SELF-CONSISTENT TRANSFORMATION OF BREMSSTRAHLUNG TO MONOCHROMATIC PHOTONEUTRON MEAN ENERGIES OF NON-MAXWELLIAN NEUTRON SPECTRA

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Abstract

The mean energy of neutrons produced in a bremsstrahlung experiment can be measured using "energy sweeping" and a multi-BF_3-counter neutron detection system. It is shown here how one can obtain from a relatively simple type of experiment the required calibration data of the detection system to permit a self-consistent transformation of the bremsstrahlung data to the equivalent monochromatic photon data for neutron spectra that are non-Maxwellian.

I. Introduction

Barrett *et al.* (1973) report a novel experiment in which the count-ratio technique of Barrett and Thies (1971) has been used to derive systematic information on nuclear level densities from measurements of bremsstrahlung photoneutron mean energies. The success of this method is mainly due to the relative speed and simplicity of the experiment, and also to the fact that level density parameters derived by this method are very little affected by the functional dependence on energy of the inverse reaction cross sections employed in their derivation. The above type of experiment can be improved in accuracy by one order of magnitude using "energy sweeping" (Thies *et al.* 1972*a*), and it becomes practical to transform the bremsstrahlung photoneutron data to data corresponding to monochromatic γ-excitation. A corresponding transformation method for the experiment using the count-ratio technique was formulated by Thies *et al.* (1972*b*). However, this latter method is only applicable if the energy spectra of the neutrons are roughly Maxwellian, i.e. it can be applied to data on heavy nuclei and possibly medium weight nuclei. In the present paper we report on a slightly more elaborate transformation formalism which permits the calculation of photoneutron mean energies of non-Maxwellian neutron spectra, i.e. it can be applied even to data on the very lightest nuclei. We show that the required calibration data can be obtained in a straightforward manner from calibration data of the simple count-ratio experiment of Barrett and Thies (1971).

II. Transformation Formalism

(*a*) Notation

For a multi-BF_3-counter neutron detection system employing a radial geometry (Thies 1963; Thies and Böttcher 1969), the number ϕ of neutrons emitted at the

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centre of the moderator and the counts $I(\boldsymbol{r}_s)$ recorded by a counter at radial distance $\boldsymbol{r}_s$ from the centre are related by

$$\phi = K \sum_s A_s I(\boldsymbol{r}_s). \tag{1}$$

In this equation the constant K depends on neutron capture cross sections of moderator and counters whereas the coefficients* A_s are independent of any physical properties of the system and can be calculated from the geometry of the system alone. According to Thies and Böttcher (1969) then the mean energy $\bar{E}_\mathrm{n}$ of the ϕ emitted neutrons is given by

$$\bar{E}_\mathrm{n} = \sum_s C_s I(\boldsymbol{r}_s) \Big/ \sum_s A_s I(\boldsymbol{r}_s), \tag{2}$$

where the coefficients* C_s are functions of the counter position $\boldsymbol{r}_s$ only but require for their calculation an accurate knowledge of the function $g(\boldsymbol{r}_s, E_\mathrm{n})$, the probability that a neutron emitted with energy E_n is recorded by a counter at position $\boldsymbol{r}_s$.

Assume now that in an idealized experiment, using *monochromatic photons* of energy $h\nu$, ${}^\mathrm{M}\phi$ neutrons were emitted and that the various counters had recorded the corresponding counts ${}^\mathrm{M}I(\boldsymbol{r}_s)$, that is, for a particular $E_i = h\nu_i$ equation (1) now reads

$${}^\mathrm{M}\phi(h\nu_i) = K \sum_s A_s {}^\mathrm{M}I(\boldsymbol{r}_s, h\nu_i).$$

We rewrite this equation using the shorthand notation

$${}^\mathrm{M}\phi_i = K \sum_s A_s {}^\mathrm{M}_sI_i, \tag{1a}$$

and equation (2) as

$$\bar{E}_{\mathrm{n}i} = \sum_s C_s {}^\mathrm{M}_sI_i \Big/ \sum_s A_s {}^\mathrm{M}_sI_i, \tag{2a}$$

where $\bar{E}_{\mathrm{n}i}$ is the mean energy of the ${}^\mathrm{M}\phi_i$ photoneutrons emitted by photons of energy $h\nu_i$.

For an actual experiment in which ${}^\mathrm{B}\phi_j$ photoneutrons were emitted by the (polychromatic) *bremsstrahlung photons* of peak photon energy E_j, analogously we denote the counts recorded by a counter at $\boldsymbol{r}_s$ by ${}^\mathrm{B}_sI_j$. In this case equation (2) becomes

$$\hat{E}_{\mathrm{n}j} = \sum_s C_s {}^\mathrm{B}_sI_j \Big/ \sum_s A_s {}^\mathrm{B}_sI_j, \tag{2b}$$

where $\hat{E}_{\mathrm{n}j}$ is the mean energy of the ${}^\mathrm{B}\phi_j$ photoneutrons emitted by bremsstrahlung of peak energy E_j.

In the following subsection we show how one can transform bremsstrahlung data for ${}^\mathrm{B}_sI_j$ into the corresponding data for ${}^\mathrm{M}_sI_i$ which would have been obtained from an equivalent idealized experiment with monochromatic photons. With these data for ${}^\mathrm{M}_sI_i$ equation (2a) can then be used to calculate the corresponding mean energies $\bar{E}_{\mathrm{n}i}$ of the photoneutrons emitted by monochromatic photons of energy

* Here KA_s and KC_s correspond to C_s and B_s respectively of Thies and Böttcher (1969).

$h\nu_i$, which is what we set out to do. We complete the definition of our notation by writing the correlation between ${}^{\mathrm{B}}_{s}I_j = {}^{\mathrm{B}}I(\boldsymbol{r}_s, E_j)$ and ${}^{\mathrm{M}}_{s}I_i = {}^{\mathrm{M}}I(\boldsymbol{r}_s, h\nu_i)$ explicitly in the form

$$ {}^{\mathrm{B}}I(\boldsymbol{r}_s, E_j) = \beta(E_j) \int_{h\nu=E_{\mathrm{th}}}^{E_j} P(E_j, h\nu) \frac{{}^{\mathrm{M}}I(\boldsymbol{r}_s, h\nu)}{{}^{\mathrm{M}}\phi(h\nu)} \sigma(h\nu)\,\mathrm{d}(h\nu) \int_{t=t_0}^{t_i} k(t)\,\mathrm{d}t, \qquad (3) $$

where $\sigma(h\nu)$ is the neutron production cross section which is zero below its threshold, i.e. for $h\nu < E_{\mathrm{th}}$; $P(E_j, h\nu)$ is the bremsstrahlung spectral distribution function (non-normalized); $\beta(E_j)$ is the relative number of bremsstrahlung photons per electron injected;* and $k(t)$ is a function which depends on target geometry and weight, and is proportional to the rate at which electrons are injected at time t. Using an analogous shorthand notation to the above, and approximating integration in equation (3) by summation of finite differences, we rewrite this equation as

$$ {}^{\mathrm{B}}_{s}I_j = \beta_j \sum_i P_{ji} \frac{{}^{\mathrm{M}}_{s}I_i}{{}^{\mathrm{M}}\phi_i} \sigma_i \Delta_i \int_{t_0}^{t_i} k(t)\,\mathrm{d}t, \qquad (3a) $$

where Δ_i is the "bin width".

(b) Analysis

Assume now that we have performed a bremsstrahlung experiment using energy sweeping (Thies *et al.* 1972*a*). The integral with respect to t in equation (3a) then has the same value for all the recorded counts ${}^{\mathrm{B}}_{s}I_j$, say

$$ K = \int_{t_0}^{t_i} k(t)\,\mathrm{d}t, $$

and hence we can write equation (3a) in the form

$$ {}^{\mathrm{B}}_{s}I_j/K\beta_j = \sum_i P_{ji}\{({}^{\mathrm{M}}_{s}I_i/{}^{\mathrm{M}}\phi_i)\sigma_i \Delta_i\}, \qquad j = 1, 2, \dots, q, \qquad (4) $$

if we recorded counts for q energies E_j. Equation (4) can then be interpreted as a system of q linear equations in the q unknowns

$$ \{({}^{\mathrm{M}}_{s}I_i/{}^{\mathrm{M}}\phi_i)\sigma_i \Delta_i\}, \qquad i = 1, 2, \dots, q. $$

Hence, if we consider P_{ji} as an element of the triangular matrix $\mathbf{P}$, where

$$ (\mathbf{P})_{ji} = P_{ji}, \qquad (5) $$

we can express the solutions of equation (4) explicitly as

$$ ({}^{\mathrm{M}}_{s}I_i/{}^{\mathrm{M}}\phi_i)\sigma_i \Delta_i = \sum_j (\mathbf{P}^{-1})_{ij}\, {}^{\mathrm{B}}_{s}I_j/K\beta_j, \qquad (6) $$

the quantity $(\mathbf{P}^{-1})_{ij}$ being an element of $\mathbf{P}^{-1}$, the inverse of $\mathbf{P}$. With equation (6),

* The quantity $\beta(E_j)$ can be determined experimentally, as discussed by Thies *et al.* (1972*a*).

we may now write equation (2a) in the form

$$\bar{E}_{\mathrm{n}i} = \frac{\sum_s C_s \sum_j (\mathbf{P}^{-1})_{ij}\, {}^{\mathrm{B}}_{s}I_j/\beta_j}{\sum_s A_s \sum_j (\mathbf{P}^{-1})_{ij}\, {}^{\mathrm{B}}_{s}I_j/\beta_j}, \tag{7}$$

which is the desired expression of $\bar{E}_{\mathrm{n}i}$ in terms of the recorded bremsstrahlung counts ${}^{\mathrm{B}}_{s}I_j$.

Equation (7) is valid whether or not the energy spectrum of the neutrons emitted by monochromatic photons is Maxwellian. For its practical application, values of the coefficients A_s and C_s are required. As indicated in subsection (*a*), the coefficients A_s can be readily calculated from the geometry of the neutron detector. The coefficients C_s can be obtained conveniently and with high relative accuracy using the following calibration experiment.

Suppose we require C_s for a detection system employing counters at five radial distances r_s. We then measure $\hat{E}_{\mathrm{n}j}$, the mean energy of the photoneutrons emitted by bremsstrahlung from a target consisting of heavy nuclei (e.g. lead) for five appropriate peak bremsstrahlung energies E_j, using the count-ratio technique of Barrett and Thies (1971). As the energy spectrum of the bremsstrahlung photoneutrons from a heavy element is Maxwellian to a good approximation (Barrett *et al.* 1973), the measured values of $\hat{E}_{\mathrm{n}j}$ should contain only negligible systematic errors. They represent five discrete values of the continuous function $\hat{E}_{\mathrm{n}}(E_j)$, which should be an extremely "well-behaved" function as it is derived from the spectrum of the corresponding photoneutrons emitted by monochromatic photons via two successive integrations; that is, if $m(h\nu, E_{\mathrm{n}})$ is the normalized spectrum of neutrons emitted by photons of energy $h\nu$ then

$$\hat{E}_{\mathrm{n}}(E_j) = \left(\int_{E_{\mathrm{th}}}^{E_j} P(E_j, h\nu)\,\sigma(h\nu)\,\mathrm{d}(h\nu) \int_{E_{\mathrm{n}}=0}^{E_j - E_{\mathrm{th}}} E_{\mathrm{n}}\, m(h\nu, E_{\mathrm{n}})\,\mathrm{d}E_{\mathrm{n}}\right) \Big/ \int_{E_{\mathrm{th}}}^{E_j} P(E_j, h\nu)\,\mathrm{d}(h\nu)\,.$$

We may now rewrite equation (2b) in the form

$$\sum_s C_s\, {}^{\mathrm{B}}_{s}I_j = \hat{E}_{\mathrm{n}j} \sum_s A_s\, {}^{\mathrm{B}}_{s}I_j\,, \tag{2c}$$

which for the five values of s and j of our experiment represents a system of five linear equations in the five unknowns C_s. The solutions C_s can be expressed explicitly in terms of elements of the 5×5 matrix ${}^{\mathrm{B}}\mathbf{I}$, defined by

$$({}^{\mathrm{B}}\mathbf{I})_{js} = {}^{\mathrm{B}}_{s}I_j\,,$$

and elements of its inverse ${}^{\mathrm{B}}\mathbf{I}^{-1}$, denoted by $({}^{\mathrm{B}}\mathbf{I}^{-1})_{sj}$, namely

$$C_s = \sum_{s'} A_{s'} \sum_j \hat{E}_{\mathrm{n}j} ({}^{\mathrm{B}}\mathbf{I}^{-1})_{sj}\, {}^{\mathrm{B}}_{s'}I_j\,. \tag{8}$$

From equation (8) it is evident that the derivation of the coefficients C_s from the calibration experiment requires no knowledge of the bremsstrahlung spectrum or the photoneutron cross section. (As it is an energy sweeping experiment, obviously no dose measurement is required.)

The self consistency of the transformations discussed above can be tested in the following manner. A heavy target, not necessarily the same as that used for the C_s calibration, is irradiated with a range of peak bremsstrahlung energies E_j. The corresponding values of $\bar{E}_{ni}$ are then computed once by the present method and once using the count-ratio technique of Thies *et al.* (1972*b*). The latter method is applicable here as the energy spectrum of the neutrons emitted by monochromatic photons from a heavy target is very nearly Maxwellian. The corresponding values $\bar{E}_{ni}$ from both methods must then agree within the limits of error.

The standard deviation $[\![\bar{E}_{ni}]\!]$ of $\bar{E}_{ni}$ due to random counting errors can readily be calculated from equation (7). Using the notation

$$\sum_s C_s \sum_j (\mathbf{P}^{-1})_{ij}/\beta_j = G, \qquad \sum_s A_s \sum_j (\mathbf{P}^{-1})_{ij}/\beta_j = F, \tag{9}$$

one obtains

$$[\![\bar{E}_{ni}]\!] = \bar{E}_{ni}\left(\frac{1}{G^2}\sum_s C_s^2 \sum_j (\mathbf{P}^{-1})_{ij}^2\, {}^{\mathrm{B}}_{s}I_j/\beta_j^2 + \frac{1}{F^2}\sum_s A_s^2 \sum_j (\mathbf{P}^{-1})_{ij}^2\, {}^{\mathrm{B}}_{s}I_j/\beta_j^2\right)^{\frac{1}{2}}. \tag{10}$$

For a Schiff spectrum, the coefficients $|(\mathbf{P}^{-1})_{ij}|$ rapidly decrease with decreasing j, in contrast to β_j and ${}^{\mathrm{B}}_{s}I_j$ which decrease only slowly with decreasing j, if the bin width Δ_i of equation (4) is chosen reasonably small. Consequently, if we denote by β_i and ${}^{\mathrm{B}}_{s}I_i$ the largest of the values β_j and ${}^{\mathrm{B}}_{s}I_j$ in equation (10), we may use the approximation

$$\sum_j (\mathbf{P}^{-1})_{ij}^2\, {}^{\mathrm{B}}_{s}I_j/\beta_j^2 \approx ({}^{\mathrm{B}}_{s}I_i/\beta_i^2) \sum_j (\mathbf{P}^{-1})_{ij}^2, \tag{11}$$

which is well approximated numerically by

$$({}^{\mathrm{B}}_{s}I_i/\beta_i^2) \sum_j (\mathbf{P}^{-1})_{ij}^2 \approx ({}^{\mathrm{B}}_{s}I_i/\beta_i^2)(20\, E_i/\Delta_i^{3/2})^2. \tag{12}$$

With equation (12), thus equation (10) may be approximated by

$$[\![\bar{E}_{ni}]\!] = \bar{E}_{ni}\frac{20\, E_i}{\beta_i \Delta_i^{3/2}}\left(\frac{1}{G^2}\sum_s C_s^2\, {}^{\mathrm{B}}_{s}I_i + \frac{1}{F^2}\sum_s A_s^2\, {}^{\mathrm{B}}_{s}I_i\right)^{\frac{1}{2}}. \tag{10a}$$

Usually equation (10a) is a good approximation, but if accurate values of $[\![\bar{E}_{ni}]\!]$ are required obviously equation (10) must be used.

III. References

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