SPACE CHARGE FIELDS IN THE DEVELOPMENT OF A TOWNSEND DISCHARGE IN NITROGEN

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Summary

As the current through a spark rises into the milliampere range, ionization due to fields produced by space charge plays an increasingly dominant role. Information on these fields can be obtained from analysis of the light intensity emitted at different points along the axis of the discharge. In an earlier publication (Doran 1968) photomultiplier records of the light output obtained at various times during the growth of a Townsend discharge in nitrogen ($pd = 600$ torr cm) were analysed by an approximate method. In the present paper a more exact relation between the light emitted and the variation in local electric field is derived, enabling the previous results to be re-analysed. This has provided a quantitative picture, spatially and temporally resolved, of the development of these fields, which are associated with luminous fronts observed to propagate back and forth across the discharge gap. The magnitude of the field variations is in general about 10–20% of the applied electric field.

I. Introduction

In a previous paper (Doran 1968) the development of a Townsend capacitor discharge in nitrogen in a uniform 2 cm gap, overvolted by a few per cent, was investigated by image converter, intensifier, and photomultiplier techniques. Streak photographs showed that the formation of the diffuse glow-like stage in the growth of the Townsend spark is preceded by a series of luminous fronts that propagate back and forth across the gap. Time-resolved photomultiplier measurements of the light output from different points along the axis of the discharge during this phase revealed that these luminous fronts were associated with increases in local electric field. The magnitude of these space charge fields was estimated from the relation of the temporal variation of the photomultiplier current $i_{\mathrm{s}}(t)$ to the field-dependent quantities of ionization coefficient $\alpha(E/p)$, excitation coefficient $\delta(E/p)$, and electron drift velocity $v_{-}(E/p)$. An equation linking these quantities has been derived by Hoffmann (1967):

$$\frac{\mathrm{d}}{\mathrm{d}t}\Big(\ln i_{\mathrm{s}}(t)\Big) = \alpha(t)\, v_{-}(t) + \frac{\mathrm{d}}{\mathrm{d}(E/p)}\Big(\ln \delta v_{-}\Big)\frac{\mathrm{d}(E/p)}{\mathrm{d}t}. \tag{1}$$

In the previous analysis the last term was neglected, and from the value of the product $\alpha(t)\, v_{-}(t)$ an approximate magnitude of the local electric field E^{*} was calculated.

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However, the previous procedure ignores two important considerations. Firstly, the last term of equation (1) is in general not negligible and in particular, where the field is low and rapidly varying, it becomes dominant. Secondly, the variations of light intensity in the discharge take place within nanoseconds; i.e. in times of the order of the time constants for the decay of the excited states responsible for the photoemission. This effect did not have to be taken into account in Hoffmann's treatment, since he applied the method in measuring space charge fields that remained substantially constant for microseconds. In the previous investigation (Doran 1968) a failure to consider the "memory effect" of earlier events, which this introduces in the light output, limits the accuracy of the analysis through the loss in time resolution. The purpose of the present work therefore is to extend the analysis to include this phenomenon and, by solving the equation derived, to gain a better measure of E/p as a function of time for a series of points along the axis of the discharge.

II. Analysis of Photomultiplier Measurements

Spectrally resolved examinations of similar discharges in nitrogen (Allen and Phillips 1964*a*, 1964*b*) and of the radiation excited by electron swarms in nitrogen, under the same E/p conditions (Legler 1963) as in the experiments reported in Doran (1968), have shown that the dominant part of the radiation consists of the molecular bands of the Second Positive Group. Wagner (1964) and Anton (1966) have measured the averaged lifetime τ of the excited states that give rise to these bands and have found, in good agreement with each other, that, due to quenching collisions, τ has a pressure dependence of the form

$$\tau(p) = \tau_0(1+p/p')^{-1}, \tag{2}$$

where

$$\tau_0 = 36\pm3\ \text{nsec} \qquad \text{and} \qquad p' = 60\pm6\ \text{torr}.$$

Thus at the pressure of 300 torr used in Doran (1968)

$$\tau = 6\pm0\cdot5\ \text{nsec}.$$

If ΔV is a volume element corresponding to the section of the discharge observed with the photomultiplier, then the number of excitations occurring in ΔV at a time t' is proportional to the total number of electrons $n(t')$ in that volume.

Assume at an arbitrary time $t = 0$ that ΔV contains n_0 electrons and that, further, in the time intervals under consideration the same number of electrons enter ΔV by drift motion as leave it by the same process. That is, any change in the electron content of ΔV is determined by ionization within the element and not by the influx of a concentration of electrons produced at another point in the gap (cf. Doran 1968). Then,

$$n(t') = n_0 \exp\left(\int_0^{t'} \alpha(\xi)\, v_-(\xi)\, \mathrm{d}\xi\right), \tag{3}$$

where ξ is a dummy variable in time.

The number of excitations per second $r_{ex}(t')$ occurring in a time interval dt' is given by

$$r_{ex}(t') = n(t')\,\delta(t')\,v_-(t')\,\mathrm{d}t', \tag{4}$$

where δ is the number of exciting collisions made by an electron drifting 1 cm in the direction of the field.

The rate of emission of photons $r_{em}(t)$ from ΔV at a later time t then becomes

$$r_{em}(t) = \frac{A}{\tau}\int_0^t r_{ex}(t)\exp\left(-\frac{t-t'}{\tau}\right)\mathrm{d}t', \tag{5}$$

A being a dimensionless constant of proportionality that accounts for the reduction in photon emission due to quenching of the excited states by collisions.

Thus, combining equations (3), (4), and (5) gives the photomultiplier current $i_s(t)$ as

$$i_s(t) = \frac{A'}{\tau}\int_0^t n_0\exp\left(\int_0^{t'}\alpha(\xi)\,v_-(\xi)\,\mathrm{d}\xi\right)\delta(t')\,v_-(t')\exp\left(-\frac{t-t'}{\tau}\right)\mathrm{d}t'. \tag{6}$$

The constant A' is related to the photomultiplier efficiency and the geometry of the optical arrangement. Differentiating with respect to time and taking logarithms gives

$$\begin{aligned}&\ln[\tau\{\tau^{-1}\,\mathrm{e}^{t/\tau}\,i_s(t)+\mathrm{e}^{t/\tau}\,\mathrm{d}(i_s(t))/\mathrm{d}t\}]\\ &\qquad = \ln(A'n_0)+\ln(\delta(t)\,v_-(t))+\frac{t}{\tau}+\int_0^t\alpha(\xi)\,v_-(\xi)\,\mathrm{d}\xi.\end{aligned} \tag{7}$$

Differentiating again and rearranging, we find

$$\frac{\mathrm{d}(E/p)}{\mathrm{d}t} = \left[\frac{\mathrm{d}}{\mathrm{d}t}\left\{\ln\left(i_s(t)+\tau\frac{\mathrm{d}(i_s(t))}{\mathrm{d}t}\right)\right\}-\alpha v_-\right]\Big/\frac{\mathrm{d}(\ln\delta v_-)}{\mathrm{d}(E/p)}, \tag{8}$$

where α, δ, and v_- are functions of the field to pressure ratio E/p that exists at the time t. On the right-hand side of (8), the first term in the square brackets may be evaluated from the experimental data, whereas the remaining terms are known functions of E/p. The experimental values of $\alpha(E/p)$, $\delta(E/p)$, and $v_-(E/p)$ used in this analysis are those of Masch (1932), Legler (1963), and Tholl (1964) respectively.

III. Results of Analysis

The differential equation (8) was solved numerically by a method of successive approximations and the results are shown in Figure 1, where $\Delta E/E_0$, the fractional change in field, is plotted as a function of time for different distances from the cathode. A comparison of these curves with the corresponding ones of Figure 5 in Doran (1968) allows the following observations.

(1) This procedure makes possible an evaluation of the electric field at all points along the $\ln i_s(t)$ curve, including regions where the slope $\mathrm{d}(\ln i_s(t))/\mathrm{d}t$ is negative. These regions were not accessible to the earlier approximate method of analysis.

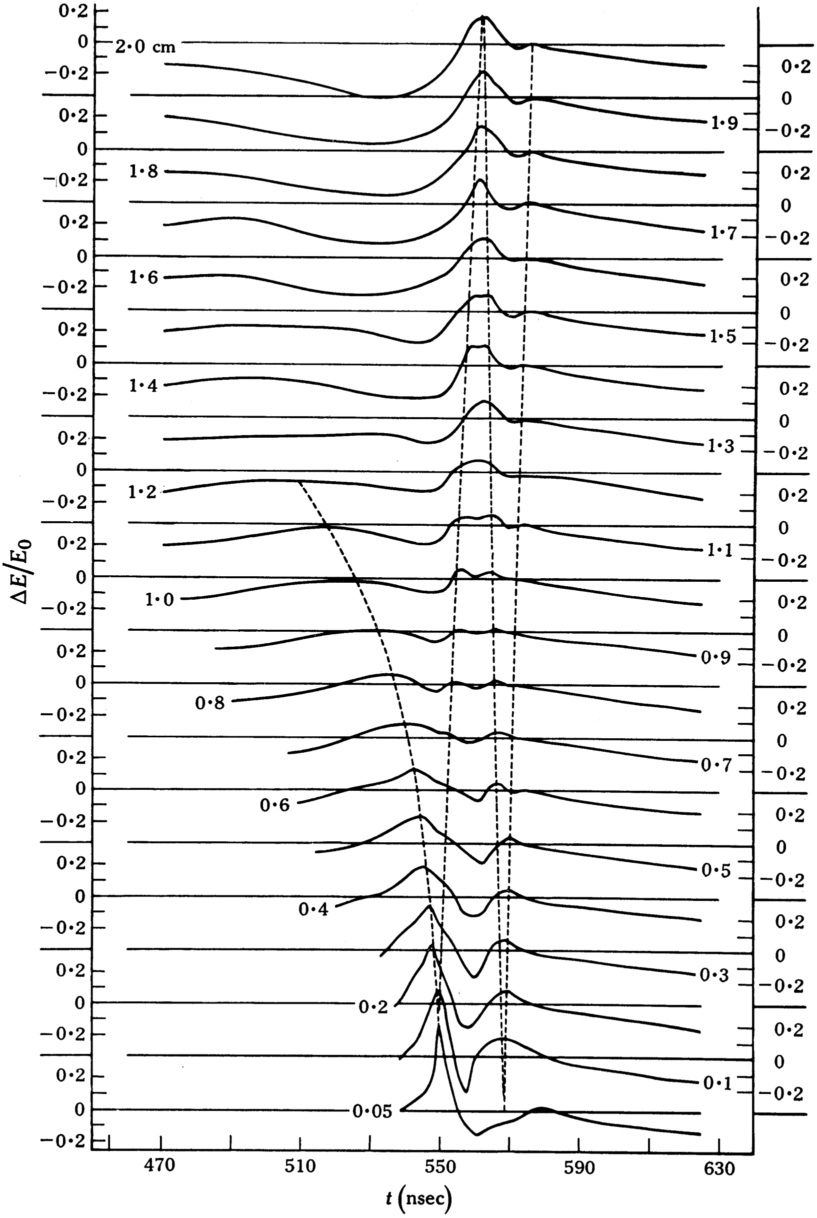

Fig. 1.—Fractional change in electric field $\Delta E/E_0$ at different points along the axis of the gap plotted as a function of time t.

(2) The qualitative appearance of the curves remains the same, with the maxima and minima in $\Delta E/E_0$ appearing at the same times on both sets of curves. The dashed lines indicating the propagating luminous fronts observed in the streak photographs were superimposed on the field–time plot and are seen to correspond to increases in the local field, as was found in the previous calculations. The magnitudes of the $\Delta E/E_0$ values at the field maxima are in general about 20% less than those calculated by the approximate method.

(3) Taking the term $\{d(\ln \delta v_-)/d(E/p)\}d(E/p)/dt$ into account leads to far less pronounced field troughs. An error was introduced into the previous estimates of the low field regions, since here the αv_- product on the right-hand side of equation (1) is small, whereas the factor $d(E/p)/dt$ in the other term is large, due to rapid variation of the field. The same applies for the modified analysis that leads to equation (8).

(4) Equation (1), which neglects the effect of the lifetimes of the excited states was also solved numerically for the same set of data. Although similar maxima in the fields appeared, these occurred up to 5 nsec later and there was a general "smearing out" of the structure in the curves. In particular, it was difficult to observe the variations of field associated with the passage of the second front.

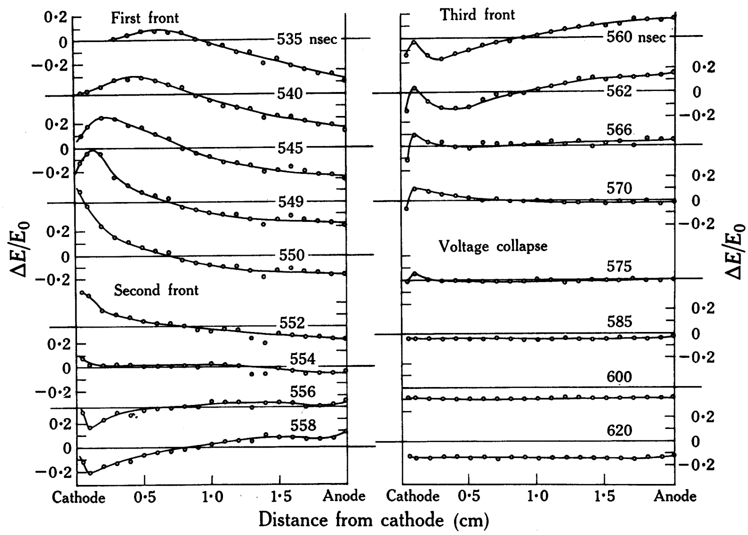

Fig. 2.—Fractional change in electric field associated with the development of the luminous fronts plotted as a function of position in the gap for different times.

As in Figure 6 of Doran (1968), the curves may be replotted to show the variation of $\Delta E/E_0$ with respect to distance along the axis of the gap, at different points in time. The results are given in Figure 2. Again it will be noticed that there is qualitative agreement with the form of the curves in the previous work, and the comments made there about the development of the three fronts remain applicable.

At times later than $t = 540$ nsec the intensity of the light output had risen sufficiently for the variation in field along the axis of the discharge to be determined for the whole length of the gap. A useful check may then be made on the accuracy of the field measurements, since

$$\int_0^d E(x,t)\,\mathrm{d}x = U(t)\,, \tag{9}$$

where $U(t)$ is the voltage applied across the gap and d is the gap distance. This implies that in Figure 2, for each $\Delta E/E_0$ curve, the areas above and below the zero line should be equal.

It can be seen that this condition applies consistently for $t > 545$ nsec, and the scatter of the experimental points indicates the magnitude of the errors in the measurements. At later times ($t > 585$ nsec), the field integral in (9) decreases steadily as the voltage across the gap, $U(t)$, collapses due to the discharging of the capacitor, which takes place as the current through the gap rises to ~ 10 A.

It is clear, however, that for $t < 545$ nsec the condition (9) is not satisfied by the experimental results. At these times the intensity of light emitted by the discharge near the cathode is very low, necessitating the operation of the photomultiplier under conditions of high noise level. Thus errors of $\sim 50\%$ in the measurement of $i_{\mathrm{s}}(t)$, resulting in errors up to 100% in the gradient $\mathrm{d}(\ln i_{\mathrm{s}}(t))/\mathrm{d}t$, are possible. As the intensity of the light output increases and the signal-to-noise ratio becomes more favourable, the condition (9) is well satisfied to within an error in field E of 6%.

A technique has thus been devised for measuring the local electric fields present in this dynamic phase of spark development, and the consistency of the results obtained justifies the initial assumptions made in the previous paper (Doran 1968) with regard to the application of this method. The former simplified method of analysis has been shown in fact to be quite a good approximation and may continue to be of use in the study of discharges in gases for which the coefficient of excitation δ and the lifetimes of excited states are unknown.

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