Shell Structure of the Even Tellurium Isotopes and Inelastic Proton Scattering

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Abstract

The isotope variation of the inelastic proton scattering cross sections leading to the 5⁻ state in tellurium is explained in terms of simple configurations of neutrons in the $s_{1/2}-d_{3/2}-h_{11/2}$ subshell.

Introduction

Direct reaction inelastic scattering is a sensitive test of nuclear structure, particularly when the transitions involved are dominated by relatively few single-particle state transitions. In practice, such transitions are noncollective and are usually associated with the excitation either of unnatural parity states or of natural parity states with large angular momentum. The spin flip requirement restricts the number of possible single-particle contributions to the excitation of unnatural parity states while, for the excitation of natural parity states with high angular momentum, there are usually few single-particle orbits of sufficient angular momentum available.

The excitation of 5⁻ states of nuclei (Whiten *et al.* 1972; Satchler 1973; Halbert and Satchler 1974) is weakly collective since, wherever they have been measured, the B(E5) values for γ -ray de-excitation are a few times the Weisskopf estimates. Hence, the recent 52 MeV proton inelastic scattering data (Matoba *et al.* 1973, 1975) to the 5⁻ states in the ¹²²Te, ¹²⁶Te, ¹²⁸Te and ¹³⁰Te nuclei not only provide excellent tests of the microscopic structure of these states, but also permit observation of shell filling effects of the h_{11/2} neutron orbit.

In a shell model for tellurium, the lowest 5^- state is formed by promoting a $3s_{1/2}(2d_{3/2})$ neutron into the $1h_{11/2}$ neutron orbit. There is little, if any, proton contribution to the 5^- state specification since this would require proton excitation out of the zero $h\omega$ valence orbits. This shell model interpretation is justified by the results of Auble and Ball (1972) for the (He³, d) and (p, t) reactions. Specifically, they found that the 5^- states were weakly populated in the proton stripping experiments but were strongly populated by the two-neutron pick-up reactions.

In the analyses of the inelastic scattering data of Matoba *et al.* (1973, 1975) reported in the present paper, the microscopic theory of direct reaction inelastic scattering was used with a ground state shell-model spectroscopy of $(s_{1/2})^2 (h_{11/2})^m$ neutrons, where m = 0, 4, 6 and 8 for ¹²²Te, ¹²⁶Te, ¹²⁸Te and ¹³⁰Te respectively. The 5⁻ state is then formed by exciting one $(s_{1/2})^1$ neutron into an $h_{11/2}$ state, so that this final state is described by an $(s_{1/2})^1 (h_{11/2})^{m+1}$ configuration.

Details of Calculations

Since complete derivations of the transition amplitude for direct reaction inelastic scattering of nucleons from nuclei have been published (Amos and Geramb 1971), only the relevant details are presented herein. In the microscopic distorted wave approximation, the inelastic scattering transition amplitude is given by

$$T_{fi} = \langle \chi_{f}^{(-)} \Psi_{J_{f}M_{f}} | t | A(\chi_{i}^{(+)} \Psi_{J_{i}M_{i}}) \rangle$$

= $\sum_{j_{1} j_{2} I m_{1} m_{2} N} S(j_{1} j_{2}; J_{i}J_{f}; I) \langle J_{i} IM_{i}N | J_{f}M_{f} \rangle (2J_{f}+1)^{-\frac{1}{2}} (-1)^{j_{1}-m_{1}}$
 $\times \langle j_{1} j_{2} m_{1} - m_{2} | I - N \rangle \langle \chi_{f}^{(-)} \phi_{j_{2}m_{2}} | t | \chi_{i}^{(+)} \phi_{j_{1}m_{1}} - \phi_{j_{1}m_{1}} \chi_{i}^{(+)} \rangle, \quad (1)$

where the $\chi^{(\pm)}$ are the distorted waves generated with an optical model and the ϕ_{jm} are single-particle bound state wavefunctions in the many-particle target wavefunctions Ψ_{JM} . The effective interaction *t* is approximated by a two-nucleon potential acting between the projectile and the target nucleon that is initially in the bound state $\phi_{j_1m_1}$. Hence the total transition amplitude is a weighted sum of single-particle transition amplitudes, with the weights being the spectroscopic amplitudes

$$S(j_1 j_2; J_i J_f; I) = \langle \Psi_{J_f} || [a_{j_2}^+ \times a_{j_1}]^I || \Psi_{J_i} \rangle.$$

$$\tag{2}$$

These amplitudes contain all the many-particle aspects of the nuclear transition.

For the transition to the 5⁻ states in the tellurium isotopes, the spectroscopic amplitudes are determined here by assuming a shell structure for the ground and 5⁻ states based upon an inert core of ¹²⁰Te, so that there are 6, 10, 12 and 14 valence neutrons in the ¹²²Te, ¹²⁶Te, ¹²⁸Te and ¹³⁰Te isotopes to be located in the $(s_{1/2} d_{3/2} h_{11/2})$ subshell. To create a 5⁻ state by scattering, either an $s_{1/2}$ or a $d_{3/2}$ neutron must be placed in an $h_{1/2}$ orbit or vice versa. Thus, for j_1 and j_2 being any of $s_{1/2}$, $d_{3/2}$ or $h_{11/2}$, the spectroscopic factors are (de Shalit and Talmi 1963)

$$\begin{split} S(j_{1} \ j_{2}; J_{i} J_{f}; I) &= \langle j_{2}^{n_{2}}(J_{2}') \ j_{1}^{n_{1}}(J_{1}'); J_{f} \parallel [a_{j_{2}}^{+} \times a_{j_{1}}]^{I} \parallel j_{2}^{n_{2}-1}(J_{2}) \ j_{1}^{n_{1}+1}(J_{1}); J_{i} \rangle \\ &= \{n_{2}(n_{1}+1)\}^{\frac{1}{2}} (-1)^{n_{1}+J_{1}'+j_{1}-J_{1}} \{(2J_{2}'+1)(2J_{1}+1)(2J_{i}+1)(2J_{f}+1)\}^{\frac{1}{2}} \\ &\times [j_{2}^{n_{2}} J_{2}'\{\mid j_{2}^{n_{2}-1}(J_{2}) \ j_{2} J_{2}'][j_{1}^{n_{1}}(J_{1}') \ j_{1} J_{1}\mid\} \ j_{1}^{n_{1}+1} J_{1}] \\ &\times \begin{pmatrix} J_{2}' \ J_{1}' \ J_{f} \\ J_{2} \ J_{1} \ J_{i} \\ j_{2} \ J_{1} \ I \end{pmatrix} \langle j_{2} \parallel [a_{j_{2}}^{+} \times a_{j_{1}}]^{I} \parallel j_{1} \rangle, \end{split}$$
(3)

where the single-particle reduced matrix element has the value $(2I+1)^{\frac{1}{2}}$. For the 5⁻ transitions in tellurium, then, $J_i = 0$ and $J_f = I = 5$. Furthermore, by demanding least seniority, we have $J_1 = J_2 = 0$, so that angular momentum restrictions determine that $J'_1 = j_1$ and $J'_2 = j_2$. The spectroscopic amplitudes in equation (1) are therefore

$$S(j_1 j_2; 05; 5) = (-1)^{n_1 + 1} \{ 11(n_1 + 1)(2j_2 + 2 - n_2)/(2j_1 + 1)(2j_2 + 1) \}^{\frac{1}{2}}.$$
 (4)

Based upon this expression, the spectroscopic amplitudes for all possible transitions within the s-d-h shell that can produce a 5^- state are summarized in Table 1. Since

the isotope variation of differential cross section magnitudes in this model is directly related to the squares of these amplitudes, the latter are also given in Table 1.

For the inelastic scattering analyses reported herein, the distorted waves were evaluated using the optical model parameters determined from pertinent elastic scattering analyses (Matoba *et al.* 1973, 1975), and the bound state wavefunctions were generated from a Woods-Saxon potential (Hodgson 1971).

Initial	Final					
configur- ation	configur- ation	$S(j_1 j_2; 05; 5)$	122 Te $(n = 1)$	126 Te (<i>n</i> = 5)	128 Te (<i>n</i> = 7)	130 Te (<i>n</i> = 9)
$(d)^4(s)^2(h)^{n-1}$	$(d)^4(s)^1(h)^n$	$\{11(13-n)/12\}^{\frac{1}{2}}$	1.00	0.67	0.50	0.33
$(d)^4(s)^2(h)^{n-1}$	$(d)^{3}(s)^{2}(h)^{n}$	$\{11(13-n)/12\}^{\pm}$	1.00	0.67	0.50	0.33
$(d)^4(s)^0(h)^{n+1}$	$(d)^{3}(s)^{0}(h)^{n+2}$	$\{11(11-n)/12\}^{\frac{1}{2}}$	0.83	0.50	0.33	0.17
$(d)^{2}(s)^{2}(h)^{n+1}$	$(d)^{2}(s)^{1}(h)^{n+2}$	$\{11(11-n)/12\}^{\frac{1}{2}}$	0.83	0.50	0.33	0.17
$(d)^{2}(s)^{2}(h)^{n+1}$	$(d)^{1}(s)^{2}(h)^{n+2}$	$\{11(11-n)/24\}^{\frac{1}{2}}$	0.42	0.25	0.17	0.08
$(d)^{0}(s)^{2}(h)^{n+3}$	$(d)^{0}(s)^{1}(h)^{n+4}$	$\{11(9-n)/12\}^{\frac{1}{2}}$	0.67	0.33	0.17	
$(d)^4(s)^0(h)^{n+1}$	$(d)^4(s)^1(h)^n$	$\{11(n+1)/12\}^{\frac{1}{2}}$	0.17	0.50	0.67	0.83
$(d)^{2}(s)^{0}(h)^{n+3}$	$(d)^{2}(s)^{1}(h)^{n+2}$	$\{11(n+3)/12\}^{\pm}$	0.33	0.67	0.83	1.00
$(d)^{2}(s)^{0}(h)^{n+3}$	$(d)^{3}(s)^{0}(h)^{n+2}$	$\{11(n+3)/24\}^{\frac{1}{2}}$	0.17	0.33	0.42	0.50
$(d)^{0}(s)^{2}(h)^{n+3}$	$(d)^{1}(s)^{2}(h)^{n+2}$	$\{11(n+3)/12\}^{\frac{1}{2}}$	0.33	0.67	0.83	1.00
$(d)^{0}(s)^{0}(h)^{n+5}$	$(d)^{0}(s)^{1}(h)^{n+4}$	$\{11(n+5)/12\}^{\frac{1}{2}}$	0.50	0.83	1.00	
$(d)^{0}(s)^{0}(h)^{n+5}$	$(d)^{1}(s)^{0}(h)^{n+4}$	$\{11(n+5)/12\}^{\frac{1}{2}}$	0.50	0.83	1.00	_

Table 1. Spectroscopic amplitudes within the s-d-h shell model for the 5⁻ transitions in tellurium

Two forms of effective interaction were used in these analyses: (1) An effective interaction consisting of the standard valence (central plus tensor) two-nucleon potential (Geramb *et al.* 1973) complemented by complex core polarization renormalization terms that were generated from a deformed optical model prescription of these correction effects (Love and Satchler 1967); (2) An effective interaction consisting of the standard valence two-nucleon potential complemented by the purely imaginary part of the core polarization renormalization, and thus approximating the complex t matrix (Satchler 1973; Geramb and Hodgson 1975). Both effective interactions can therefore be expressed as

$$t = c V(12) - \{Y_5(\hat{r}_1), Y_5(\hat{r}_2)\}\{y_5^{\text{Re}}f_U(r_1)f_{\text{BD}}(r_2) + i y_5^{\text{Im},1}f_W(r_1)f_{\text{BD}}(r_2) + i y_5^{\text{Im},2}f_{\text{WD}}(r_1)f_{\text{BD}}(r_2)\}.$$
(5)

The form factors $f_{\rm U}$, $f_{\rm W}$ and $f_{\rm WD}$ are respectively proportional to the radial derivatives of the real, volume imaginary and surface imaginary parts of the optical model potential, which are required by the deformed potential model (Love and Satchler 1967). Likewise $f_{\rm BD}$ refers to the bound state Woods–Saxon potential. The two forms of our interaction constrain the values of the parameters c, $y_5^{\rm Re}$, $y_5^{\rm Im}$,¹ and $y_5^{\rm Im}$,².

Results and Discussion

The experimental differential cross sections for the inelastic scattering of 52 MeV protons to the 5⁻ states in the four isotopes of tellurium are structurally equivalent to within the experimental errors. Their magnitudes vary in the ratio 1:0.67:0.5:0.34 for ¹²²Te:¹²⁶Te:¹²⁸Te:¹³⁰Te to within 10% (Matoba *et al.* 1973, 1975). This same



Fig. 1. Comparison with the experimental data of Matoba *et al.* (1973, 1975) of our predicted differential cross sections for the 5^- excitations in the tellurium isotopes following inelastic scattering with 51.9 MeV protons.

variation is evident in the present shell model prediction for the square of the spectroscopic amplitudes as given in the first two entries in Table 1. However, all transitions listed in Table 1 involve exclusively single-neutron excitations between either the $3s_{1/2}$ or $2d_{3/2}$ and the $1h_{11/2}$ states, and therefore inelastic scattering analyses cannot distinguish their exact relative contributions. Nevertheless, since the isotope dependence of the magnitudes of the cross sections is so well predicted by the $(d)^4(s)^2(h)^{n-1}$ ground state specification, and since all other configurations lead to observably different variations, these other configurations must be minor in their effect. Of course, it is essential that a complete shell model diagonalization using the s-d-h basis be made to determine the exact eigenvectors.

The results of our calculations are presented in Fig. 1. The topmost differential cross section was calculated using the interaction form 1, whereas the other results were obtained using interaction form 2. The parameter values that yield these results are specified below:

	с	y_5^{Re} (MeV ⁻¹)	$y_5^{Im,1}$ (MeV ⁻¹)	$y_5^{\rm Im,2} ({\rm MeV^{-1}})$	e _{eff} (e)
Interaction 1	1.00	0.0011	0.0011	0.0011	0.95
Interaction 2	2.12		0.0041	0.0010	1.12

These results were obtained by using only the $s_{1/2}$ to $h_{11/2}$ single-particle transition (first entry in Table 1), since the $d_{3/2}$ to $h_{11/2}$ transition (second entry in Table 1) gave no sensible difference in the structure or magnitude predictions of the cross sections. Again, it requires a proper shell model evaluation to determine the relative $s_{1/2}$ and $d_{3/2}$ mixtures.

The collective model core polarization corrections, as used in interaction form 1, are related to the effective charges (Love and Satchler 1967) required in the renormalization of electromagnetic transition rates. From our analyses, an effective charge of 0.95e is predicted for the neutrons, which is in qualitative agreement with those values used in structure analyses of the 2^+ and 4^+ states in tellurium (Sorensen 1970). This renormalization is a consequence of the restricted Hilbert space used in our analyses as well as in all practical shell model calculations.

The use of interaction form 2 is based upon an alternative viewpoint of the reaction mechanism, namely, that the transition is mediated by a complex t matrix. The resultant cross sections must again be renormalized to account for the restricted Hilbert space used, and equating this renormalization to an effective charge (Atkinson and Madsen 1970) yields a value of $1 \cdot 12e$.

In summary, the isotope variation of the scattering cross sections to the 5^- states in tellurium reflects the expectation of a simple shell model structure. However, it is desirable to have further inelastic scattering data, particularly to the 7^- states whose isotope variation of cross sections is similar to that of the 5^- transitions (Matoba *et al.* 1973, 1975). Such experiments should be complemented by transfer and pickup reactions leading to these same negative parity states, so that a complete assessment of the tellurium wavefunctions can be made.

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Manuscript received 20 May 1975