1$ and that (2) is satisfied. An example of such fits is given in Table 3 for $a_2 = 7 \cdot 0$ fm, the level parameters being specified by the values of E_1 and the corresponding X_2. Table 3 also includes values of g_2/g_1 and g_3/g_1 for both of the cases $B_2 = 0$ and $-0 \cdot 5$; these are approximately the values of $S_2(E_2)$ and $S_2(E_1)$ and so form a reasonable range of B_2 values (see Appendix III of BHT). It is seen that variation of B_2 within this range produces only small changes in both g_2/g_1 and g_3/g_1.

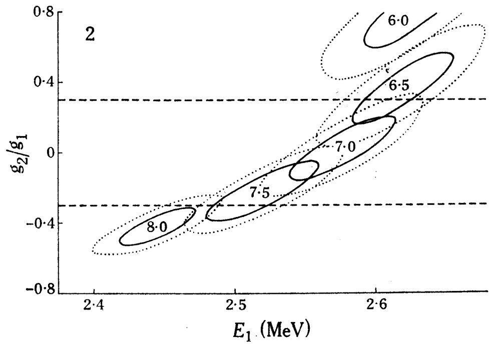

Fig. 2.—Acceptable regions for R-matrix three-level fits to δ_2^{exp} and to the ^{9}Be(p, d)^{8}Be data for various channel radii a_2 and for $B_2 = 0$. The values of a_2 (in fm) are indicated within the sets of contours, which are for $Z_2 = 1\cdot5$ (solid curves) and $Z_2 = 2\cdot0$ (dotted curves). The acceptable regions are within the contours $Z_2 = 1\cdot5$ and between the dashed lines $g_2/g_1 = \pm0\cdot3$.

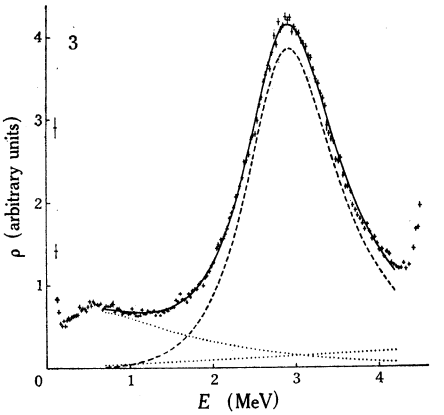

Fig. 3.—Spectral density ρ for the reaction ^{9}Be(p, d)^{8}Be as a function of ^{8}Be channel energy E. The points are experimental values and the solid curve is the fit over the region $E = 0\cdot7$–$4\cdot2$ MeV. The dashed curve shows the contribution from 2^+ states of ^{8}Be, with $a_2 = 6\cdot75$ fm and other parameters as in Section VII, and the dotted curves show the two background contributions due to 0^+ states of ^{8}Be and to competing reaction modes.

TABLE 5

PARAMETER VALUES FOR BEST FITS TO δ_2^{exp} AND THE ^{9}Be(p, d)^{8}Be DATA IN THE THREE-LEVEL APPROXIMATION FOR VARIOUS CHANNEL RADII

$B_2 = 0$, $g_3/g_1 = 0$

a_2 (fm)	E_1 (MeV)	γ_1^2 (MeV)	E_2 (MeV)	γ_2^2 (MeV)	E_3 (MeV)	γ_3^2 (MeV)	g_2/g_1	X_2	Y_2	Z_2
6·0	2·632	0·489	10·75	0·958	123·0	11·37	0·85	0·58	3·5	1·28
6·5	2·617	0·402	8·84	0·815	41·4	3·36	0·36	0·57	3·4	1·24
6·75	2·600	0·371	8·08	0·748	33·2	2·55	0·18	0·54	3·4	1·22
7·0	2·580	0·345	7·41	0·682	27·2	1·93	0·03	0·48	3·4	1·15
7·5	2·518	0·308	6·34	0·557	20·1	1·19	−0·23	0·40	3·2	1·03
8·0	2·444	0·286	5·56	0·458	16·3	0·84	−0·43	0·53	3·4	1·22

Table 4 gives the smallest values of Y_2 obtained for various channel radii together with the corresponding values of E_1 and X_2. The values of g_2/g_1 and g_3/g_1 for both $B_2 = 0$ and $-0\cdot5$ satisfy (2) only for channel radii between about $6\cdot5$ and $7\cdot8$ fm, but this applies to the smallest Y_2 values and a wider range of channel radii could yield fits with acceptable values of Y_2 and of g_2/g_1 and g_3/g_1. In order to obtain a best simultaneous fit to δ_2^{exp} and the ^{9}Be(p, d)^{8}Be data, we introduce the quantity $Z_2 = X_2 + 0 \cdot 2 Y_2$ and take the smallest Z_2 as giving the best fit provided (2) is satisfied. The conditions $X_2 \lesssim 1$ and $Y_2 \lesssim 5$ for acceptable fits are replaced by $Z_2 \lesssim 1 \cdot 5$, which is the mean of the smallest possible and the largest acceptable values of Z_2.

The complete sets of parameter values for best fits, in this sense, are given in Table 5 for various channel radii, including some for which $|g_2/g_1|$ does not satisfy (2). Complementary to Table 5 is Figure 2, where contours of constant Z_2 are shown as functions of g_2/g_1 and of E_1 for various channel radii. Acceptable fits correspond to regions within the contours $Z_2 = 1 \cdot 5$ and the lines $g_2/g_1 = \pm 0 \cdot 3$.

From Table 5 and Figure 2, it is seen that the best overall fit to the α–α scattering and the ^{9}Be(p, d)^{8}Be data is obtained for a_2 near $7 \cdot 1$ fm, and that acceptable fits can be obtained for a_2 values between about $6 \cdot 3$ and $8 \cdot 0$ fm. The fit to the ^{9}Be(p, d)^{8}Be data is shown in Figure 3 for $a_2 = 6 \cdot 75$ fm and the parameters of Section VII, the same as are used in Figure 1.

IV. Properties of the 16·6 and 16·9 MeV Levels of ^{8}Be

Restrictions on the R-matrix parameter values may also be obtained by fitting the α-particle spectrum following ^{8}Li β-decay, but significant contributions to this spectrum come from the narrow 2^+ levels of ^{8}Be at $16 \cdot 6$ and $16 \cdot 9$ MeV, and it is necessary to consider first the properties of these levels. These may be obtained from other reactions, in particular ^{10}B(d, α)^{8}Be, ^{7}Li(d, n)^{8}Be, ^{9}Be(p, d)^{8}Be, ^{9}Be(^{3}He, α)^{8}Be, and ^{8}B(β^+)^{8}Be.

In the reaction ^{10}B(d, α)^{8}Be, the $16 \cdot 6$ and $16 \cdot 9$ MeV levels have been observed as two prominent distinct peaks, and analysis has been based on the assumption that contributions come only from these two levels (perhaps interfering with each other) together with a non-interfering background (Browne, Callender, and Erskine 1966). It is possible, however, that the broad 2^+ levels can contribute an appreciable interfering background in this energy region, so that the experimental data should be analysed on this basis in order to obtain parameters for the two levels.

We use equation (9) of BHT, with $l = 2$, and with $\lambda = 1$, 2, and 3 referring to the three broad 2^+ levels included in the phase-shift analysis of Section II and $\lambda = a$ and b referring to the $16 \cdot 6$ and $16 \cdot 9$ MeV levels respectively. Justification for the use of this formula is given in Appendix I of BHT, and some additional comments are given in Appendix I of this paper. In application to the 16–17 MeV region, constant values may be used for S_2 and P_2, and for the contributions of the broad levels to the sums over λ. Also we put $G_{\lambda x} = g^2_{\lambda x} P_x$, where the energy dependence of $G_{\lambda x}$ is contained in the factor P_x which does not necessarily depend on x; generally P_x may be taken as constant, but in ^{8}B(β^+)^{8}Be its energy dependence is significant. For the particular choice $B_2 = S_2 \approx 0$ (see Table 6), the cross section may then be written

$$\sigma_\alpha \propto \sum_x P_x \frac{\left| \sum\limits_{\lambda=a}^{b} \{g_{\lambda x} \Gamma^{\frac{1}{2}}_\lambda / (E_\lambda - E)\} + J_x \right|^2}{1 + \left| \sum\limits_{\lambda=a}^{b} \{\tfrac{1}{2}\Gamma_\lambda / (E_\lambda - E)\} + K \right|^2}, \tag{4}$$

where

$$\left.\begin{aligned} \Gamma_\lambda &= 2\gamma_\lambda^2 P_2\,, \qquad \lambda = a,b\,, \\ J_x &= (2P_2)^{\frac{1}{2}} \sum_{\lambda=1}^{3} \{g_{\lambda x}\gamma_\lambda/(E_\lambda - E_{\rm M})\}\,, \\ K &= P_2 \sum_{\lambda=1}^{3} \{\gamma_\lambda^2/(E_\lambda - E_{\rm M})\}\,, \end{aligned}\right\} \tag{5}$$

with $E_{\rm M}$ a mean value of E, say $E_{\rm M} = 16\cdot8$ MeV. For other choices of B_2, the same energy dependence of σ_α may be obtained by using a generalization of (4) with parameter values obtained from the relations of Appendix II.

Equation (4) is a generalization of equation (13) of Barker (1967), the essential modification being the inclusion of the real constants J_x and K which give the contributions of the background due to the broad 2^+ levels. We omit the J_x terms,* as the feeding factors for the levels 2 and 3 are assumed to be relatively small and the level 1 is far away. On the other hand K may not be omitted, as it is $P_2 R_2(E_{\rm M})$ where R_2 is the R-function of Section II with the narrow 2^+ levels omitted, so that equation (1) of BHT with $B_2 = S_2$ gives

$$K = \tan(\delta_2 + \phi_2)\,, \tag{6}$$

where $\delta_2 = \delta_2^{\rm exp}(E_{\rm M}) \approx 74^\circ$ (see Fig. 1) and $\phi_2 = \phi_2(E_{\rm M})$. Since ϕ_2 is a sensitive function of the channel radius a_2, K becomes very large for certain values of a_2 (see Table 6).

The dependence on E of the right-hand side of equation (4), with $J_x = 0$, can be made independent of the value of K by appropriate choice of the parameters E_λ, Γ_λ, and $g_{\lambda x}$ ($\lambda = a,b$). The necessary relations are given in Appendix II. Fits to the ^{10}B(d, α)^{8}Be data of Browne, Callender, and Erskine (1966) have been obtained previously with $K = 0$ (Barker 1967). The corresponding parameter values, with the superscript 0 denoting $K = 0$, are

$$E_b^0 - E_a^0 = 303\ \text{keV}\,, \qquad \Gamma_a^0 = 108\ \text{keV}\,, \qquad \Gamma_b^0 = 79\ \text{keV}\,. \tag{7}$$

These values in the formula (4), with the appropriate feeding factors, give the same energy dependence as the weighted mean parameters of Browne, Callender, and Erskine give with their form of the differential cross section. Identical fits can be obtained for any value of K, but the required values of the level parameters depend on K as shown in Table 6. In particular, from the relations of Appendix II,

$$\Gamma_a + \Gamma_b = (1+K^2)(\Gamma_a^0 + \Gamma_b^0)\,, \tag{8}$$

while approximately, for $|K| \lesssim 3$,

$$\Gamma_a/\Gamma_b \approx (\Gamma_a^0/\Gamma_b^0)\exp\{(\Gamma_a^0+\Gamma_b^0)K/(E_b^0 - E_a^0)\} \approx 1\cdot37\exp(0\cdot6K)\,. \tag{9}$$

* These terms may be needed to explain some energy- and angle-dependent effects observed recently by Callender (personal communication) in the ^{10}B(d, α)^{8}Be reaction.

Now the 16·6 and 16·9 MeV states appear to be well described as mixtures of basic $T = 0$ and $T = 1$ states, which we label by 0 and 1′ respectively

$$\Psi_a = \alpha\Psi_0+\beta\Psi_{1'}, \qquad \Psi_b = \beta\Psi_0-\alpha\Psi_{1'}, \tag{10}$$

with $\alpha^2+\beta^2 = 1$ (Barker 1966). Thus we can write

$$\Gamma_a+\Gamma_b = \Gamma_0 \tag{11}$$

TABLE 6

VALUES OF VARIOUS QUANTITIES, MOSTLY CONNECTED WITH THE 16·6 AND 16·9 MeV STATES OF ^{8}Be, FOR VARIOUS CHANNEL RADII

$B_2 = S_2 = S_2(E_M)$, $E_M = 16\cdot8$ MeV

a_2 (fm)	ϕ_2 (deg)	K	E_b-E_a (keV)	Γ_a (keV)	Γ_b (keV)	$\lvert\alpha\rvert$	$\lvert\beta\rvert$	S_2	P_2	γ_1^2 (MeV)	$\mathscr{S}_0/\mathscr{S}_1$
5·0	191	11·2	1134	23200	420	0·991	0·133	−0·106	5·6	0·85	2·46
5·5	223	−1·93	328	262	620	0·545	0·838	−0·088	6·3	0·603	0·115
6·0	257	−0·56	300	123	123	0·707	0·707	−0·073	7·0	0·471	0·037
6·5	290	0·07	304	111	77	0·767	0·642	−0·062	7·6	0·390	0·031
6·75	307	0·38	309	136	78	0·796	0·605	−0·058	8·0	0·364	0·037
7·0	324	0·78	323	207	94	0·828	0·560	−0·055	8·3	0·339	0·053
7·5	358	3·10	451	1728	250	0·934	0·356	−0·050	9·0	0·306	0·357
8·0	392	−3·38	402	386	1931	0·408	0·913	−0·045	9·6	0·286	0·421

for the width of the basic $T = 0$ state, and

$$\Gamma_a/\Gamma_b = \alpha^2/\beta^2, \tag{12}$$

so that different values of K lead to different values of Γ_0 and of α and β. Also from (10), (11), and (12) we may write the feeding amplitudes for the states a and b in terms of those for the states 0 and 1′:

$$g_{ax} = \Gamma_0^{-\frac{1}{2}}(\Gamma_a^{\frac{1}{2}}g_{0x}+\Gamma_b^{\frac{1}{2}}g_{1'x}), \qquad g_{bx} = \Gamma_0^{-\frac{1}{2}}(\Gamma_b^{\frac{1}{2}}g_{0x}-\Gamma_a^{\frac{1}{2}}g_{1'x}). \tag{13}$$

From the ^{10}B(d,α)^{8}Be reaction, the approximate equalities of the widths of the observed peaks and of their intensities have been used as arguments that $|\alpha/\beta| \approx 1$ (Barker 1966). This result, however, is based on the assumption that $K = 0$,* and the same fit obtained with other values of K leads to different values of α/β (see Table 6). No selection of a best value of K is possible from arguments based on isobaric spin conservation, as equations (A11) in Appendix II show that changing K does not change the values of the combinations of feeding amplitudes

$$\sum_x g_{0x}^2, \qquad \sum_x g_{1'x}^2, \qquad \text{and} \qquad \sum_x g_{0x}g_{1'x}$$

* It was also assumed that there is no interference between the 16·6 and 16·9 MeV levels. Inclusion of interference does not change the result. This is contrary to the conclusion of Marion *et al.* (1967); however, their discussion on the effect of interference on the intensity ratio of these levels in the ^{10}B(d,α)^{8}Be reaction is incorrect, as they assumed that $A/B = \alpha/\beta$ in their formula (21), whereas their formula (9) implies $A/B = \alpha^2/\beta^2$ (cf. Browne, Callender, and Erskine 1966).

which occur in (4) when (13) is used. Relative values of these combinations are 1, 0·092, and 0·017 respectively.

Similarly, arguments based on the relative yields of the 16·6 and 16·9 MeV levels observed in ^{7}Li(d, n)^{8}Be, ^{9}Be(p, d)^{8}Be, and ^{9}Be(^{3}He, α)^{8}Be, assumed to proceed as direct stripping or pickup transitions, have been used to suggest that $|\alpha/\beta| \approx 1$ (Barker 1966; Marion and Wilson 1966; Paul 1966). Again previous analyses assumed $K = 0$, and used theoretical estimates of g_{0x} and $g_{1'x}$ obtained from shell-model calculations of spectroscopic factors with x labelling channel spin (Barker 1966). These estimates provide consistent fits to all the data (Dietrich and Cranberg 1960; Dorenbusch and Browne 1963; Marion, Ludemann, and Roos 1966). Exactly the same fits can be obtained, however, for any value of K, with the same values of g_{0x} and $g_{1'x}$ and with other parameters satisfying the relations (A10), so that again no choice of a best value of K is possible. The same applies to other reactions populating the 16·6 and 16·9 MeV levels, including ^{6}Li(^{3}He, p)^{8}Be (Erskine and Browne 1961), ^{7}Li(^{3}He, d)^{8}Be (Marion *et al.* 1967), and ^{7}Li(p, γ)^{8}Be (Marion and Wilson 1966; Paul, Kohler, and Snover 1968). Thus it is not possible to obtain unique values of Γ_a and Γ_b by analysis of these experimental data alone.

The contribution of the 16·6 and 16·9 MeV levels to the phase shift δ_2 is also independent of the channel radius a_2, when the level parameters are constrained to fit the ^{10}B(d, α)^{8}Be data. This follows from the relations of Appendix II, which show that in the energy region near 17 MeV the 16·6 and 16·9 MeV levels give an additive contribution to δ_2 of

$$\arctan[\tfrac{1}{2}\{\Gamma_a^0/(E_a^0-E)+\Gamma_b^0/(E_b^0-E)\}].$$

Some restrictions on the values of K and of a_2 may be sought by using additional results of shell-model calculations. Thus from (6), (8), and (11)

$$\Gamma_0 = (1+K^2)\Gamma_0^0 = \sec^2(\delta_2+\phi_2)\Gamma_0^0, \tag{14}$$

where $\Gamma_0^0 = 187$ keV from (7). Also we can write $\Gamma_0 = 2\gamma_0^2 P_2$. Thus for each value of a_2, γ_0^2 can be calculated and compared with the value of γ_1^2 given in Table 1 (and in Table 6). With the reasonable approximations that the denominators

$$1+\sum_c \gamma_{\lambda c}^2 S_c'(E_\lambda)$$

occurring in (A10) of BHT are the same for each of the states 0 and 1, and that the single-particle (α-particle) reduced widths are the same for these two states, then the ratio of the spectroscopic factors of these levels for the $\alpha+\alpha$ channel can be written

$$\mathscr{S}_0/\mathscr{S}_1 = \gamma_0^2/\gamma_1^2 = \sec^2(\delta_2+\phi_2)\,\Gamma_0^0/2\gamma_1^2 P_2, \tag{15}$$

and values of this ratio are given in the last column of Table 6 for various channel radii. These values may be compared with those obtained from shell-model calculations by assuming that for a level of the lowest configuration the spectroscopic factor is proportional to the intensity of the basic state $\Psi(1s^4\,1p^4[4]^{11}D_2)$ in the wavefunction. Thus from Barker (1966), $\mathscr{S}_0/\mathscr{S}_1 = (-0\cdot064/0\cdot987)^2 = 0\cdot004$, while the potential (POT) fit of Cohen and Kurath (1965) gives $\mathscr{S}_0/\mathscr{S}_1 = 0\cdot024$. Both of these values lie below the minimum of $0\cdot030$ obtained from (15) for any a_2 in the range covered in Table 6. The values in Table 6 are for $B_2 = S_2$. For general B_2, the value $\Gamma_0(B_2)$ of Γ_0 is given in terms of the quantity $\Gamma_0 \equiv \Gamma_0(S_2)$ occurring in equation (14) by (see equations (A8) and (A9) of Appendix II)

$$\Gamma_0(B_2) = \Gamma_0(S_2)\{1-(B_2-S_2)K/P_2\}^{-2}. \tag{16}$$

From the values of S_2 and P_2 given in Table 6, it is seen that the values of $\mathscr{S}_0/\mathscr{S}_1$ in Table 6 are not altered significantly for reasonable values of B_2 ($0 \gtrsim B_2 \gtrsim -0\cdot5$). Balashov and Rotter (1965) had previously noted the disagreement between the shell-model value of $\mathscr{S}_0/\mathscr{S}_1$ and that obtained (with $K = 0$) from the observed widths, and attributed it to the mixing of shell-model levels with levels of a collective nature. In our description this corresponds to an admixture of say Φ_2 in the eigenstate Ψ_0, where Φ_2 is the pure state of higher configuration that forms the main part of the eigenstate Ψ_2, and Ψ_0 consists mainly of the pure state Φ_0 of the lowest configuration. As Φ_2 has a large α-particle reduced width compared with Φ_0, the calculated value of $\mathscr{S}_0$ is very sensitive to the intensity of Φ_2 in Ψ_0; thus 10% intensity gives $\mathscr{S}_0/\mathscr{S}_1 \approx 0\cdot2$. Very little restriction can therefore be placed on the value of K, especially as large values of $|K|$ correspond to E_2 (or E_3) near 17 MeV, in which case larger admixtures of Φ_2 (or Φ_3) in Ψ_0 are probable (this is consistent with the trend of $\log(ft)_2$ values in Table 7).

If the isobaric spin mixing in the levels a and b is attributed to the Coulomb interaction (Barker 1966), then it would seem that the Coulomb matrix elements required to produce this mixing would depend on K, since α and β depend on K. Actually the relations of Appendix II are such that

$$H^c_{01'} \equiv \alpha\beta(E_a-E_b) \qquad \text{and} \qquad H_{1'1'} \equiv \beta^2E_a+\alpha^2E_b$$

are independent of the choice of K, while

$$H_{00} \equiv \alpha^2E_a+\beta^2E_b = H^0_{00}-\tfrac{1}{2}K\Gamma^0_0$$

(the dependence on B_2 is similar). The accuracy of shell-model calculations is not such as to restrict severely the value of K or a_2 on this account.

The region of ^{8}Be excitation energies near 16 MeV has also been investigated by Matt *et al.* (1964), who measured the spectrum of high energy α-particles following ^{8}B β-decay, covering the region $E = 15$–17 MeV. We fit their data by using formula (4) with $J_x = 0$, $P_x = f_{\beta+}$ (the integrated Fermi function, which is $f(-4, 17\cdot563$ MeV $-E)$ in the notation of Bahcall 1966), and $x = F$ or G corresponding to Fermi and Gamow–Teller transitions. The feeding amplitudes $g_{\lambda x}$ can be written in terms of $g_{1'F}$, g_{0G}, and $g_{1'G}$ (since $g_{0F} = 0$), and these can be related in an approximate way to the more usual β-decay matrix elements $|\int 1|^2_{1'}$, $|\int \boldsymbol{\sigma}|^2_0$, and $|\int \boldsymbol{\sigma}|^2_{1'}$. This cannot be done rigorously as the levels are interfering; a fuller discussion is given in Appendix III.

The values of these matrix elements required for a given fit to the data are independent of the value of K (and of the value of B_2). We choose E_b-E_a, Γ_a, and

Γ_b to fit the $^{10}B(d,\alpha)^8Be$ data as mentioned above, so that for $K = 0$ they have the values (7), and also take $E_a^0 = 16\cdot620+0\cdot095$ MeV $= 16\cdot715$ MeV, which corresponds to the excitation energy $16\cdot627$ MeV obtained by Marion *et al.* (1967) from $^7Li(^3He,d)^8Be$. Then we put $|\int 1|_{1'}^{2} = 2$, take $|\int \boldsymbol{\sigma}|_{1'}^{2} = 0$, which is reasonable from shell-model calculations (Barker 1966), and vary $|\int \boldsymbol{\sigma}|_0^2$ to minimize a quantity Y_{2+} defined in a similar manner to (3). The experimental data are those of Matt *et al.* (1964). In the theoretical fit, allowance is made for the experimental energy resolution and source thickness. The effect of the latter is uncertain as the geometry of the target is not known; we assume a uniform α-particle energy loss between 0 and Δ, and choose Δ to minimize Y_{2+}. This gives $\Delta = 140$ keV and a mean energy loss of 70 keV, which may be compared with the values quoted by Matt *et al.* of a source thickness of 160 keV and a mean energy loss of about 50 keV. Then the best fit, which is shown in Figure 4, gives

$$|\int \boldsymbol{\sigma}|_0^2 = 2\cdot60, \qquad Y_{2+} = 0\cdot87. \tag{17}$$

The dependence of Y_{2+} on $|\int \boldsymbol{\sigma}|_0^2$ is quadratic, so that

$$Y_{2+} = 0\cdot87+51(|\int \boldsymbol{\sigma}|_0^2-2\cdot60)^2. \tag{18}$$

A shell-model estimate gave $|\int \boldsymbol{\sigma}|_0^2 = 2\cdot26$ (Barker 1966). The $\log(ft)$ values for the $16\cdot6$ and $16\cdot9$ MeV levels do not depend sensitively on the value of a_2 or of K; for $a_2 = 6\cdot75$ fm, one obtains from (17) the values $3\cdot31$ and $3\cdot38$ respectively.*

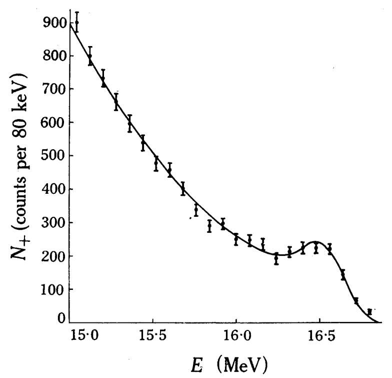

Fig. 4.—Spectrum of high energy α-particles from $^8B(\beta^+)^8Be(\alpha)^4He$ as a function of ^8Be channel energy E. The points are experimental values and the curve is the best fit corresponding to the values (17).

From the discussion of this section, we conclude that experimental data from reactions proceeding through the $16\cdot6$ and $16\cdot9$ MeV levels of ^8Be are not sufficient by themselves to pick out a best value of K or of a_2, and therefore do not lead to unique values of the widths Γ_a and Γ_b of these levels. For each value of a_2, however, well-defined values of Γ_a and Γ_b, as well as of E_a and E_b, can be obtained, by fitting, for example, the $^{10}B(d,\alpha)^8Be$ data of Browne, Callender, and Erskine (1966), and these are used in the $^8Li(\beta^-)^8Be$ fits of the next section. Also the $^8B(\beta^+)^8Be$ data

* Matt *et al.* (1964) obtained $\log(ft) = 3\cdot03$ for the $16\cdot6$ MeV level, if its position and width were taken from Erskine and Browne (1961), but we find that their formula in this case gives $\log(ft) = 3\cdot33$.

may be fitted for any channel radius to give values of the Gamow–Teller matrix element $|\int \boldsymbol{\sigma}|_0^2$, and these are also used for the ^{8}Li β-decay.

V. Restriction of R-matrix Parameters from $^8\text{Li}(\beta^-)^8\text{Be}$

The β-decays of ^{8}Li and ^{8}B are suitable means for studying the 2^+ states of ^{8}Be for several reasons: allowed decays populate only the 2^+ states, there is no competing mode in which the α-particle is emitted before the β-particle, and the energy dependence of the feeding factors for the β-emission is well known (the integrated Fermi function). A possible disadvantage is that the β-decay matrix elements to the broad 2^+ levels ($\lambda = 1$, 2, and 3) are expected to be small compared with those to the narrow 2^+ levels at 16·6 and 16·9 MeV ($\lambda = a$ and b). For the $\lambda = 1$ level this follows from shell-model calculations (Cohen and Kurath 1965; Barker 1966) and for the $\lambda = 2$ and 3 levels from the fact that β-decay matrix elements vanish between states of different shell-model configurations.

The most accurate published experimental results appear to be those of Alburger, Donovan, and Wilkinson (1963), who measured the α-spectrum following ^{8}Li β-decay for energies corresponding to ^{8}Be excitation energies $E_x \lesssim 7{\cdot}4$ MeV ($E \lesssim 7{\cdot}5$ MeV). Pfander (see Lipperheide 1966) obtained agreement with these measurements and extended the spectrum to $E \approx 14$ MeV.

Previous one-level approximations did not adequately fit the observed spectrum for $E \gtrsim 3{\cdot}5$ MeV (Alburger, Donovan, and Wilkinson 1963, and references quoted therein), and it was suggested that the contribution from the 16·6 MeV level could account for the discrepancy (Alburger, Donovan, and Wilkinson 1963; Matt *et al.* 1964). No such detailed fit has been published, apart from an early one by Griffy and Biedenharn (1960), who used a form for the yield that is probably incorrect (see e.g. Alburger, Donovan, and Wilkinson 1963), and an indirect treatment by Lipperheide (1966) using dispersion relations.

The form of α-spectrum used here is taken from (A12) and (A14) of Appendix III, with $\lambda = 1$, 2, 3, a, and b, and $f_\beta = f_{\beta-} \equiv f(4, 16{\cdot}608 \text{ MeV} - E)$. Values of most of the parameters are obtained from fits to other data. Thus for various channel radii a_2 and for $B_2 = 0$, values of E_λ and γ_λ^2 are used which for $\lambda = 1, 2$, and 3 give acceptable fits to δ_2^{exp} (Tables 1 and 2) and for $\lambda = a$ and b give the best fit to the $^{10}\text{B}(\text{d},\alpha)^8\text{Be}$ data (Table 6). Of the feeding amplitudes $g_{\lambda x}$, we take $g_{\lambda F} = 0$ ($\lambda = 1$, 2, and 3) since these levels are assumed to be pure $T = 0$. We also take $g_{3G} = 0$ since level 3 is not expected to be populated strongly. For the levels a and b, we make the same assumptions as in Section IV, and restrict g_{0G} or $|\int \boldsymbol{\sigma}|_0^2$ so that good fits are obtained to the $^8\text{B}(\beta^+)^8\text{Be}$ data according to equation (18). Thus the only free parameters are g_{1G} and g_{2G}, or rather A_{1G} and A_{2G} defined in (A15). The formula so obtained for the α-spectrum is

$$N_-(E) = C^2 \frac{Nt_{\frac{1}{2}}}{\ln 2} f_{\beta-} P_2 \left(\left| \frac{g_{1'F}\gamma_a\gamma_b}{\gamma_0} \frac{E_b - E_a}{(E_a - E)(E_b - E)} \right|^2 + \left| \sum_{\lambda=1}^{2} \frac{g_{\lambda G}\gamma_\lambda}{E_\lambda - E} + \frac{g_{0G}}{\gamma_0} \sum_{\lambda=a}^{b} \frac{\gamma_\lambda^2}{E_\lambda - E} \right|^2 \right)$$
$$\div \left| 1 - (S_2 + \mathrm{i}P_2) \sum_{\lambda=1}^{b} \frac{\gamma_\lambda^2}{E_\lambda - E} \right|^2 . \tag{19}$$

The Fermi transitions do not contribute much in the region of interest ($E \lesssim 14$ MeV).

The quality of fit to the experimental data is measured by a quantity Y_{2-} defined in terms of $N_-(E)$ in a manner similar to (3). The experimental data are taken from Figure 8 of Alburger, Donovan, and Wilkinson (1963) for $E = 2\cdot0$–$7\cdot5$ MeV, with errors assigned on the basis of statistical errors in their thin target yields, and from Figure 2 of Lipperheide (1966) for $E = 7\cdot5$–14 MeV, with errors assigned rather arbitrarily. As these experimental values lie on smooth curves, there is no expectation that Y_{2-} need be near unity. The data of Alburger, Donovan, and Wilkinson for $E < 2\cdot0$ MeV are not used, as this part of their spectrum is very sensitive to their subtraction procedure. A linear least squares programme is used to obtain A_{1G} and A_{2G}, and then (A22) and (A21) are used to give values of $|\int \boldsymbol{\sigma}|_\lambda^2$ and $(ft)_\lambda$ for $\lambda = 1$ and 2.

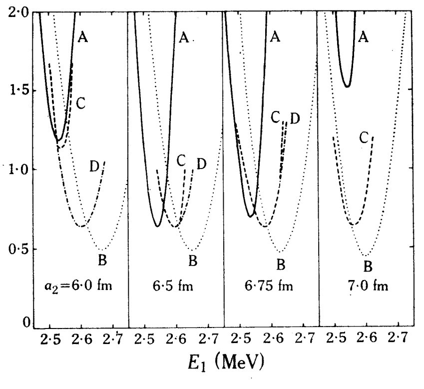

Fig. 5.—Minimum values of
(A) Y_{2-},
(B) X_2,
(C) $0\cdot2\,Y_2$, $|g_2/g_1| \leqslant 0\cdot3$,
(D) $0\cdot2\,Y_2$, $|g_2/g_1| > 0\cdot3$,
as functions of E_1 for various channel radii a_2.

Because the dependence on $|\int \boldsymbol{\sigma}|_0^2$ of Y_{2+} is much more sensitive than that of Y_{2-} (a change of $|\int \boldsymbol{\sigma}|_0^2$ by 13% from its optimum value of $2\cdot60$ doubles Y_{2+} but changes Y_{2-} by less than 10%) we give results only for $|\int \boldsymbol{\sigma}|_0^2 = 2\cdot60$. In Figure 5, for various channel radii, the minimum values of Y_{2-} are shown as functions of E_1, the remaining level parameters being chosen to minimize X_2 (as in Table 2 for $a_2 = 7\cdot0$ fm). Also shown are the corresponding values of X_2 and Y_2, which characterize the fits to the α–α scattering data and the ^{9}Be(p, d)^{8}Be data respectively. For each channel radius, the E_1 value giving the smallest Y_{2-} is lower than that giving the smallest Y_2, and this is lower than that giving the smallest X_2, the differences each being about 50 keV for $a_2 = 6\cdot75$ fm. The smallest Y_{2-} drop to a fairly sharp minimum of about $0\cdot6$ at $a_2 \approx 6\cdot6$ fm. If values of $Y_{2-} \lesssim 1\cdot2$ are regarded as acceptable, then it is only for channel radii in the range $6\cdot3$–$7\cdot0$ fm that one can obtain acceptable simultaneous fits to $\delta_2^{\exp}$ and to the ^{8}B(β^+)^{8}Be and ^{8}Li(β^-)^{8}Be data. In order to obtain the best simultaneous fit to these data, we introduce the quantity $Z_{2-} = X_2 + Y_{2-}$ and take the smallest Z_{2-} as giving the best fit. Such values of Z_{2-} are given in Table 7 for various channel radii, together with the corresponding values of other quantities. The best overall fit is obtained for $a_2 = 6\cdot7$ fm.

The corresponding log(ft) values for levels 1 and 2 are $5\cdot79$ and $5\cdot23$ respectively. For the $2\cdot9$ MeV level, this value is somewhat larger than the values $5\cdot62$ and $5\cdot64$ obtained previously on the assumption that all the ^{8}Li and ^{8}B decays go to a sharp level at $2\cdot9$ MeV (Bahcall 1966), and therefore further from values obtained in shell-model calculations, which are 4·77, 5·04, and 5·26 (Cohen and Kurath 1965) and 5·45 (Barker 1966). The value of $\log(ft)_2$ may be attributed to about 2% intensity admixture of the state Φ_0 (of the lowest configuration) into the $\lambda = 2$ state Ψ_2, which belongs mainly to higher configurations. This may be compared with about 20% admixture in the similar 7·66 MeV state of ^{12}C required to account for its observed $\log(ft)$ value (Cohen and Kurath 1965).

Table 7

PARAMETER VALUES FOR BEST FITS TO $\delta_2^{\exp}$ AND THE ^{8}Li AND ^{8}B β-DECAY DATA FOR VARIOUS CHANNEL RADII

$|\int \sigma|_0^2 = 2\cdot60$, $Y_{2+} = 0\cdot87$

a_2 (fm)	E_1 (MeV)	γ_1^2 (MeV)	E_2 (MeV)	γ_2^2 (MeV)	E_3 (MeV)	γ_3^2 (MeV)	$\log(ft)_1$	$\log(ft)_2$	X_2	Y_{2-}	Z_{2-}
5·5	2·500	0·714	13·54	1·223	177·7	18·4	5·70	4·43	1·23	2·16	3·40
6·0	2·554	0·536	10·73	1·000	73·6	6·66	5·74	4·95	1·33	1·34	2·67
6·5	2·561	0·424	8·78	0·824	37·0	2·82	5·77	5·18	1·13	0·77	1·90
6·75	2·551	0·390	8·07	0·746	33·6	2·62	5·79	5·23	1·02	0·79	1·81
7·0	2·551	0·352	7·37	0·676	26·3	1·77	5·80	5·28	0·67	1·53	2·20
7·1	2·554	0·339	7·14	0·651	25·0	1·65	5·80	5·29	0·52	2·92	3·44

The fit to the ^{8}Li(β^-)^{8}Be data is shown in Figure 6 for $a_2 = 6\cdot75$ fm and the parameters of Section VII, the same as used in Figures 1 and 3.

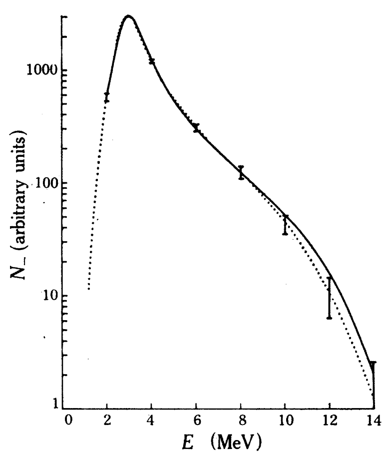

Fig. 6.—Spectrum of α-particles from ^{8}Li(β^-)^{8}Be(α)^{4}He as a function of ^{8}Be channel energy E. The dotted curve gives the experimental values and the solid curve is the fit over the region $E = 2\cdot0$–$14\cdot0$ MeV for $a_2 = 6\cdot75$ fm and other parameters as in Section VII. The vertical bars indicate typical errors assigned to the experimental data.

VI. Data from Other Reactions

The 2·9 MeV level of ^{8}Be has been observed in many reactions (Lauritsen and Ajzenberg-Selove 1966). The position and particularly the width of the level obtained from fits based on a one-level approximation appear to differ for different reactions, as was pointed out, for example, by Berkowitz (1964). We attribute these variations to interference between the $2 \cdot 9$ MeV level and the other broad 2^+ levels.

In some reactions this interference is expected to be small, because the broad levels belong mainly to higher configurations and are weakly fed. These include reactions in which ^{8}Be is produced in a direct transition from a heavier target nucleus by the extraction of nucleons, e.g. ^{9}Be(p, d)^{8}Be, ^{9}Be(d, t)^{8}Be, and ^{9}Be(^{3}He, α)^{8}Be. In these cases the observed position and width of the $2 \cdot 9$ MeV peak should be close to the values $2 \cdot 84$ and $1 \cdot 30$ MeV given in Section VII. The ^{9}Be(p, d)^{8}Be data of Hay (personal communication) are discussed in Section III; other measurements on this reaction are less useful, being at too low a bombarding energy (Beckner, Jones, and Phillips 1961), having poorer resolution (Kull 1967), or being over an energy range not sufficiently wide to allow determination of the background (Laugier *et al.* 1966). Data from ^{9}Be(d, t)^{8}Be (Vlasov and Ogloblin 1959) have low resolution and from ^{9}Be(^{3}He, α)^{8}Be (Dorenbusch and Browne 1963) have a large background, but in each case the position and width of the $2 \cdot 9$ MeV peak appear to be consistent with those obtained from Hay's data.

In reactions in which ^{8}Be is produced in a direct transition from a lighter target nucleus by the addition of nucleons, states of higher configurations may be formed. The addition of one nucleon, as in ^{7}Li(d, n)^{8}Be, can populate 1p–1h configurations (relative to the lowest configuration $1s^4\,1p^4$), while the addition of two nucleons, as in ^{6}Li(^{3}He, p)^{8}Be, can populate both 1p–1h and 2p–2h configurations. If it is assumed that the higher broad 2^+ states belong to 2p–2h configurations, they should not be populated by one-nucleon transfer reactions. Fits to preliminary data obtained by Kean and Spear (personal communication) from magnetic analysis of the ^{7}Li(^{3}He, d)^{8}Be reaction at a beam energy of 15 MeV and an angle of 10° support this assumption. The deuteron spectrum is assumed to have three contributions as described in Section III, the 2^+ level parameters being fixed at the values given in Section VII so that the only free parameters are g_2/g_1 and the normalizations of the three contributions. The best fit is obtained for $g_2/g_1 = 0 \cdot 15$, corresponding to a peak energy of $2 \cdot 82$ MeV and a width of $1 \cdot 27$ MeV for the 2^+ contribution. The data of Vlasov *et al.* (1960) from ^{7}Li(α, t)^{8}Be at 40 MeV and 7° appear to be consistent with such values.

If ^{8}Be is produced in a reaction that proceeds through an intermediate compound nucleus, formation of states of higher configurations is not necessarily inhibited, and appreciable interference effects and hence different positions and widths of the $2 \cdot 9$ MeV peak are possible. We illustrate this by fitting data obtained from two reactions at energies and angles where direct contributions are probably small, and for which larger values of the width have been quoted. Johnson and Trail (1964) measured the neutron spectrum from ^{7}Li(d, n)^{8}Be at $1 \cdot 98$ MeV and 120°. With the same procedure as for ^{7}Li(^{3}He, α)^{8}Be above, except that allowance is made for the experimental energy resolution and a neutron penetration factor is assumed the same as that of Johnson and Trail, the best fit to the data, shown in Figure 7, is obtained for $g_2/g_1 = -0 \cdot 90$, corresponding to a position and width of the 2^+ contribution of $2 \cdot 97$ MeV and $1 \cdot 62$ MeV respectively. Thus appreciable feeding of the second 2^+ state can lead to a much larger width of the $2 \cdot 9$ MeV peak. Kavanagh

(1960) measured the proton spectrum from ^{7}Be(d, p)^{8}Be at $1\cdot475$ MeV and 60°. In this case, the best fit is obtained with negligible feeding of the upper 2^+ level. This fit is shown in Figure 8.

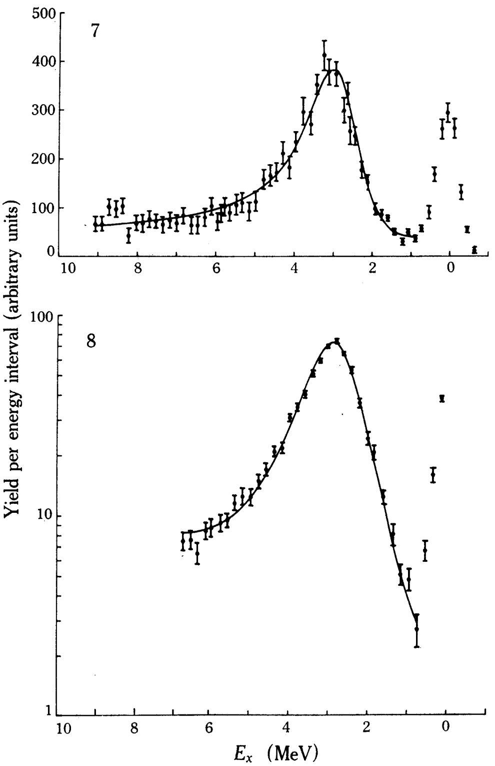

Fig. 7.—Neutron spectrum from ^{7}Li(d, n)^{8}Be, as a function of ^{8}Be excitation energy E_x. The experimental points are from Johnson and Trail (1964). The curve is the best fit over the region $E_x = 0\cdot9$–$9\cdot0$ MeV for the 2^+ level parameters given in Section VII, with background contributions from 0^+ states of ^{8}Be and from competing reaction modes, and with allowance for the experimental energy resolution.

Fig. 8.—Proton spectrum from ^{7}Be(d, p)^{8}Be, as a function of ^{8}Be excitation energy E_x. The experimental points are from Kavanagh (1960). The curve is the best fit over the region $E_x = 0\cdot8$–$6\cdot6$ MeV for the 2^+ level parameters given in Section VII, with background contributions from 0^+ states of ^{8}Be and from competing reaction modes, and with allowance for the experimental energy resolution.

VII. Summary of Results

Restrictions on the values of parameters in R-matrix fits to the d-wave α–α scattering phase shift have been obtained by requiring simultaneous fits to experimental data from various reactions. In Section III, the best value of the channel radius a_2 was found to be $7\cdot1$ fm, while values in the range $6\cdot3$–$8\cdot0$ fm were acceptable. In Section V, the best value was $6\cdot7$ fm and the acceptable range $6\cdot3$–$7\cdot0$ fm.

The best overall channel radius* may be taken as

$$a_2 = 6\cdot75 \text{ fm}, \tag{20}$$

* Recently Kermode (preprint 1968) has used one-level approximations to fit both scattering and reaction data for 0^+, 2^+, and 4^+ levels of ^{8}Be, and has obtained a best channel radius of $4\cdot6$ fm, but it is not clear over what energy range he gets a good fit to the scattering data, as effective range expansions are used as intermediate stages in the fitting.

and from Figure 5 the best value of E_1 is then about $2\cdot56$ MeV. The values of the level parameters are then

$$\left.\begin{array}{lll} E_1 = 2\cdot56\ \mathrm{MeV}, & E_2 = 8\cdot07\ \mathrm{MeV}, & E_3 = 34\cdot1\ \mathrm{MeV}, \\ \gamma_1^2 = 0\cdot387\ \mathrm{MeV}, & \gamma_2^2 = 0\cdot746\ \mathrm{MeV}, & \gamma_3^2 = \ 2\cdot68\ \mathrm{MeV}, \end{array}\right\} \tag{21}$$

while the feed factors for ^{9}Be(p, d)^{8}Be are

$$g_2/g_1 = 0\cdot03, \qquad g_3/g_1 = 0\cdot0, \tag{22}$$

and for ^{8}Li(β^-)^{8}Be

$$\log(ft)_1 = 5\cdot79, \qquad \log(ft)_2 = 5\cdot23. \tag{23}$$

The corresponding qualities of fit are given by

$$X_2 = 0\cdot90, \qquad Y_2 = 3\cdot3, \qquad Y_{2-} = 0\cdot97. \tag{24}$$

Quantities connected with the $16\cdot6$ and $16\cdot9$ MeV levels are given in Section IV and particularly in Table 6. The positions and widths of the first two 2^+ levels, defined as in BHT except that here we use excitation energies, are

$$\left.\begin{array}{ll} E_{1\mathrm{m}} = 2\cdot84\ \mathrm{MeV}, & E_{2\mathrm{m}} = \ 8\cdot5\ \mathrm{MeV}, \\ \Gamma_1 = 1\cdot30\ \mathrm{MeV}, & \Gamma_2 = 10\cdot5\ \mathrm{MeV}. \end{array}\right\} \tag{25}$$

The above values are all for $B_2 = 0$. For $B_2 = -0\cdot5$, the changes in g_2/g_1, g_3/g_1, $\log(ft)_1$, $\log(ft)_2$, $E_{1\mathrm{m}}$, Γ_1, $E_{2\mathrm{m}}$, and Γ_2 are $0\cdot05$, $0\cdot02$, $-0\cdot04$, $-0\cdot02$, $0\cdot01$ MeV, $0\cdot01$ MeV, $0\cdot3$ MeV, and $-0\cdot1$ MeV.

The values (25) for the position and width of the first excited state of ^{8}Be are both smaller than the mean values given by Lauritsen and Ajzenberg-Selove (1966), which were based on one-level analyses.

By comparison with the results of BHT for the same channel radius, the second 2^+ level is found to lie about 2 MeV above the second 0^+ level and to have about the same width. Similar 2^+ levels are found in other light even nuclei, about 1–2 MeV above the second 0^+ level, and apparently not belonging to the lowest shell-model configuration. Thus there are 2^+ levels in ^{10}Be at $5\cdot96$ and $7\cdot55$ MeV (Lauritsen and Ajzenberg-Selove 1966), in ^{14}C at $7\cdot01$ and $8\cdot32$ MeV and in ^{14}O at $6\cdot59$ and $7\cdot78$ MeV (Ball and Cerny 1967), and in ^{16}O at $6\cdot92$ MeV (Lauritsen and Ajzenberg-Selove 1962). Shell-model calculations with only the lowest configuration predict* only one 2^+ level in this energy region for $A = 10$ and $A = 14$ and none for $A = 16$. The ratio of α-particle reduced widths obtained here for the second 2^+ and second 0^+ states of ^{8}Be is similar to that found for the $6\cdot92$ and $6\cdot06$ MeV states of ^{16}O by Loebenstein *et al.* (1967) (see also Bethge *et al.* 1967; Meier-

* Boyarkina (1964) predicts 2^+, $T = 1$ levels for $A = 10$ at energies $3\cdot4$, $5\cdot9$, and $8\cdot6$ MeV above the lowest $T = 1$ level, but the calculated expectation value of the energy of the [222]^{35}S state is in error and the third 2^+ state should be much higher.

Ewart, Bethge, and Pfeiffer 1968). A similar 2^+ state might be expected in ^{12}C about 2 MeV above the 0^+ state at 7·66 MeV (see also Balashov and Rotter 1965). Morinaga (1966) has given arguments for a 2^+ assignment to the level at 10·3 MeV, previously taken to be 0^+ because of its large width of about 3 MeV (Ajzenberg-Selove and Lauritsen 1968). A 2^+ state of the type considered here, with an α-particle reduced width the same as that of the second 2^+ state of ^{8}Be for the same channel radius (say 6·75 fm) and situated at 10·3 MeV, would have an observed width of about 2·2 MeV. The ghost of the 7·66 MeV state (Barker and Treacy 1962) could also contribute in this region.

The analysis of high energy α–α scattering data by Darriulat *et al.* (1965) suggests 6^+ and 8^+ levels of ^{8}Be at about 28 and 57 MeV respectively. The suggestion that these belong to a rotational band based on the 0^+ ground state becomes open to question because of the existence of 0^+ and 2^+ excited states at about 6 and 9 MeV.

VIII. Acknowledgments

The author wishes to thank Dr. H. J. Hay and Dr. R. H. Spear for making available their unpublished experimental data.

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Appendix I

Discussion of One-channel Approximation for 2^+ States of ^{8}Be

In Appendix I of BHT, use of the R-matrix one-channel approximation for α–α scattering and for reactions involving ^{8}Be was justified provided that only the $\alpha+\alpha$ channel is open ($E \lesssim 17$ MeV) and that for all other channels c the shift factors are linear functions of E. The parameters entering the formulae are then $\bar{E}_\lambda$ and $\bar{\gamma}^2_{\lambda l}$ rather than the original eigenenergies E_λ and reduced widths $\gamma^2_{\lambda l}$.

One case was mentioned in which the $\bar{E}_\lambda$ and $\bar{\gamma}^2_{\lambda l}$ are simply related to the E_λ and $\gamma^2_{\lambda l}$, namely, no channel c has non-vanishing $\gamma_{\lambda c}$ for more than one level λ. This was relevant for the 0^+ levels of ^{8}Be studied in BHT, but in the present case there are three 2^+ levels of ^{8}Be below 17 MeV that appear to belong to the lowest shell-model configuration, and for some nucleon channels more than one of these have large reduced widths, e.g. both the 2·9 and 16·6 MeV levels for the ^{7}Li(0)+p channel (Barker 1966). Thus in this case ξ is apparently not diagonal but couples the states $\lambda = 1$, a, and b. For example,

$$\xi_{1a} = \sum_c \gamma_{1c}\gamma_{ac}(S_c - B_c) \tag{A1}$$

for $E \lesssim 17$ MeV. We take c to include only nucleon channels, and put

$$S_c - B_c = \alpha_c(E-\epsilon)\,, \tag{A2}$$

where B_c is chosen to make ϵ independent of c. The orthogonality of the states 1 and a requires that

$$\sum_c \gamma_{1c}\gamma_{ac} = 0\,, \tag{A3}$$

so that if α_c = constant, then

$$\xi_{1a} = 0\,. \tag{A4}$$

Actually (A4) is approximately true, as the main contributions to the left-hand side of (A3) are from channels with approximately the same threshold energies and for these the α_c are about equal, so that more or less complete cancellation still occurs in

$$\sum_c \gamma_{1c}\gamma_{ac}\alpha_c\,.$$

Similarly ξ_{1b} and ξ_{ab} are expected to be small, so that (A10) of BHT is expected to be approximately true for all the 2^+ levels $\lambda = 1, 2, 3, a$, and b, and $\xi_{aa} \approx \xi_{bb}$ so that

$$\bar{\gamma}_a/\bar{\gamma}_b \approx \gamma_a/\gamma_b \tag{A5}$$

(omitting the subscript $l = 2$). Similarly for reactions the feeding factors satisfy

$$\bar{G}^{\frac{1}{2}}_{ax}/\bar{G}^{\frac{1}{2}}_{bx} \approx G^{\frac{1}{2}}_{ax}/G^{\frac{1}{2}}_{bx}\,, \tag{A6}$$

so that ratios of feeding factors $G_{\lambda x}$ obtained from shell-model calculations via spectroscopic factors can also be used for the barred quantities $\bar{G}_{\lambda x}$.

As in BHT, the quantities E_λ, γ^2_λ, and $G_{\lambda x}$ used in the main body of this paper correspond to the barred quantities of this appendix rather than to the unbarred quantities.

We note that the one-channel approximation for α–α scattering below the ^{7}Li+p threshold would obviously be invalid if an antiresonance of the type described by Kermode (preprint 1968) exists in the s-wave channel. This follows from the Wigner (1955) condition on the energy derivative of the phase shift. It can be shown, however, that also in the two-channel approximation considered by Kermode the existence of an antiresonance is inconsistent with R-matrix theory if only one of the channels is open.

Appendix II

Variations of Level Parameters for 16·6 and 16·9 MeV Levels of ^{8}Be *with Changes of* B_2 *and of* K

The generalization of equation (4) for the case $B_2 = B_2' \neq S_2$ may be written

$$\sigma_\alpha \propto \sum_x \Bigg[P_x\left|\sum_{\lambda=a}^{b}\frac{g'_{\lambda x}\Gamma'^{\frac{1}{2}}_\lambda}{E'_\lambda-E}+J'_x\right|^2$$

$$\div\left\{\left|1+\frac{B_2'-S_2}{P_2}\left(\sum_{\lambda=a}^{b}\frac{\frac{1}{2}\Gamma'_\lambda}{E'_\lambda-E}+K'\right)\right|^2+\left|\sum_{\lambda=a}^{b}\frac{\frac{1}{2}\Gamma'_\lambda}{E'_\lambda-E}+K'\right|^2\right\}\Bigg]. \tag{A7}$$

The dependence on E of σ_α can be made independent of the choice of B_2' by satisfying the general relations of Appendix II of BHT. For the special case considered in Section IV, these simplify to give for the primed quantities ($B_2 = B_2'$) in terms of the unprimed quantities ($B_2 = S_2$)

$$E'_{a,b} = \tfrac{1}{2}\{E_a+E_b-\mu(\Gamma_a+\Gamma_b)\pm(\{E_a-E_b-\mu(\Gamma_a-\Gamma_b)\}^2+4\mu^2\Gamma_a\Gamma_b)^{\frac{1}{2}}\}, \tag{A8a}$$

$$\Gamma'_a = \lambda^2\{\Gamma_a(E_b-E'_a)+\Gamma_b(E_a-E'_a)\}/(E'_b-E'_a), \tag{A8b}$$

$$\Gamma'_b = \lambda^2\{\Gamma_a(E'_b-E_b)+\Gamma_b(E'_b-E_a)\}/(E'_b-E'_a), \tag{A8c}$$

$$K' = \lambda K, \tag{A8d}$$

$$g'_{ax} = \lambda\frac{(g_{ax}\Gamma_a^{\frac{1}{2}}+\mu J_x\Gamma_a)(E_b-E'_a)+(g_{bx}\Gamma_b^{\frac{1}{2}}+\mu J_x\Gamma_b)(E_a-E'_a)}{\Gamma_a'^{\frac{1}{2}}(E'_b-E'_a)}, \tag{A8e}$$

$$g'_{bx} = \lambda\frac{(g_{ax}\Gamma_a^{\frac{1}{2}}+\mu J_x\Gamma_a)(E'_b-E_b)+(g_{bx}\Gamma_b^{\frac{1}{2}}+\mu J_x\Gamma_b)(E'_b-E_a)}{\Gamma_b'^{\frac{1}{2}}(E'_b-E'_a)}, \tag{A8f}$$

$$J'_x = \lambda J_x, \tag{A8g}$$

where

$$\lambda = \{1-(B'_2-S_2)K/P_2\}^{-1}, \qquad \mu = \tfrac{1}{2}(B'_2-S_2)\lambda/P_2. \tag{A9}$$

Similarly the dependence on E of σ_α given by equation (4) can be made independent of the choice of K. If the parameter values corresponding to $K = 0$ are denoted by E^0_λ, Γ^0_λ, $g^0_{\lambda x}$, and J^0_x, then their values for any other value of K are given by

$$E_{a,b} = \tfrac{1}{2}\{E^0_a+E^0_b-\tfrac{1}{2}K(\Gamma^0_a+\Gamma^0_b)\pm(\{E^0_a-E^0_b-\tfrac{1}{2}K(\Gamma^0_a-\Gamma^0_b)\}^2+K^2\Gamma^0_a\Gamma^0_b)^{\frac{1}{2}}\}, \tag{A10a}$$

$$\Gamma_a = (1+K^2)\{\Gamma^0_a(E^0_b-E_a)+\Gamma^0_b(E^0_a-E_a)\}/(E_b-E_a), \tag{A10b}$$

$$\Gamma_b = (1+K^2)\{\Gamma^0_a(E_b-E^0_b)+\Gamma^0_b(E_b-E^0_a)\}/(E_b-E_a), \tag{A10c}$$

$$g_{ax} = (1+K^2)^{\frac{1}{2}}\frac{g^0_{ax}\Gamma_a^{0\frac{1}{2}}(E^0_b-E_a)+g^0_{bx}\Gamma_b^{0\frac{1}{2}}(E^0_a-E_a)+J^0_x(E^0_b-E_a)(E^0_a-E_a)}{\Gamma_a^{\frac{1}{2}}(E_b-E_a)}, \tag{A10d}$$

$$g_{bx} = (1+K^2)^{\frac{1}{2}}\frac{g^0_{ax}\Gamma_a^{0\frac{1}{2}}(E_b-E^0_b)+g^0_{bx}\Gamma_b^{0\frac{1}{2}}(E_b-E^0_a)-J^0_x(E_b-E^0_b)(E_b-E^0_a)}{\Gamma_b^{\frac{1}{2}}(E_b-E_a)}, \tag{A10e}$$

$$J_x = (1+K^2)^{\frac{1}{2}}J^0_x. \tag{A10f}$$

For the feeding amplitudes defined in (13) and for the particular case $J'_x = J_x = J^0_x = 0$, these relations give

$$g'_{0x} = g_{0x} = g^0_{0x}, \qquad g'_{1'x} = g_{1'x} = g^0_{1'x}. \tag{A11}$$

Appendix III

Formulae for β-decay Analysis

For application to the β-decay of ^{8}Li or ^{8}B, the formula (9) of BHT can be used with $l = 2$, $G_{\lambda x} = g^2_{\lambda x} f_\beta$, where f_β is the integrated Fermi function, and $x = F$ or G corresponding to Fermi and Gamow–Teller transitions. Then

$$w(E) = C^2 f_\beta P_2 \frac{\left|\sum\limits_\lambda \{g_{\lambda F}\gamma_\lambda/(E_\lambda - E)\}\right|^2 + \left|\sum\limits_\lambda \{g_{\lambda G}\gamma_\lambda/(E_\lambda - E)\}\right|^2}{\left|1-(S_2-B_2+\mathrm{i}\,P_2)\sum\limits_\lambda \{\gamma^2_\lambda/(E_\lambda - E)\}\right|^2} \tag{A12}$$

with the constant C^2 chosen so that the transition probability is

$$w \equiv \int_0^\infty w(E)\,\mathrm{d}E = (\ln 2)/t_{\frac{1}{2}}, \tag{A13}$$

where $t_{\frac{1}{2}}$ is the half-life of the decay. Then the α-particle energy spectrum $N(E)$ is given by

$$N(E) = (Nt_{\frac{1}{2}}/\ln 2)w(E), \tag{A14}$$

where

$$N \equiv \int_0^\infty N(E)\,\mathrm{d}E$$

is the total number of counts. Thus the fit to the observed spectrum leads to values of

$$A_{\lambda x} = C(Nt_{\frac{1}{2}}/\ln 2)^{\frac{1}{2}} g_{\lambda x}\gamma_\lambda, \qquad x = F, G. \tag{A15}$$

In order to relate these to the more commonly used matrix elements $|\int 1|^2_\lambda$ and $|\int \boldsymbol{\sigma}|^2_\lambda$, and to obtain $(ft)_\lambda$ values for the transitions to the various levels, we define a transition probability to level λ as

$$w_\lambda = \int_0^\infty w_\lambda(E)\,\mathrm{d}E = (\ln 2)/t_{\lambda\frac{1}{2}},$$

where $w_\lambda(E)$ is obtained from (A12) by omitting all contributions from levels other than λ (this is not the same as assuming that only level λ is fed). Then

$$w_\lambda = C^2(g^2_{\lambda F} + g^2_{\lambda G})\gamma^2_\lambda \int_0^\infty \frac{f_\beta P_2\,\mathrm{d}E}{|E_\lambda - E - (S_2 - B_2 + \mathrm{i}P_2)\gamma^2_\lambda|^2}. \tag{A16}$$

This is not a partial transition probability and $t_{\lambda\frac{1}{2}}$ is not a partial half-life as the summation over λ of w_λ does not equal w, due to the interference of the levels. We also define f_λ as the mean value of f_β for the level λ, writing

$$\int_0^\infty \frac{f_\beta P_2\,\mathrm{d}E}{|E_\lambda - E - (S_2 - B_2 + \mathrm{i}P_2)\gamma_\lambda^2|^2} = f_\lambda \int_0^\infty \frac{P_2\,\mathrm{d}E}{|E_\lambda - E - (S_2 - B_2 + \mathrm{i}P_2)\gamma_\lambda^2|^2} = f_\lambda I_\lambda, \tag{A17}$$

so that*

$$w_\lambda = C^2(g_{\lambda F}^2 + g_{\lambda G}^2)\gamma_\lambda^2 f_\lambda I_\lambda. \tag{A18}$$

We note that for a narrow level ($P_2 \to 0$),

$$I_\lambda = \pi\gamma_\lambda^{-2}(1-\gamma_\lambda^2\,\mathrm{d}S_2/\mathrm{d}E)^{-1}_{E_{\lambda r}}, \tag{A19}$$

where

$$E_{\lambda r} = E_\lambda + \gamma_\lambda^2\{B_2 - S_2(E_{\lambda r})\}.$$

Now the usual formula for the transition probability to a single isolated narrow level λ is (Wu and Moszkowski 1966)

$$w_\lambda = \tfrac{1}{2}\pi^{-3}(mc^2/\hbar)f_\lambda(G_F^2|\textstyle\int 1|_\lambda^2 + G_G^2|\int \boldsymbol{\sigma}|_\lambda^2), \tag{A20}$$

where $G_F = 2\cdot 98\times 10^{-12}$ and $G_G = 3\cdot 51\times 10^{-12}$. This gives

$$(ft)_\lambda = B/(|\textstyle\int 1|_\lambda^2 + R|\int \boldsymbol{\sigma}|_\lambda^2), \tag{A21}$$

with $B = 6240$ and $R = 1\cdot 39$. By equating (A18) and (A20), and using (A15), we get

$$\left.\begin{aligned} |\textstyle\int 1|_\lambda^2 &= 2\pi^3(\hbar/mc^2)\{(\ln 2)/Nt_{\frac{1}{2}}\}(A_{\lambda F}^2 I_\lambda/G_F^2),\\ |\textstyle\int \boldsymbol{\sigma}|_\lambda^2 &= 2\pi^3(\hbar/mc^2)\{(\ln 2)/Nt_{\frac{1}{2}}\}(A_{\lambda G}^2 I_\lambda/G_G^2)\end{aligned}\right\} \tag{A22}$$

and these may be used in (A21) to obtain $(ft)_\lambda$.

* Matt *et al.* (1964) used a formula similar to (A18) but with the integration in I_λ going from 0 to $E_{\max}$, where $f_\beta(E_{\max}) = 0$. This does not appear to be reasonable as the integrand of I_λ does not involve f_β.