SHORT COMMUNICATIONS

SOME COMMENTS ON THE ANALYSIS OF EXCITATION FUNCTIONS USING DIFFERENT TRANSFORMS*

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The statistical theory of nuclear reactions (Brink and Stephen 1963; Ericson 1963) has been used successfully for some years. When it comes to determining properties like the mean level width Γ several authors (Bőhning 1965; Ericson 1965, personal communication; Hellstrőm and Dallimore 1969) have suggested and used a Fourier analysis instead of the autocorrelation function $C(E+\epsilon, E)$ because the former is associated with smaller errors, particularly when the average cross section has a slow variation with energy. The aim of this note is to elucidate some of the properties of the Fourier transform applied to analysis of excitation functions and also to point out that other transforms are possible. To fix notation, we take the Fourier transform F of the cross section $\sigma(E)$ over an energy interval I with middle point E_0 as

$$F_k(\sigma(E_0)) = \int_I \exp\{2\pi i k(E-E_0)/I\}\sigma(E)\,\mathrm{d}E\,.$$

It is well known that the expectation value $\bar{S}_k$ of the squares of the real and imaginary parts of F_k depends on the mean level width Γ and the multiples k of the basic frequency, that is,

$$\bar{S}_k = \text{const.} \times \exp(-2\pi\Gamma k/I)\,, \tag{1}$$

where I is the energy interval. This fact has been used by several authors (e.g. Richter *et al.* 1966; Hellstrőm and Dallimore 1969) to extract Γ from excitation functions. Some interesting properties of the real and imaginary parts themselves have been pointed out by Ericson (1965, personal communication) but have been neglected in the above-mentioned analyses. The real and imaginary parts are both expected to have normal Gaussian distributions with the same dispersion. For

$$X = \mathrm{Re}\{F_k(\sigma(E))\}/\mathrm{Im}\{F_k(\sigma(E))\},$$

we have

$$P(X) = (2\pi\sigma_S^2)^{-\frac{1}{2}}\exp(-X^2/2\sigma_S^2)\,.$$

The real and imaginary parts of $F_k(\sigma(E))$ have been studied for a set of eight artificial excitation functions. Figure 1(*a*) shows X as a function of k for one of them and Figure 1(*b*) shows $P(X)$ as a function of X for all of them, i.e. for each interval $X+\Delta X$ contributions from all excitation functions have been added. This was done to achieve better statistics. The curve is a least squares fitted Gaussian.

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As was mentioned earlier, it is possible to deduce the mean level width from a Fourier analysis and equation (1). The most frequently used method so far, however, has been to calculate the average number of maxima per unit energy interval and set them equal to $0{\cdot}55/\Gamma$ (Dallimore and Hall 1966) or to use

$$C(E+\epsilon, E) = \langle\sigma(E+\epsilon)\,\sigma(E)\rangle - \langle\sigma(E)\rangle^2 = \Gamma^2/(\Gamma^2+\epsilon^2)\,. \tag{2}$$

It is possible to write this expression in a few other ways. One is to use the convolution integral and Laplace transforms and another to multiply the Fourier transform by its

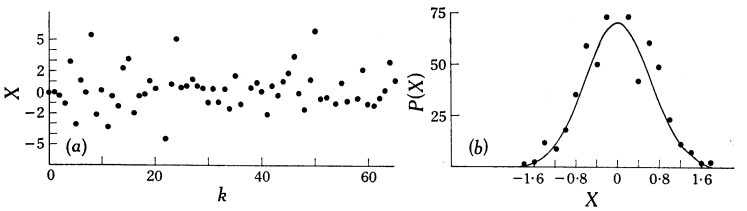

Fig. 1.—With X the ratio of the real to the imaginary parts of the Fourier transform of the cross section, (a) shows the distribution of X as a function of multiples of the basic frequency k, and (b) the probability distribution $P(X)$ as a function of X (the zero component is not included).

complex conjugate and then to take its inverse of the result. Writing (e.g. Gardner and Barnes 1942)

$$(f*g)(t) = \int_0^t f(\tau)\,g(t-\tau)\,\mathrm{d}\tau = \int_0^t g(\tau)f(t-\tau)\,\mathrm{d}\tau = (g*f)(t)\,,$$

we see that $f*g = g*f$ is a commutative operation. We also have

$$L(f*g) = \tilde{f}(s)\,\tilde{g}(s)\,,$$

where L indicates Laplace transform, $\tilde{f}(s)$ is the Laplace transform of $f(t)$, and $\tilde{g}(s)$ is the Laplace transform of $g(t)$. The rather complex operation of the convolution integral is, when Laplace transformed, simply represented by multiplication of two Laplace transforms. The autocorrelation function then becomes

$$C(E+\epsilon, E) = L^{-1}\,L(f*g) = L^{-1}\big(\tilde{f}(s)\,\tilde{g}(s)\big)$$

where L^{-1} is the inverse of L. Here of course $f = g$.

If, instead, we use Fourier transforms we can write (e.g. Lee 1960)

$$\langle\sigma(E+\epsilon)\,\sigma(E)\rangle = F_\epsilon^{-1}\big(F_k^*\,F_k(\sigma(E))\big)\,. \tag{3}$$

Knowing the theoretical form of the autocorrelation function as a function of ϵ, we can easily determine Γ (from the right-hand side of (2)), even in this case. This is

shown in Figure 2. Thus when properly normalized we can write the autocorrelation function in a very simple operator form as

$$F^{-1}(F^*F).$$

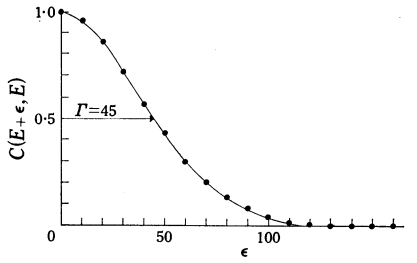

Fig. 2.—The autocorrelation function as a function of ϵ for $C(E+\epsilon, E) = (F^{-1}F^*F)_\varepsilon$.

TABLE 1

COMPARISON OF METHODS FOR CALCULATING Γ FOR EIGHT ARTIFICIAL EXCITATION FUNCTIONS

Method	Mean Level Width Γ								$\bar{\Gamma}$
(1)	43	46	50	58	42	56	55	44	49±2
(2)	43	71	47	45	43	64	51	54	52±4
(3)	45	67	47	44	67	49	57	45	52±3
(4)*	52	48	47	54	55	52	58	50	52±2

* Dallimore and Hall (1966).

Equation (3) was normalized to unity for convenience. However, if this equation is normalized in the following way

$$\frac{\langle\sigma(E+\epsilon)\,\sigma(E)\rangle}{\langle\sigma(E+\epsilon)\rangle\langle\sigma(E)\rangle},$$

equation (3) as well as equation (2) may be written

$$\frac{\langle\sigma(E+\epsilon)\,\sigma(E)\rangle}{\langle\sigma(E+\epsilon)\rangle\langle\sigma(E)\rangle} - 1 = \frac{1-y_D^2}{N}\,\frac{\Gamma^2}{\Gamma^2+\epsilon^2}$$

where all the symbols are the same as in Stephen (1963). The ratio y_D is defined as $\langle\sigma_{\text{direct}}/\sigma_{\text{total}}\rangle$ and N, the effective number of open channels, can be calculated.

The mean level width Γ was calculated and compared for a series of eight artificial excitation functions using methods (1), (2), (3), and that given by Dallimore and Hall (1966). The results and their errors are listed in Table 1. The artificial excitation functions were generated on a computer using the expression

$$\sigma = \left|\sum_\lambda A_\lambda/(E-E_\lambda+\tfrac{1}{2}\mathrm{i}\Gamma)\right|^2,$$

with the matrix elements A_λ taken as real random numbers with a Gaussian distribution of mean zero and a standard deviation of unity and $\Gamma = 50$. Part of one of them is shown in Figure 3.

The estimation of Γ from artificial excitation functions has been found to give exactly the same results within the errors for the four different methods employed. The conclusion therefore must be that each method equally well may be applied when properties like Γ are under consideration.

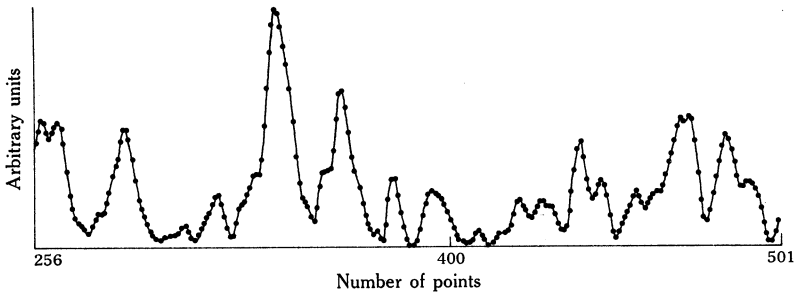

Fig. 3.—Part of an artificial excitation function generated on a computer.

It is also possible to extract the amount of the direct reaction mechanism. This is a highly desirable quantity to know since it is model dependent and so can be used to compare the statistical fluctuation model with other theories like the optical model, the Hauser–Feshbach theory, or the distorted-wave Born approximation. For highly excited nuclei this still remains to be done.

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