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Intensity Distribution of Interplanetary Scintillation at 408 MHz

R. G. Milne

Chatterton Astrophysics Department, School of Physics, University of Sydney, Sydney, N.S.W. 2006.

Abstract

It is shown that interplanetary scintillation of small-diameter radio sources at 408 MHz produces intensity fluctuations which are well fitted by a Rice-squared distribution, better so than is usually claimed. The observed distribution can be used to estimate the proportion of flux density in the core of 'core–halo' sources without the need for calibration against known point sources.

1. Introduction

The observed intensity of a radio source of angular diameter $\lesssim 1''$ arc shows rapid fluctuations when the line of sight passes close to the Sun. This interplanetary scintillation (IPS) is due to diffraction by electron density variations which move outwards from the Sun at high velocities ($\sim 350\ \mathrm{km\,s^{-1}}$).

In a typical IPS observation the fluctuating intensity $S(t)$ from a radio source is recorded for a few minutes. This procedure is repeated on several occasions during the couple of months that the line of sight is within $\sim 20°$ (for observations at 408 MHz) of the direction of the Sun. For each observation we derive a mean intensity $\bar{S}$; a scintillation index m defined by

$$m^2 = \langle (S-\bar{S})^2 \rangle / \bar{S}^2 ,$$

where $\langle\ \ \rangle$ denote the expectation value; intensity moments of order q

$$Q_q = \langle (S-\bar{S})^q \rangle ;$$

and the skewness parameter $\gamma_1 = Q_3\,Q_2^{-3/2}$, hereafter referred to as γ. A histogram, or probability distribution, of the normalized intensity $S/\bar{S}$ is also constructed. The present paper is concerned with estimating the form of this distribution.

There is some debate as to what statistics the intensity fluctuations from IPS actually follow. The three suggested probability distributions are the Rice-squared, log normal and gaussian distributions. From theoretical studies using a thin screen model for the solar wind, Mercier (1962) deduced that the intensity has a Rice-squared distribution. Uscinski (1968) analysed the problem of multiple scattering by a thick nonabsorbing medium and demonstrated how to calculate the probability distribution of the fluctuations from a set of integro–differential equations which may be solved analytically for some limiting cases. Salpeter (1967) has shown that the gaussian distribution is applicable for very weak scattering.

However, Young (1971) proposed that the radio intensity should be distributed log normally for $m > 0 \cdot 3$ (strong scattering); otherwise a gaussian distribution was adequate. This proposal was based on a study of turbulence by Tatarski (1961), and it has been successfully used to interpret the optical scintillation of stars (Reiger 1963; Young 1969). Experiments on laser beam propagation through an extended turbulent atmospheric medium by Ochs and Lawrence (1969) and Fried *et al.* (1967) showed that the intensity fluctuations were very well described by a log normal distribution.

Lang (1971) has demonstrated from histogram plots of *interstellar* scintillation of pulsar radiation that both Rice-squared and log normal functions describe the observed data reasonably well, with the latter usually showing somewhat better agreement (111–606 MHz). Observation of strong IPS at 74 MHz of the Crab pulsar by Armstrong *et al.* (1972) showed that the intensity fluctuations are quite unambiguously described by a log normal law. However, pulsar data, particularly for the Crab, are complicated by intrinsic variations on various time scales.

One of the few published histogram plots of radio scintillation (not including pulsars) by Cohen *et al.* (1967) showed, from weak scattering observations of 3C 298, that a gaussian probability is best suited. However, from strong scattering observations of 3C 273 they concluded that a Rice-squared distribution fitted the data moderately well, even though the experimental distribution was more peaked than the theoretical fit. In fact, on close examination of their histogram, the data suggest that a log normal distribution would be more satisfactory.

The results given in the present paper show that the Rice-squared distribution gives the best description of the statistics of the scintillating small-diameter component. The log normal distribution *appears* to apply for $p \lesssim 0 \cdot 7$, where p is the ratio of the core flux density to the total source flux density. Once the nonscintillating fraction of the intensity is removed from the data, the fluctuating component is quite well described by the Rice-squared distribution.

Two methods of analysis have been used:

(1) Individual histograms of the intensity fluctuations for a given source during a four minute observing period are compared with the various theoretical probability distributions.

(2) For each observation the scintillation index m and the skewness parameter γ are derived. Results for a particular source observed at various angles from the Sun are plotted on a γ, m diagram and compared with theory.

2. Probability Distributions

The Rice distribution (for amplitudes) developed by Rice (1954) was modified to describe intensities by Cohen *et al.* (1967) who defined the Rice-squared probability distribution for the intensity S as

$$\begin{aligned} P(S) &= \tfrac{1}{2}\sigma^{-2}\,\mathrm{I}_0(bS^{\frac{1}{2}}\sigma^{-2})\exp\{-\tfrac{1}{2}\sigma^{-2}(S+b^2)\} && \text{for} \quad S \geqslant 0 \\ &= 0 && \text{for} \quad S < 0 \,. \end{aligned}$$

Here, $\mathrm{I}_0(x)$ is a modified Bessel function of the first kind of order zero, b is the length of a constant vector and σ^2 is the variance of the gaussian component. Physically, the Rice distribution consists of the sum of a constant vector b and a Rayleighdistributed vector, the latter being a vector that is the sum of two perpendicular vectors the lengths of which have identical and independent gaussian distributions with zero means. The mean intensity and modulation index for the Rice-squared distribution are respectively

$$\langle S\rangle = b^2+2\sigma^2 \qquad \text{and} \qquad m^2 = 4\sigma^2(b^2+\sigma^2)/\langle S\rangle^2\,.$$

If the mean intensity is normalized to unity, so that $b^2+2\sigma^2 = 1$, the Rice-squared distribution can be expressed in terms of the single parameter m, and we have (Bourgois and Cheynet 1972)

$$\gamma = 2m(3-2m^2)(2-m^2)^{-3/2}\,.$$

For the log normal distribution, namely (Eadie *et al.* 1971),

$$\begin{aligned} P(S) &= (2\pi)^{-\frac{1}{2}}(\sigma S)^{-1}\exp\{-\tfrac{1}{2}\sigma^{-2}(\log S-\mu)^2\} && \text{for} \quad S>0 \\ &= 0 && \text{for} \quad S\leqslant 0\,, \end{aligned}$$

we have

$$\langle S\rangle = \exp(\mu+\tfrac{1}{2}\sigma^2) \qquad \text{and} \qquad m^2 = \exp(2\mu+\sigma^2)\{\exp(\sigma^2)-1\}\,.$$

If $\langle S\rangle = 1$, the log normal distribution can also be defined in terms of m alone, and

$$\gamma = 3m+m^3\,.$$

The simple gaussian distribution is defined by

$$P(S) = (2\pi)^{-\frac{1}{2}}\sigma^{-1}\exp\{-\tfrac{1}{2}\sigma^{-2}(S-\mu)^2\}\,.$$

For weak scintillation this formula is adequate and we have

$$\langle S\rangle = \mu, \qquad m = \sigma \qquad \text{and} \qquad \gamma = 0\,.$$

However, for $m > 0{\cdot}3$ the gaussian distribution must be appreciably truncated at $S = 0$, and the mean and variance are no longer given by μ and σ^2. The relevant formulae for the truncated gaussian are rather complicated and only plots of the distribution (Figs 1 and 2) and the γ, m curve (Fig. 3) are given for purposes of comparison, since this particular distribution is not well suited to describe the data, as is discussed below. Plots of all three distributions together with the truncated gaussian are given for $m = 0{\cdot}2$, $0{\cdot}5$ and $0{\cdot}7$ in Fig. 1.

3. Observations

A comprehensive survey of IPS of radio sources selected from the Parkes catalogue in the declination range $+20°$ to $-70°$ has been undertaken using the 1·6 km cross-type radio telescope at the Molonglo Radio Observatory. This transit instrument operates at 408 MHz ($\lambda = 0{\cdot}73$ m), has a bandwidth 2·5 MHz, and each arm has a collecting area of $\sim$18 000 m^2 (Mills *et al.* 1963). Full details of this work will be published later and only the probability distribution of the intensity fluctuations is discussed here.

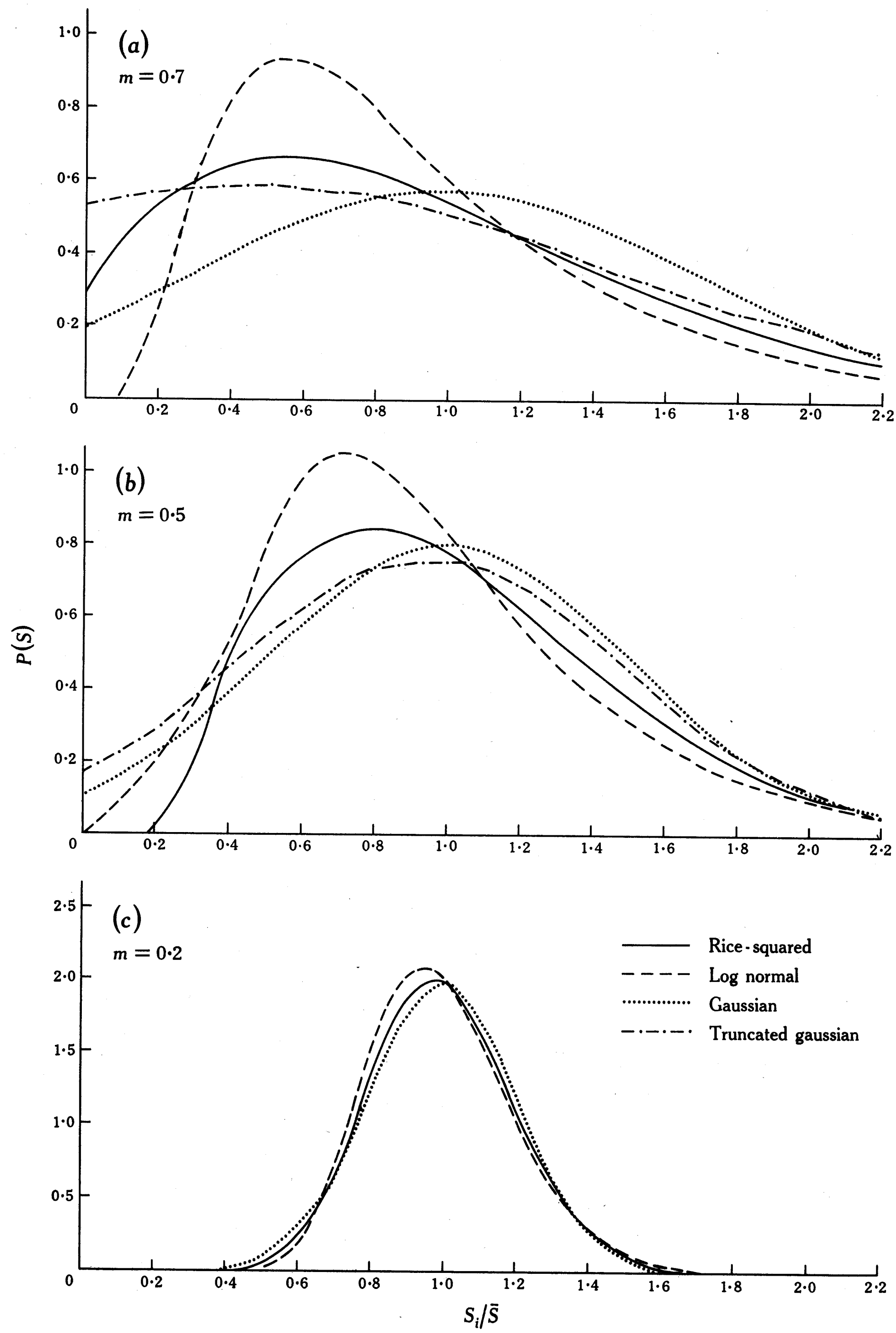

Fig. 1. Comparisons of the Rice-squared, log normal, gaussian and truncated gaussian probability distributions $P(S)$ for the indicated values of the scintillation index m.

Observations of the following five 'point' radio sources were made for solar elongations $\varepsilon > 7°$: 0019−00, 0056−00, 0316+16, 1148−00 and 2203−18. For smaller values of ε the scintillation index begins to decrease at this observing frequency because of the finite diameter of the source, due partly to interstellar scattering (Zeissig and Lovelace 1972; Readhead and Hewish 1972). More than ten observations of each source were made. For each observation a histogram of the intensity fluctuations about the mean was constructed, and values of γ and m were computed.

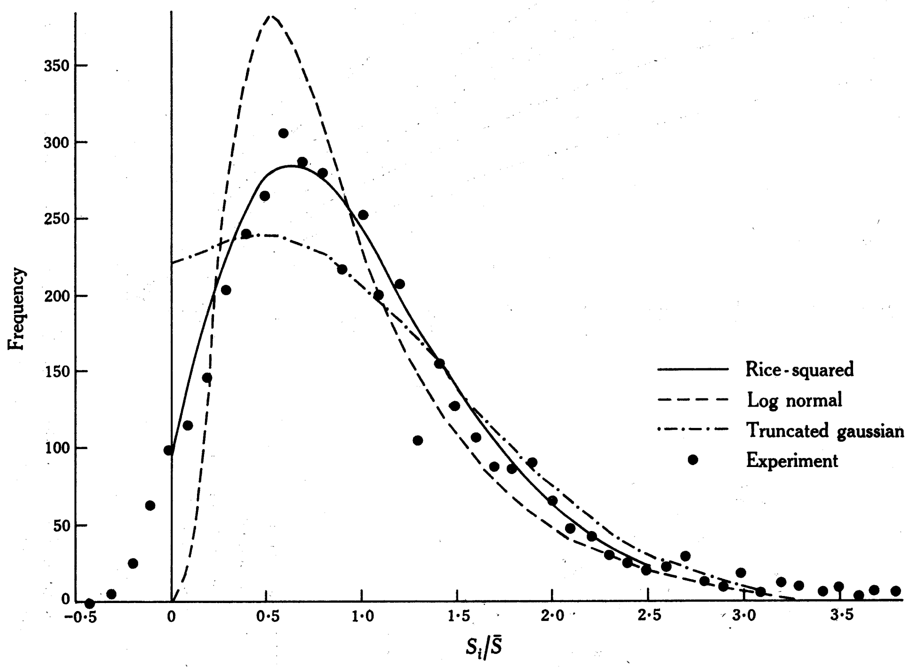

Fig. 2. Comparison of the experimental intensity histogram obtained for 0316+16 ($\varepsilon = 5°{\cdot}9$ and $m = 0{\cdot}7$) with fitted Rice-squared, log normal and truncated gaussian distributions.

A one-parameter (m) least squares fit was made to the histogram data using the three probability distributions discussed above, and in each case an estimate of m was obtained from the best fit. The 'goodness' of each fit was determined by computing χ^2 and plotting simultaneously the observed and expected histogram points for each of the three proposed distributions. A typical histogram for the source 0316+16 ($m \approx 0{\cdot}7$) is shown in Fig. 2 together with the three fitted distributions.

The overall results for the five point sources demonstrated that the Rice-squared distribution gave a reasonable fit for $\varepsilon < 20°$. The log normal distribution provided a better fit to about 10% of the observations, although in these cases the value of χ^2 was usually comparable with that computed from the Rice-squared fit, and inspection of the histograms indicated that either distribution was adequate. For $m > 0{\cdot}7$ the truncated gaussian distribution provided as good a fit as the Rice-squared distribution, and this point is discussed further in Section 5 below.

For $\varepsilon > 20°$ the gaussian distribution appeared from the χ^2 estimate to provide a marginally better fit to the data. However, inspection of Fig. 1*c* for $m = 0{\cdot}2$ illustrates that the three probability distributions do not differ markedly. In fact the theoretical

Fig. 3

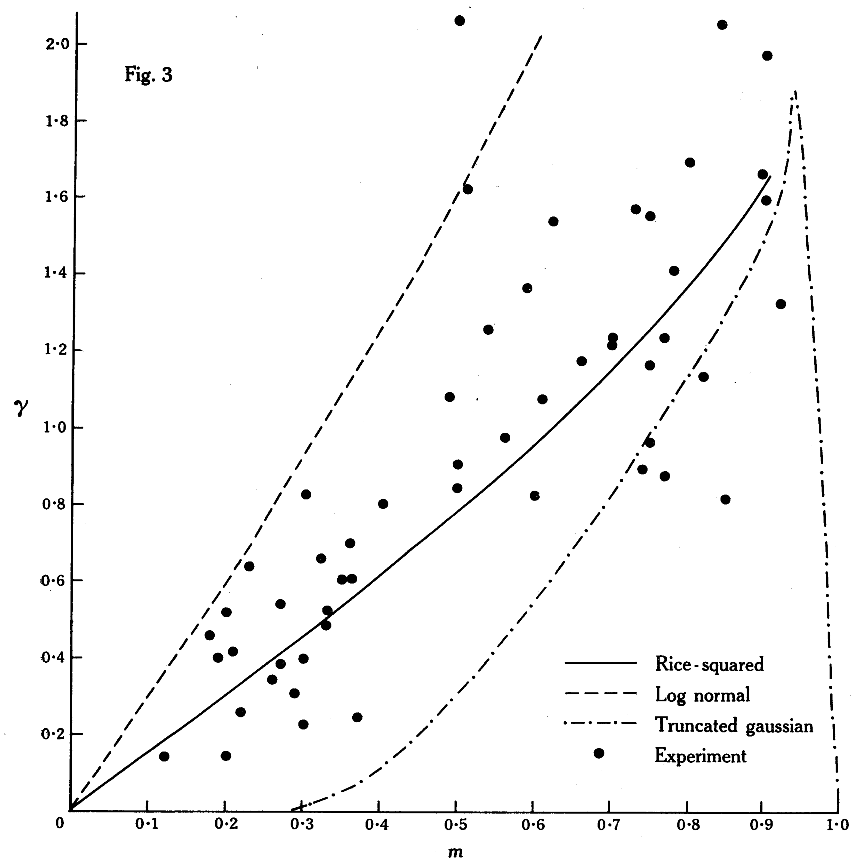

Fig. 4

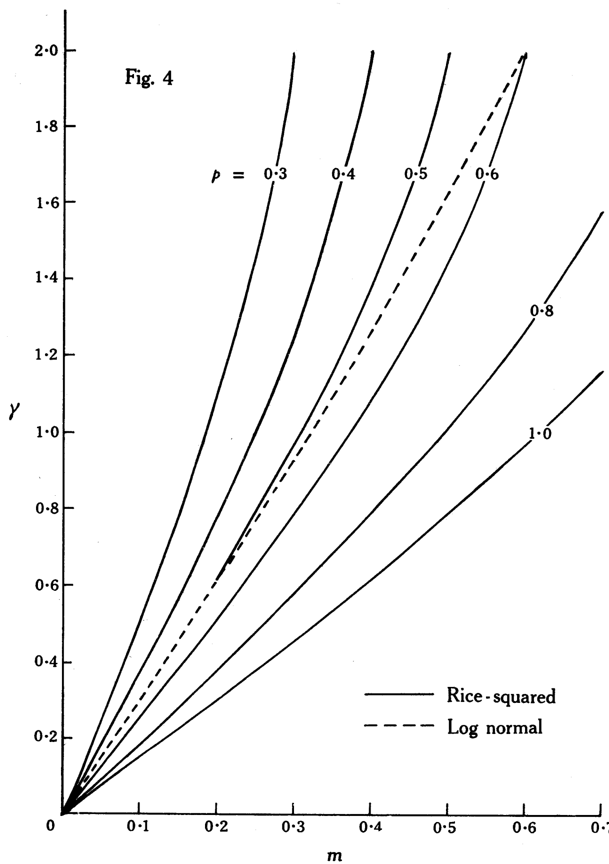

Fig. 3. Comparison of experimental γ, m results for the five point sources 0019−00, 0056−00, 0316+16, 1148−00 and 2203−18 with predicted curves from Rice-squared ($p = 1$) and log normal statistics, together with theoretical truncated gaussian values.

Fig. 4. Plots of γ, m curves based on the Rice-squared functions are shown for the indicated values of the core–halo parameter p. The corresponding curve for the log normal distribution is also shown.

estimate of the scintillation index was identical for the three fitted distributions for $m \lesssim 0\cdot25$.

Following the work of previous authors, the γ, m results for these observations have been plotted (Fig. 3) together with the theoretical curves predicted from the Rice-squared, log normal and truncated gaussian functions. This plot resembles a similar graph by Pramesh Rao *et al.* (1974) at 327 MHz, but their points are more scattered between the two theoretical curves than about the Rice-squared curve as shown here. However, their plot includes three sources (out of a total of nine) which are 'core–halo' type objects, although a scaling factor was used to normalize the scintillation index for these sources. As shown in the following section, such sources appear to satisfy a log normal distribution. Similar results to Fig. 3 have been obtained by Bourgois and Cheynet (1972) at 1420 MHz from observations of 0316+16 and 1148−00.

The γ, m results again indicate that the Rice-squared distribution is a more appropriate model than the log normal. The scatter of points about the theoretical Rice-squared curve in Fig. 3 is due to the large error associated with the computation of γ. The fractional standard error in γ varied between $0\cdot2$ and $0\cdot4$ (obtained by dividing the original intensity record into eight segments), whereas for m the fractional standard error was $\sim 0\cdot1$. The large error in γ is caused by spikes, three or more times the relative source intensity. These occur quite randomly, with possibly none occurring in a period of 10 or more seconds. The appearance of these spikes is discussed in Section 5 below. Further, there are the day to day variations in the interplanetary medium (Houminer and Hewish 1972) and this is reflected by the points moving along the Rice-squared curve in the γ, m plane.

4. Core–Halo Type Objects

An ideal core–halo type object can be represented by a single point source component together with a component whose angular extent is too large to produce scintillations. Most extragalactic sources contain several compact and extended components. For these objects the value p defines the ratio of the point source flux density to the total source flux density. Two examples are 0003−00 and 1226+02. For 1226+02, over 20 observations were made for elongations extending from 5° to 40°. For both sources the histogram analysis showed unequivocally that the log normal distribution provided the best description of the fluctuating intensity. However, this conclusion is quite misleading. A more satisfactory interpretation is that the small diameter component of the source is always described by a Rice-squared distribution, but for $p \lesssim 0\cdot6$ the fluctuations about the mean of the total source intensity appear to be better described by the log normal distribution, as is shown below. On this interpretation it is possible to estimate the value of p from the histogram for one observation only.

Let

$$X_i = S_i/\bar{S}$$

describe the bin centres of the original histograms. Then, if the contant $\bar{S}(1-p)$ is subtracted from the intensities, we have a new set of bins centred on

$$Y_i = X_i p^{-1} - (1-p)p^{-1}.$$

The theoretical Rice-squared distribution can now be fitted to this new histogram where the scintillation index is now given by m/p, m being the observed scintillation index for the source. A least squares analysis yields an estimate of p.

Table 1. Comparisons of evaluations of core–halo parameter p

(1) Source name Molonglo	(2) 3C	(3) p (Histogram)	(4) p (γ, m)	(5) p (m/m_o)	(6) Other IPS observations* $\mu = m/m_o$	(7) f(MHz)
0003−00	3C2	0·6±0·05	0·6–0·7	0·5±0·1	0·6	327[a]
					0·6	430[b]
0430+05	3C120	0·7±0·1	0·6	0·6±0·1	0·8±0·2	430[b]
0518+16	3C138	0·6±0·1	0·7	0·6±0·1	0·8	430[b]
1005+07	3C237	0·6±0·1	0·5–0·6	0·5±0·1	0·7	430[b]
1040+12	3C245	0·4±0·15	0·4	0·3±0·05	0·4	430[b]
1226+02	3C273	0·3±0·1	0·2–0·3	0·23±0·05	0·3	430[b]
1253−05	3C279	0·4±0·1	0·4	0·4±0·05	0·5	327[a]
1938−15		0·6±0·1	0·6	0·5±0·1	0·6	327[a]
2223−05	3C446	0·6±0·1	0·6	0·5±0·1	0·8	327[a]
2251+15	3C454·3	0·5±0·2	0·5	0·6±0·1	1·0	430[c]

* References are: [a] Bhandari *et al.* (1974); [b] Harris and Hardebeck (1969); [c] Harris *et al.* (1970).

The computed value of p from a histogram transformation, for one observation only, would not be sufficient to determine p accurately because of the statistical nature of the properties of the solar wind. The average value for p from five or more observations of a particular source is given for 10 sources in column 3 of Table 1. The r.m.s. error in p is $\sim 0 \cdot 1$ for each source.

It is also possible to establish the value of p for a source from the γ, m data, as proposed by Bourgois and Cheynet (1972). The value of γ for core–halo type sources should be the same as that for a single point source at the same elongation. However, the value of m, which represents the ratio of scintillating intensity to total source intensity, will be smaller by the factor p compared with that for a simple point source. Consequently, in plots of γ, m, the experimental points for these sources will lie to the left of the theoretical Rice-squared curve for point sources, i.e. will approach the log normal curve. This is illustrated in Fig. 4 where the theoretical γ, m curves based on a Rice-squared function are plotted for various values of p.

The value of p is determined from the γ, m plots in the following way. For an ideal core–halo type object

$$p = m/m_0,$$

where m is the observed scintillation index and m_0 is the scintillation index for a point source at the same solar elongation. For these objects the equation for γ in Rice-squared statistics can be written as

$$\gamma = 2(m/p)\{3-2(m/p)^2\}\{2-(m/p)^2\}^{-3/2}.$$

A one-parameter least squares analysis is performed on the γ, m data to estimate p. The results from this analysis appear in column 4 of Table 1 under the heading p (γ, m), and they agree excellently with the measured values from the histogram analysis (column 3).

Previous methods of estimating p have involved comparison of the scintillation index with that for known point sources at the same elongation. However, this required many observations of point sources at varying distances from the Sun to establish a point source calibration curve of m as a function of ε. Such an analysis has been made on the Molonglo data and the details will be presented elsewhere. For comparison, the results obtained are presented in column 5 of Table 1 under the heading p (m/m_0).

The results of all three methods of estimating p are in close agreement with one another and they also agree with other IPS results obtained at Ooty and Arecibo (column 6 of Table 1). The one major discrepancy is for the source 2251+15 which, however, is variable at 408 MHz (Hunstead 1972).

The Arecibo results (430 MHz) for p (usually calculated from one observation of $\mu = m/m_0$) are somewhat higher than those obtained here. Inspection of their point source calibration curve (derived from observations of 0316+16) shows that their values of m_0 are about $0 \cdot 1$ less than the corresponding values derived from the Molonglo data. Arecibo m_0 values may be lower due to observations at a different level of solar activity. The values of μ obtained at Ooty (327 MHz) are also slightly higher but this probably reflects the difference in observing frequency. In any case the differences are hardly significant in view of the r.m.s. uncertainties.

Power spectrum measurements of the sources 0003−00, 1938−15 and 2223−05 show that the core of these objects has a finite extent. No account of this was considered when calculating the value $p = m/m_0$ for these sources (these values of p should be incremented by $\sim 0 \cdot 1$). Diameter effects on the histogram and γ, m results are small and will be treated elsewhere.

5. Discussion

From the histogram and γ, m methods of analysis it appears that the Rice-squared distribution provides the best description for the intensity fluctuations of point sources at 408 MHz for $\varepsilon \lesssim 20°$. This conclusion is further strengthened by the use of Rice-squared statistics to estimate the value of p for core–halo objects, the results of which are in very good agreement with the values obtained by the standard method.

Examination of Fig. 3 shows that the truncated gaussian distribution may provide a fit to the data for $0 \cdot 7 \lesssim m \lesssim 0 \cdot 9$. However, since the range of applicability of the truncated gaussian is small and the fit certainly no better than the Rice-squared distribution, the latter is favoured. Fig. 3 further shows that for the truncated gaussian $\gamma = 0$ at $m = 1$, even though the probability distribution for this function practically coincides with (at least out to $S/\bar{S} \approx 5 \cdot 0$) that of the Rice-squared for which $\gamma = 2 \cdot 0$ at $m = 1$. (However, for $S/\bar{S} > 6 \cdot 0$ the truncated gaussian falls off very much faster than the Rice-squared distribution, making $\gamma = 0$ at $m = 1$ for the former distribution.) This implies that interpretation from the γ, m diagram can be misleading, whereas the histogram analysis provides an accurate comparison between the various distributions and the data. However, since the mean of the maximum value of m for any one source was between $0 \cdot 7$ and $0 \cdot 8$ (as is generally found by all observers) the truncated gaussian distribution is not really appropriate to describe the data (see Fig. 1*a*).

For $\varepsilon > 20°$ it is difficult to differentiate between the three distributions, and a gaussian distribution is quite adequate to describe the weak intensity fluctuations.

Observations close to the Sun ($\varepsilon \approx 5°$) have also been made on a number of occasions. In this case, even though the scintillation index had decreased from its peak value at $\varepsilon \approx 8°$, the intensity fluctuations were still adequately described by the Rice-squared function (details of these observations will appear elsewhere).

The assumption of a Rice-squared distribution allows the value of p for core–halo sources to be found, preferably from five or more observations. These estimates are more accurate than those obtained from γ, m data, especially near the Sun where scintillation spikes can influence the value of γ but have only a small effect on model fitting to the observed histograms. Both these methods of estimating p eliminate the need for calibration on known point sources.

Some minor features of the intensity distribution are not compatible with a pure Rice-squared function. For instance, values of $\gamma > 2$ have been observed at all frequencies and this is outside the range predicted by Rice-squared statistics. These high values of γ are caused by large scintillation spikes, first reported by Cohen and Gundermann (1969). Spikes can easily occur up to 5 times the mean source intensity, and they start to become apparent for $\varepsilon < 10°$ (at 408 MHz), i.e. in the transition from weak to strong scattering. They are most likely caused by a lens-like action in the interplanetary medium close to the Sun, producing very strong focusing at the Earth for a short period as the lens-like system is swept out from the Sun by the solar wind (Armstrong *et al.* 1972).

The existence of large spikes is also seen in histogram plots, where they produce a small but finite probability of very large intensities at the high end. The number of very large spikes greater than five times the mean intensity is relatively small (about 5 in 4000 samples); consequently they have a small effect on the model fitting to the histogram data, and it is virtually impossible to include them in any theoretical fitting analysis.

The problem of spikes is not removed by taking the logarithm of the intensity, as suggested by Young (1971). Reference to Fig. 3 immediately shows that the log normal distribution is clearly not compatible with point source data. Even at high values of γ and m where spikes are more significant, the log normal distribution is definitely no more satisfactory than the Rice-squared.

One further problem in the construction of histograms is that of negative intensities which are revealed in observations close to the Sun (see Fig. 2). Observation of negative intensities has been reported by Cohen *et al.* (1967) and Armstrong *et al.* (1972). This phenomenon appears to occur after a large scintillation spike has been observed, and is most probably caused by some part of the receiving equipment saturating on the large positive spike. Alternatively, the intensity baseline of the observation can be wrongly determined, especially when observing close to the Sun, and noise fluctuation could send the apparent intensity negative. Because of their relatively rare occurrence they were ignored in the histogram fitting.

Finally, why do some investigators find that a log normal distribution provides the best description of certain scintillation data (optical and radio)? It seems that the important quantity is the effective thickness L of the scattering screen compared with the distance z of the screen from the observer. If $L \approx z$ the full three-dimensional analysis is applicable (Tatarski 1961; Young 1971). The results for the thick screen ($L \approx z$) are similar to those for the thin screen ($L \ll z$) but involve the logarithm of the intensity rather than the intensity itself.

The theoretical predictions for $L \approx z$ are supported by low frequency radio observations of the Crab pulsar (Armstrong *et al.* 1972) and optical scintillation. At low radio frequencies (<80 MHz) the maximum fluctuations occur at solar elongation $\sim 30°$ and effective screen thickness $L \approx z$. The thickness of the screen is usually taken to be $(2\sin\varepsilon)/\sqrt{3}$. This corresponds to a 60° sector centred on the Sun (Cohen and Gundermann 1969). Most of the scattering is inside this sector if the electron density fluctuations vary as rapidly as $(\sin\varepsilon)^{-2}$. Furthermore, Young (1969) has shown that optical scintillation of stars and planets is produced throughout the terrestrial atmosphere rather than primarily in a thin layer (that is, $L \approx z$ for optical scintillation of stars, planets and laser beams along an extended path length in the air). All these phenomena are apparently well described by a log normal distribution and have a Kolmogorov spectrum.

Preliminary power spectrum measurements from the Molonglo data suggest that a power law with asymptotic index $\sim 8/3$ describes the intensity fluctuations for $\varepsilon > 10°$. Such spectra also correspond to Kolmogorov turbulence.

Hence it appears that the important condition in determining the intensity probability distribution of scintillation phenomena is not simply the state of turbulence in the scattering screen but the screen's thickness relative to its distance from the observer. This conclusion could be tested at low radio frequencies by a histogram analysis similar to that described above.

Acknowledgments

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