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A Dual Absorptive Model for Backward Hadron Scattering

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Abstract

It is shown that a dual absorptive model can explain the dip structure of backward differential cross sections of hadronic processes. The polarizations in those backward processes in which the dominant contribution comes from non-flip amplitude are also found to be qualitatively consistent with experiment.

Introduction

The ability to explain the structure of differential cross sections in various hadronic processes is a desirable feature for any phenomenological model. Previous attempts to account for the dip structure in various backward reactions have achieved only partial success, however, with models which are able to predict the dip structure in some reactions failing to do so in others. Thus although explanations of various backward reactions are available on a piecemeal pattern, neither the class I models, such as the weak-cut model (Arnold 1967; Capella and Tranh Thanh Van 1969), the Regge pole model and the Veneziano model, nor the class II models, such as the strong-cut model (Henyey *et al.* 1969) and the Dar–Weisskopf model (Dar *et al.* 1969), give a unified picture of the dip structure in backward differential cross sections. Another difficulty that most of the models encounter is the erratic behaviour of polarization in various forward and backward processes.

Recently Harari (1971*a*, 1971*b*) has proposed a dual absorptive model which seems to have remarkable success in predicting the dip structure of several forward hadronic processes. Although this model is still only qualitative and has also met certain difficulties in explaining polarizations in forward processes (Harari and Schwimmer 1971; Barger and Halzen 1972), its overall results are quite encouraging. Originally the model was aimed at explanations of forward scattering processes of hadrons, but Aye (1972) has shown that it can be equally applied to pion–nucleon backward scattering, for which it reads:

(i) $\operatorname{Im} f^s_{\Delta\lambda}(s,u)$ is proportional to $\hat{J}_{\Delta\lambda}(r\surd(-u))$, where $f^s(s,u)$ is an s-channel hadronic amplitude, $\hat{J}_{\Delta\lambda}$ has the same general structure (dips, bumps etc.) as the Bessel function $J_{\Delta\lambda}(r\surd(-u))$, $\Delta\lambda$ is the magnitude of the total s-channel helicity flip, and $r \sim 1$ fm. For exotic s-channel processes $\operatorname{Im} f^s(s,u) = 0$.

(ii) The u-channel description of $\operatorname{Im} f^s(s,u)$ is given by a combination of Regge poles and cuts. A weak (or no) cut is needed if the impact parameter representation of $\operatorname{Im} f^s(s,u)$ is dominated by partial waves with $l \sim qr$, where q is the c.m. momentum, while a strong cut is required if the impact parameter representation of $\operatorname{Im} f^s(s,u)$ is dominated by partial waves with $l \ll qr$.

(iii) $\mathrm{Re}\, f^s(s,u)$ is unknown in the s-channel picture. In the u-channel description $\mathrm{Re}\, f^s(s,u)$ gets contributions from the same resonances and cuts which contribute to $\mathrm{Im}\, f^s(s,u)$. If a weak (or no) cut is needed for $\mathrm{Im}\, f^s(s,u)$ then $\mathrm{Re}\, f^s(s,u)$ is given by the usual signature factor, while if a strong cut is required then the phase of the amplitude approaches the signature factor as $s \to \infty$. Since this may happen very slowly, $\mathrm{Re}\, f^s(s,u)$ remains undetermined for practical purposes.

For πN backward amplitudes, Aye (1972) has found that according to the above model $\Delta\lambda = 0$ amplitude needs a strong-cut contribution for Δ exchange, while $\Delta\lambda = 1$ amplitude needs a strong-cut contribution for N exchange. It may happen that for certain processes like $K^+p \to K^+p$ strong cuts contribute to all the amplitudes. In such cases the predictive power of the above model is diminished considerably. The Harari model can thus give useful information only for those processes in which at least some dominant amplitudes do not need a strong-cut contribution.

In the present work we show that if, in addition to the above restriction on the applicability of the Harari model, we assume that a strong-cut contribution is needed only for $\Delta\lambda \neq 0$ amplitudes then we are able to derive certain qualitative information about the dip structure of backward differential cross sections and polarization in those processes in which a $\Delta\lambda = 0$ amplitude dominates. In this case the u-channel description of $f^s(s,u)$ is given by

$$\mathrm{Im}\, f^s_{\Delta\lambda=0}(s,u) = \hat{J}_0(r\sqrt{(-u)}), \tag{1a}$$

$$\mathrm{Re}\, f^s_{\Delta\lambda=0}(s,u) = \hat{J}_0(r\sqrt{(-u)})\begin{Bmatrix} \tan\frac{1}{2}\pi\{\alpha(u)-\frac{1}{2}\} \\ -\cot\frac{1}{2}\pi\{\alpha(u)-\frac{1}{2}\} \end{Bmatrix} \tag{1b}$$

and

$$\mathrm{Im}\, f^s_{\Delta\lambda\neq0}(s,u) = (-1)^{\Delta\lambda}\, \hat{J}_{\Delta\lambda}(r\sqrt{(-u)}), \tag{2a}$$

$$\mathrm{Re}\, f^s_{\Delta\lambda\neq0}(s,u) = a = \; ? \tag{2b}$$

The tangent term in equation (1b) is associated with the contributions from odd trajectories while the cotangent term is associated with even trajectories.

Dip Structure of Backward Differential Cross Sections

Although actual fitting of the experimental data must be carried out to finally decide whether a certain helicity amplitude dominates a given process, for our qualitative analysis it will be sufficient to just examine the trend of the data for differential cross sections near $u = 0$ and make a rough guess as to the dominance of the various amplitudes involved in the process. For example, if the differential cross section $d\sigma/du$ for a particular reaction decreases rapidly with increasing $|u|$ (i.e. there is a sharp peak at $u = 0$) then the $\Delta\lambda = 0$ amplitude(s) is (are) dominant in the vicinity of $u = 0$. On the other hand, if $d\sigma/du$ has a small value at $u = 0$ and increases rapidly with $|u|$ (i.e. a dip at $u = 0$) then the indication is that the helicity flip ($\Delta\lambda \neq 0$) amplitude(s) is (are) dominant near $u = 0$. If there is no sharp backward peak or dip in the differential cross section, the flip ($\Delta\lambda \neq 0$) and non-flip ($\Delta\lambda = 0$) amplitudes are of comparable strength. Let us now consider certain backward scattering processes.

(i) The backward processes $\pi^+p \to \pi^+p$, $\pi^-p \to \pi^0n$ and $K^-n \to \pi^-\Lambda^0$ are dominated by the even-signatured N_α trajectory. Also, all of these reactions show a sharp backward peak at $u = 0$ which indicates that the $\Delta\lambda = 0$ amplitude is dominant. Thus we may write for each differential cross section

$$\begin{aligned} \mathrm{d}\sigma/\mathrm{d}u &\propto \{\hat{J}_0(r\surd(-u))\}^2\{1+\cot^2\tfrac{1}{2}\pi(\alpha-\tfrac{1}{2})\} \\ &= \{\hat{J}_0(r\surd(-u))\}^2/\sin^2\tfrac{1}{2}\pi(\alpha-\tfrac{1}{2}). \end{aligned} \tag{3}$$

The N_α trajectory has been parameterized by Barger and Cline (1967) as $\alpha(\mathrm{N}_\alpha) = -0\cdot38 + 0\cdot88\,u$, with an error which is less than 10%, so that for $u = -0\cdot2$ we have $\alpha \approx -\frac{1}{2}$. Since this gives a nonzero value of $\sin\frac{1}{2}\pi(\alpha-\frac{1}{2})$ for $u = -0\cdot2$, the double zero of $\{\hat{J}_0(r\surd(-u))\}^2$ must cause a dip in the differential cross sections given by equation (3). This agrees with the experimental findings of E. W. Anderson *et al.* (1968), Orear *et al.* (1968), Crennell *et al.* (1969) and Boright *et al.* (1970).

(ii) The backward processes $\pi^-\mathrm{p} \to \pi^-\mathrm{p}$ and $\pi^-\mathrm{p} \to \rho^-\mathrm{p}$ have Δ_δ as the exchange trajectory. There is neither a sharp backward peak nor a dip in their differential cross sections at $u = 0$ and consequently the flip and non-flip amplitudes make comparable contributions. Thus, since $\hat{J}_{\Delta\lambda\neq0}$ does not vanish at $u = -0\cdot2$, no dip can appear. This is also in agreement with experiment (E. W. Anderson *et al.* 1968, 1969; Orear *et al.* 1968; Crennell *et al.* 1969; Boright *et al.* 1970).

(iii) The backward process $\mathrm{K}^+\mathrm{p} \to \mathrm{K}^+\mathrm{p}$ is dominated by Λ_α exchange and the differential cross section is of the same form as in (ii) above. Thus we expect no dip, in agreement with experiment (Baker *et al.* 1968).

(iv) The backward photoproduction processes $\gamma\mathrm{p} \to \pi^0\mathrm{p}$ and $\gamma\mathrm{p} \to \pi^+\mathrm{n}$ do not have sharp peaks or dips in their differential cross sections at $u = 0$, so we can conclude that the flip and non-flip amplitudes make comparable contributions and consequently that there should be no dips at $u = -0\cdot2$. This is again consistent with experimental observation (R. L. Anderson *et al.* 1968, 1969; Tompkins *et al.* 1969).

(v) In the process $\mathrm{K}^-\mathrm{p} \to \pi^+\Sigma^-$ there is a sharp peak in the backward differential cross section at $u = 0$, so that the dominant contribution from the $\Delta\lambda = 0$ amplitude would cause the experimentally observed dip at $u = -0\cdot2$ (Barger *et al.* 1972).

(vi) In the process $\mathrm{K}^-\mathrm{p} \to \pi^-\Sigma^+$ the absence of either a dip or a sharp peak at $u = 0$ in the backward differential cross section indicates comparable contributions from flip and non-flip amplitudes and so no dip is to be expected for any value of u, which is just what is found experimentally (Barger *et al.* 1972).

Thus we see that the dip characteristics in the differential cross sections for these 10 backward reactions are consistent with the predictions of the modified dual absorptive model. We shall now consider polarization phenomena in some backward processes.

Polarizations

From equations (1) and (2), for the scattering processes describable by $\Delta\lambda = 0$ and $\Delta\lambda = 1$ amplitudes we have

$$f_0^s(s,u) = \hat{J}_0(r\surd(-u))\left\{\begin{matrix} \tan\frac{1}{2}\pi\{\alpha(u)-\frac{1}{2}\} \\ -\cot\frac{1}{2}\pi\{\alpha(u)-\frac{1}{2}\} \end{matrix}\right\} + \mathrm{i}\,\hat{J}_0(r\surd(-u)), \tag{4a}$$

$$f_1^s(s,u) = a - \mathrm{i}\,\hat{J}_1(r\surd(-u)), \tag{4b}$$

where the subscript to f denotes the total s-channel helicity flip for that amplitude; we choose $r = 1$ fm. For such processes the polarization P is given by

$$P \propto \frac{\mathrm{Im}(f_0 f_1^*)}{\mathrm{d}\sigma/\mathrm{d}u} = \frac{J_0\{a - J_1 \cot \frac{1}{2}\pi(\alpha - \frac{1}{2})\}}{J_0^2 \operatorname{cosec}^2 \frac{1}{2}\pi(\alpha - \frac{1}{2}) + a^2 + J_1^2}, \tag{5a}$$

if the exchange trajectory is even, and

$$P \propto \frac{J_0\{a + J_1 \tan \frac{1}{2}\pi(\alpha - \frac{1}{2})\}}{J_0^2 \sec^2 \frac{1}{2}\pi(\alpha - \frac{1}{2}) + a^2 + J_1^2}, \tag{5b}$$

if the exchange trajectory is odd.

Let us now consider the backward process $\pi^- \mathrm{p} \to \mathrm{K}^0 \Lambda^0$. This reaction is dominated by the Σ_α trajectory (Barger *et al.* 1969), which is of even signature and is given by $\alpha(\Sigma_\alpha) = -1 + u$. Also, the backward differential cross section (Bellettini 1968) has a peak at $u = 0$ so that the scattering is dominated by the non-flip amplitude. The polarization (5a) thus takes the form

$$P \propto \frac{a - J_1(r\sqrt{(-u)}) \cot \frac{1}{2}\pi(u - 1\cdot 5)}{J_0(r\sqrt{(-u)})} \sin^2 \tfrac{1}{2}\pi(u - 1\cdot 5). \tag{6}$$

Since at $u = 0$ the polarization is small (Beusch *et al.* 1970), it follows that a is small near $u = 0$ and thus

$$P \propto \{-J_1(r\sqrt{(-u)})/J_0(r\sqrt{(-u)})\} \sin \pi(u - 1\cdot 5). \tag{7}$$

When this form is assumed for small values of $|u|$, the characteristic features of P are found to be in qualitative agreement with experiment (Beusch *et al.*).

As noted in (i) of the previous section, the process $\pi^+ \mathrm{p} \to \pi^+ \mathrm{p}$ has a sharp backward peak in its differential cross section at $u = 0$ and is therefore dominated by a $\Delta\lambda = 0$ amplitude. In addition, the dominant exchange trajectory N_α is even and is parameterized as $\alpha(\mathrm{N}_\alpha) = -0\cdot 38 + 0\cdot 88\,u$. Thus the polarization is given by

$$P \propto \frac{a - J_1(r\sqrt{(-u)}) \cot 0\cdot 44\,\pi(u - 1)}{J_0(r\sqrt{(-u)})} \sin^2 0\cdot 44\,\pi(u - 1). \tag{8}$$

Since the polarization is again small at $u = 0$, it follows that even in this case a is negligible near $u = 0$, and the relation (8) reduces to

$$P \propto \{-J_1(r\sqrt{(-u)})/J_0(r\sqrt{(-u)})\} \sin 0\cdot 88\,\pi(u - 1). \tag{9}$$

For small values of $|u|$, this form is also found to be qualitatively consistent with the observed polarizations (Dick *et al.* 1972), giving a maximum in the vicinity of $u = -0\cdot 2$ and changing sign for $u \approx -0\cdot 3$.

From (ii) above, for the process $\pi^- \mathrm{p} \to \pi^- \mathrm{p}$, in which the flip and non-flip amplitudes make comparable contributions and the dominant exchange trajectory Δ_δ is odd, the polarization is given by

$$P \propto \frac{J_0\{a + J_1 \tan \frac{1}{2}\pi(\alpha - \frac{1}{2})\}}{J_0^2 \sec^2 \frac{1}{2}\pi(\alpha - \frac{1}{2}) + a^2 + J_1^2} = \frac{J_0\{a + J_1 \tan \pi(0\cdot 49\,u - 0\cdot 16)\}}{J_0^2 \sec^2 \pi(0\cdot 49\,u - 0\cdot 16) + a^2 + J_1^2}. \tag{10}$$

At $u = 0$ the polarization is small and so, with a neglected, the relation (10) becomes

$$P \propto \frac{J_0 J_1 \tan \pi(0\cdot 49\, u - 0\cdot 16)}{J_0^2 \sec^2 \pi(0\cdot 49\, u - 0\cdot 16) + J_1^2}. \tag{11}$$

This form gives zero polarization at $u = -0\cdot 2$, in disagreement with experiment (CERN–IPN(Orsay)–Oxford collaboration results as quoted by Barger *et al.* 1972, p. 212). However, if we make the additional assumption that a $\Delta\lambda = 0$ amplitude dominates the process near $u = -0\cdot 2$, the theoretical polarization curve is found to be qualitatively consistent with the experimental results.

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