

Interplanetary Scintillation Power Spectra

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Abstract

Power spectrum measurements of interplanetary scintillation at 408 MHz show that an inverse power law spectrum provides the best description for all scintillating radio sources. The inverse power law index is reasonably constant at ~ 2.4 for solar elongation angles $\epsilon > 10^\circ$, and this agrees well with spacecraft observations. For $\epsilon < 10^\circ$ the index apparently decreases with decreasing ϵ , and this appears to be consistent with recent strong scattering theory. A Bessel analysis attempted in order to detect Fresnel structure proved unsuccessful because of noise on the power spectra.

1. Introduction

Interplanetary scintillation (IPS) is observed when radiation from small diameter radio sources (usually less than $\sim 1''$ arc) passes close to the Sun. The electron density fluctuations moving outwards from the Sun at $\sim 350 \text{ km s}^{-1}$ randomly modulate the phase of the radio waves through changes in refractive index, and consequently produce random intensity diffraction patterns on the ground. The radio intensity fluctuations can be described by a power spectrum which is related to the spatial spectrum of the electron density irregularities in the solar wind.

The very early observations of IPS power spectra were assumed to imply that the spectra were close to gaussian (Cohen *et al.* 1967; Dennison and Wiseman 1968), although this conclusion was based on a qualitative rather than a quantitative estimate. Hewish (1971, 1972) argued forcibly from IPS observations that a gaussian spectrum adequately described the intensity fluctuation for both weak and strong scattering. This assumption was used by Cohen and Gundermann (1969) and others to derive various parameters of the solar wind. Furthermore, a gaussian function has *always* been assumed when deriving radio source structure from IPS measurements (Harris and Hardebeck 1969; Bhandari *et al.* 1974; Readhead and Hewish 1974). However, Ekers and Little (1971) and Pramesh Rao *et al.* (1974) have reported spectra varying from exponential to gaussian as the scattering went from weak to strong at 2295 and 430 MHz respectively, while Lovelace *et al.* (1970), Zeissig (1971) and Coles *et al.* (1974) have observed power law spectra (i.e. spectra of the form $P(f) \propto f^{-n}$, where f is the scintillation frequency and n the power law index) at observation frequencies of 430, 430 and 74 MHz respectively. The present results show that a power law spectrum provides the best description for the observed spectra. Consequently use of a power law spectrum should make it possible to improve diameter estimates of radio sources derived from IPS.

Cronyn (1970) has demonstrated the important result that, if the three-dimensional spatial power spectrum $P_{ns}(K)$ of the electron density fluctuations follows a power law with index β (that is, $P_{ns}(K) \propto K^{-\beta}$, where K is the wavenumber), then the spectrum $P_{nt}(f)$ of the *temporal* fluctuations of density as seen by a space probe also follows a power law but with index $\beta-2$. The subscript convention adopted here is: the first subscript refers to the quantity being measured (n for density, I for radio intensity, ϕ for phase) and the second subscript refers to the spectral domain (s for spatial frequencies, t for temporal frequencies). The wavenumber K can be converted to a frequency through the relation $K = 2\pi f/U$, where U is the solar wind velocity normal to the line of sight. Since $U \approx 350 \text{ km s}^{-1}$, it is assumed to first order that the Earth and the spaceprobe are stationary with respect to the Sun. Furthermore, the spectrum $P_{It}(f)$ of the radio intensity fluctuations from IPS is also power law but with index $\beta-1$ (Cronyn).

In the case of *weak* scattering (i.e. for solar elongation angles $\varepsilon > 10^\circ$ at $\sim 400 \text{ MHz}$) Salpeter (1967) derived the following important relationship between the two-dimensional phase fluctuation power spectra $P_{\phi s}(K_x, K_y)$ and the spatial intensity power spectra $P_{Is}(K_x, K_y)$:

$$P_{Is}(K_x, K_y) = P_{\phi s}(K_x, K_y) \mathcal{F}(K_x, K_y). \quad (1)$$

In this expression,

$$\mathcal{F}(K_x, K_y) = 4 \sin^2(K_r^2/K_f^2) \quad (2)$$

is the Fresnel filter factor, which arises from the application of Fresnel diffraction theory to a thin diffracting screen,

$$K_f^2 = 4\pi/\lambda z$$

is the Fresnel wavenumber, where λ is the wavelength of observation and z is the distance of the scattering screen from the Earth, while $K_r^2 = K_x^2 + K_y^2$. The Fresnel filter seriously distorts all IPS spectra below the Fresnel frequency

$$f_f = U/(\pi z \lambda)^{\frac{1}{2}},$$

which has a value of $\sim 0.7 \text{ Hz}$ for $z \approx 1 \text{ A.U.}$ and at an observation frequency of 408 MHz . The observed temporal frequency spectrum $P_{It}(f)$ is given by the strip integral over K_y of equation (1) (Cronyn 1970).

Until recently there was no adequate theory to explain the observations in the strong scattering region. However, Rumsey (1975) has recently presented a form of $P_{Is}(K_x, K_y)$ based on the solution of the parabolic wave equation, in which $P_{Is}(K_x, K_y)$ is defined by a double integral that must be evaluated numerically. In a companion paper, Marians (1975) investigated the form of $P_{Is}(K_x, K_y)$ for various levels of scattering and various source diameters using the power law spectrum $P_{ns}(K) \propto K^{-3}$. The results of this analysis are compared with the present observations in Section 4b.

2. Observations and Analysis

The IPS observations reported here were made during the years 1970–73 using the north–south (NS) arm of the Molonglo cross-type radio telescope situated

~ 250 km SW. of Sydney. The NS arm has a collecting area of $\sim 18\,000$ m², and operates at 408 MHz with a bandwidth of 2.5 MHz. Eleven fan beams were available from the NS arm, and one of them was chosen as the 'on-source' beam, while another (usually one or two beamwidths away) was taken as the 'off-source' beam. These two beams were simultaneously recorded on magnetic tape. The analogue data were subsequently sampled every 50 ms and recorded digitally on a second magnetic tape. The telescope allowed a maximum useful observing period ~ 3.5 min per day for each source.

The power spectra of the IPS data were computed using a fast Fourier transform algorithm, and the real and imaginary components were squared and added to give the power. The spectra were normally computed from successive 26 s periods of data and added, thereby giving a spectral resolution of 0.04 Hz. Division of the original record (~ 3.5 min) into eight segments gave an estimate of the statistical uncertainty in the power of $\sim 30\%$ (varying from 25% to 35%) for both the on-source and off-source beams at all frequencies.

From the difference $P(f)$ between the on-source and off-source spectra, an estimate of the width of the power spectrum was made by computing the second moment f_2 using the definition

$$f_2^2 = \int_0^{f_c} f^2 P(f) df / \int_0^{f_c} P(f) df,$$

where f_c is the frequency at which the on-source spectrum meets the noise given by the off-source spectrum. Fig. 1 presents a scatter plot of f_2 values for observations over a series of elongations $p = \sin \epsilon$ of the five 'point' radio sources: PKS 0019-00, 0056-00, 0316+16, 1148-00 and 2203-18. None of these sources showed a systematic bias, indicating that they all contained equivalent 'point source' components. The scintillation index (defined as the r.m.s. value of the intensity fluctuations divided by the average radio source intensity) was also plotted against p for observations of these sources, and this showed that all were point sources (i.e. with angular diameters $\lesssim 0''.02$) and not core-halo type objects (Milne 1975). The statistical uncertainty of each observation in Fig. 1 was less than 2% (computed from the ensemble average of eight segments). However, the day-to-day variations as indicated by the large scatter of points, were much greater due to variation in the interplanetary medium, e.g. rotating sector structure (Houminer and Hewish 1972).

The shape of the observed power spectra (Fig. 2) was typically a relatively flat region out to about 0.7 Hz (the Fresnel frequency at 408 MHz) followed by a rapid or slow fall-off, depending upon whether the scintillation was weak or strong respectively. To determine the exact form of the power spectra, gaussian, exponential and power law models were each applied to the spectral data, a two-parameter least squares fit was performed and a value of χ^2 for each fit was computed. The observed and computed power for each fit were also simultaneously plotted and this allowed a visual inspection to be made of the quality of the fit.

The gaussian and exponential fits were made from 0.2 Hz, below which frequency ionospheric effects are important (Rufenach 1971), to a frequency f_p which was defined by $P_{\text{on}}(f_p) \approx 2P_{\text{off}}(f_p)$, and consequently we have $f_p < f_c$. This procedure prevented the differential power spectra $P(f)$ from approaching zero at high frequencies, where the power law fit could become invalid. On the other hand, the

power law fit was made from 0.8 Hz to f_p , as the spectra were obviously not power law for $f < 0.8$ Hz because of the high-pass filtering effects of the Fresnel filter factor. Tests on simulated gaussian and power law spectra showed that the Fresnel filter factor does not change the form of the intensity spectrum $P_{If}(f_x, f_y)$ above the Fresnel frequency but simply imposes a cosinusoidal modulation, which is usually not observed (see Section 5 below). To ensure against bias, the gaussian and exponential models were again fitted from 0.8 Hz to f_p to enable direct comparisons to be made with the power law fit. Examples of the model fitting are presented in Fig. 2.

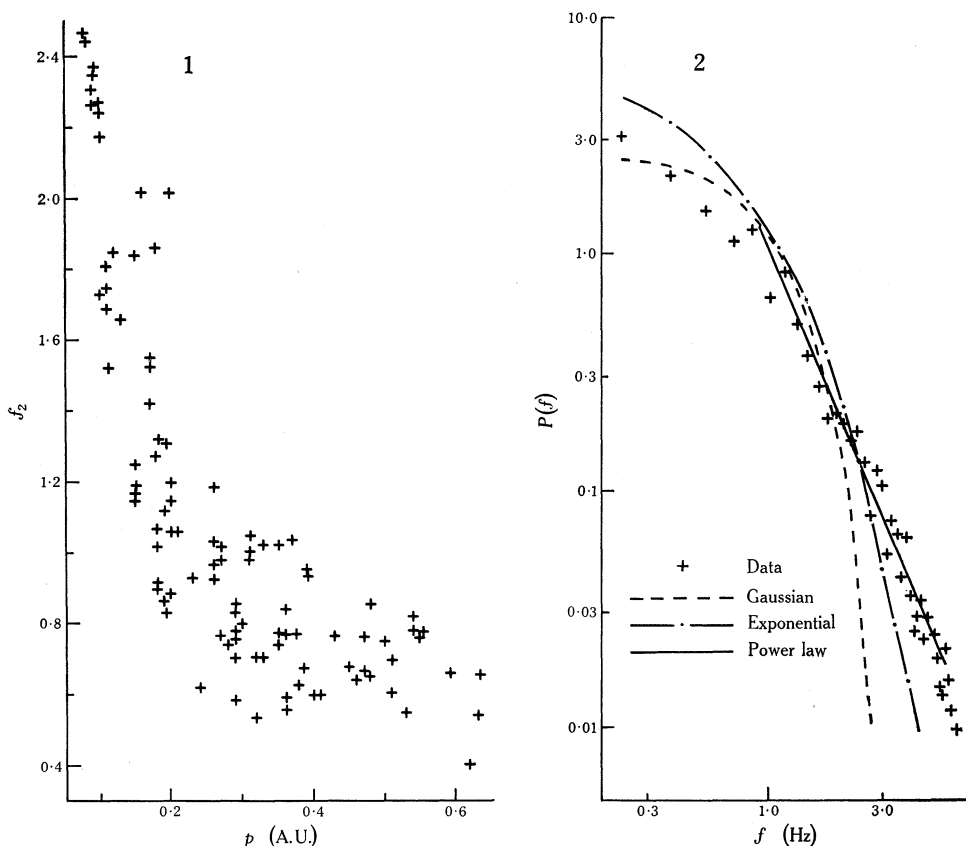


Fig. 1. Values of the second moment f_2 for the five point sources PKS 0019-00, 0056-00, 0316+16, 1148-00 and 2203-18 plotted against solar elongation p ($= \sin \epsilon$) in A.U. There were no systematic differences between the values of f_2 for any particular source.

Fig. 2. Example of model fitting to the power spectra $P(f)$ of 1148-00 for $\epsilon = 10^\circ.4$. The straight line represents the power law fit above 0.8 Hz. The lowest four data points were ignored in the model fitting.

For reasonably strong scintillation the subtraction of $P_{\text{off}}(f)$ had a negligible effect on the model fitting. Finally, it was found that the model fitting was not critically dependent on the value of f_p chosen. The estimates of the power law index n were practically identical over the frequency ranges 1-4 and 4-7 Hz for most of the strong scintillating sources observed, and this implies that the index was a well-determined parameter for a given observation.

3. Results

The power law model proved by far the best fit for all sources (point and extended) at all elongation angles. For $\epsilon > 10^\circ$ the exponential model proved satisfactory for $\sim 10\%$ of the observations but even here the fit was no better than for the power law model, while the gaussian model fit was *always* the least satisfactory for all the observed spectra. Furthermore, the fitting of the gaussian and exponential models was, for most of the sources, practically identical in the two cases: $f > 0.8$ Hz and $f > 0.2$ Hz. This demonstrates that the fitting procedures were not critically dependent on the frequency interval used. As illustrated in Fig. 2 the obvious difference between the power law and the gaussian and exponential models was that, for the latter two models, the power decreased rapidly with increasing frequency above ~ 3 Hz when compared with the observational data.

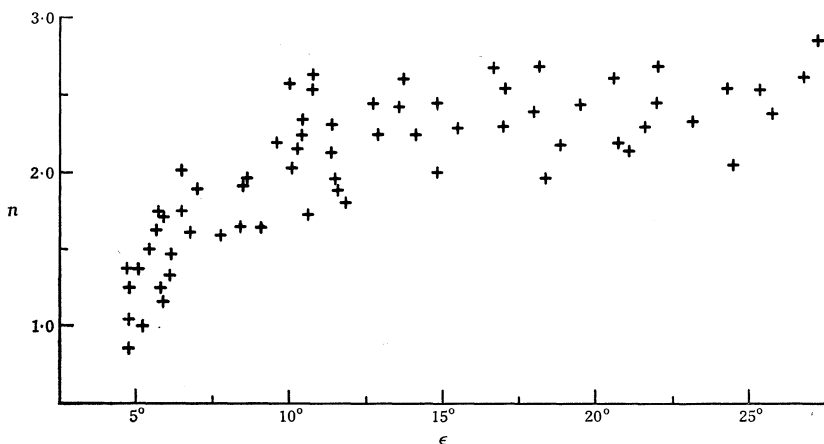


Fig. 3. Power law indices n plotted against solar elongation angle ϵ for the five point sources PKS0019-00, 0056-00, 0316+16, 1148-00 and 2203-18.

For the five point sources mentioned in Section 2, the value of the power law index n varied from ~ 1 to ~ 3 for $5^\circ < \epsilon < 35^\circ$. The spread in n is illustrated in Fig. 3, where it is plotted against solar elongation. At elongation angles greater than $\sim 10^\circ$ the indices remain essentially independent of ϵ but vary between 2 and 3, due to daily variation in the interplanetary medium. The mean value of n for $\epsilon > 10^\circ$ is 2.4 ± 0.2 . For $\epsilon < 10^\circ$ the observed spectra appear to be still power law, but the index begins to decrease rapidly.

Core-halo type objects with a 'resolved' core component (from IPS measurements at Molonglo), such as 0003-00, 1334-17 and 1938-15, were still best described by an inverse power law spectrum. However, for these objects the index was a little higher, and the power law fit was not as good as for 'point' sources, as the spectra exhibited a more rapid fall-off because of the product of the point source IPS spectrum with the brightness distribution of the source (Cohen *et al.* 1967).

Fitting procedures were also applied to weakly scintillating sources, i.e. sources which had a small signal-to-noise ratio for scintillations with $f_c < 2$ Hz. This low signal-to-noise ratio was caused by one or more of the following conditions: (1) a low intrinsic flux density of a small-diameter component, (2) an angular diameter

greater than $\sim 0'' \cdot 5$, (3) a large elongation angle or (4) a high ecliptic latitude (it should be noted that the minimum solar elongation angle is equal to the ecliptic latitude). For weak scintillation, a power law still provided the best description.

4. Discussion

(a) Weak Scattering

Early measurements by spacecraft of the fluctuating interplanetary magnetic field (Jokipii and Coleman 1968), the number density of protons in the solar wind (Intriligator and Wolfe 1970) and the radial solar wind velocity (Coleman 1968; Goldstein and Siscoe 1972) all suggested that the observed temporal power spectra were best described by an inverse power law spectrum with an index $\beta - 2 \approx 1 \cdot 5$. These spectra, however, were all measured for frequencies below $\sim 10^{-2}$ Hz, that is, for frequencies much less than for IPS spectra, which are in the range 0.1–10 Hz.

However, Unti *et al.* (1973) measured the proton density fluctuation in the frequency range 0.0048–13.3 Hz from the Earth satellite OGO-5 and found that the average slope of the power spectrum was 1.6, ranging from 1.12 to 2.17, and that these spectra also appeared to be reasonable extrapolations of the earlier results mentioned in the previous paragraph. Furthermore Cronyn (1972), using the observed IPS results of Dennison (1969), Lovelace *et al.* (1970) and Hewish (1971), showed that the electron density fluctuation spectrum may be deduced from the IPS spectrum. The agreement with the spacecraft measurements of Intriligator and Wolf (1970) was remarkably good, although there was a small difference in the power law indices (1.3 for the proton density spectra compared with 1.6 for the extrapolated IPS spectra).

The present IPS spectra measurements of the index n at 408 MHz for $\varepsilon > 10^\circ$ are in excellent agreement with extrapolated spaceprobe measurements which were made at $\sim 1 \cdot 0$ A.U. Furthermore, the fluctuation in the power law index is roughly the same as that obtained by Unti *et al.* (1973). However, Unti *et al.* found that 7 out of 32 of their spectra showed a flattening at between 0.1 and 0.5 Hz, although the statistical significance of this flattening was apparently marginal. Using a much larger set of OGO-5 spectra, Neugebauer (1975) found a small but statistically significant power enhancement at a frequency of $\sim 0 \cdot 7$ Hz, corresponding to the proton gyroradius. Long-term averages of the power spectra smear out the enhancement owing to the time variation of both the solar wind velocity and the size of the proton gyroradius. Irregularity scales are inferred from the point at which the spatial power spectrum turns over, or flattens out, and hence the flattening could possibly be used to derive a second 'inner' scale of ~ 100 km, as distinct from the 10^6 km 'outer' scale of the electron density irregularities measured by Goldstein and Siscoe (1972) and Jokipii and Coleman (1968) from spacecraft observations. Such spectra have been suggested by Matheson and Little (1971) and more recently by Rickett (1973). The significance of such a 'bump' in the spectra is that it could represent the dominance of a particular plasma wave mode (e.g. the proton gyroradius). In fact a turbulent plasma is likely to be dominated by a series of plasma wave modes (Sarf 1970). Unfortunately the observed spectra recorded at Molonglo were distorted below $\sim 0 \cdot 7$ Hz because of Fresnel filtering effects (see equation 1).

Other IPS measurements (Lovelace 1970; Zeissig 1971) at 430 MHz yield values of $n = 2 \cdot 9 \pm 0 \cdot 3$ and $n \approx 2 \cdot 6$ respectively for $10^\circ < \varepsilon < 35^\circ$, while Coles *et al.*

(1974) found a value of n in the range $2.5\text{--}3.5$ at 74 MHz. These results are also in good agreement with the present estimate $n = 2.4 \pm 0.2$. All spacecraft and IPS observations show a spread in the values of n , even though the estimate for a given observation seems well determined. Changes in the solar wind velocity do not affect n . However, if the electron density fluctuations are not described by a stationary process, one would expect to observe a range of values for n .

(b) Strong Scattering

For $\varepsilon < 10^\circ$ the index n starts to decrease, slowly at first then more rapidly as $\varepsilon \rightarrow 4^\circ$, as illustrated in Fig. 3. This behaviour accompanies an increase in the second moment f_2 , as shown in Fig. 1, i.e. the spectrum becomes wider and flatter. A simultaneous turnover occurs in the scintillation index which reaches a maximum value of approximately 0.9 at $\varepsilon \approx 10^\circ$ and then starts to decrease for observations made closer to the Sun. The turnover of the scintillation parameters n , f_2 and the scintillation index occur in the transition from weak to strong scattering (where phase fluctuations exceed ~ 1 rad).

The behaviour of the temporal intensity spectra for a point source as the scattering goes from weak to strong has been derived by Marians (1975) for a value of $\beta = 3$ (see also Coles *et al.* 1974). In the case of strong scattering the spectrum broadens out beyond the Fresnel frequency and slowly turns over to finally attain an asymptotic slope of $\beta - 1$. The width of the flat portion of the spectrum depends on the strength of turbulence in the solar wind and the velocity U . The level of power at low frequencies is suppressed relative to that just before the transition from weak to strong scattering, and all Fresnel structure disappears in strong scattering.

The observed strong scattering spectra taken at Molonglo certainly show a decrease in the low frequency power (also observed by Cohen and Gundermann 1969) and a general broadening, but no spectra were ever observed to have a power law slope at high frequencies similar to that found in weak scattering, namely 2.4, before they met the noise level. However, some of the spectra did show a steepening of the slope at high frequencies and might have approached a power law slope of 2.4 if the signal to noise had permitted an extension to higher frequencies. The theoretical strong scattering spectra of Marians (1975) do not reach an asymptotic slope of $\beta - 1$ until about 10 db below the power level at low frequencies. In weak scattering the $\beta - 1$ slope is attained just above the Fresnel frequency.

A further complication arises in the strong scattering case where the spectra depend *markedly* on the radio source structure. Finite diameter sources cause the observed spectra to steepen more rapidly at high frequencies. The actual effect on the spectrum depends on the nature of the brightness distribution, which further depends strongly on the observing frequency. The present results indicate that the 'point' sources used here are no larger than $\sim 0''.02$. Such a small diameter has no appreciable effect on the power spectra below ~ 8 Hz (assuming a gaussian brightness distribution).

Marians (1975) also showed that the scintillation index saturates at unity for point sources in strong scattering. However, a diameter of only $0''.01$ will limit the scintillation index to ~ 0.9 , and then it decreases for $\varepsilon < 10^\circ$ (at 408 MHz). Larger diameter sources will show a turnover at approximately the same solar elongation as the strength of scattering increases but will attain relatively lower maximum values of the scintillation index. The scintillation indices for the five 'point' sources mentioned

in Section 2 reached a maximum value between 0.8 and 0.9 at $\varepsilon \approx 10^\circ$ before turning over.

It is possible that the observed spectra recorded at 408 MHz are influenced by real changes in the interplanetary medium for $\varepsilon \lesssim 10^\circ$. Ekers and Little (1971) observed the presence of a high random component of the solar wind velocity ($\sim 200 \text{ km s}^{-1}$) which persists out to $\sim 0.14 \text{ A.U.}$, just where the acceleration of the interplanetary plasma is decreasing. This random component was insignificant beyond $\sim 0.19 \text{ A.U.}$ ($\varepsilon \approx 11^\circ$). Furthermore, the results of Mariner 9 (Callahan 1974) show that the spectral index for the columnar electron density fluctuations is essentially constant at $\beta = 3.9 \pm 0.2$ for solar elongation angles from ~ 0.03 to $\sim 0.4 \text{ A.U.}$ However, the Mariner results do imply that in the region near the Sun ($0.07 \lesssim p \lesssim 0.2 \text{ A.U.}$) the electron density fluctuations decline more slowly with distance than they do on the average for $0.15 \lesssim p < 1.0 \text{ A.U.}$

5. Bessel Analysis

Lovell *et al.* (1970) pointed out that the observed spectra will display deep minima, corresponding to the zeros in the sine function of the Fresnel filter factor, only if (1) the fluctuations are almost isotropic; (2) scattering occurs in a 'thin screen' and is weak; (3) deviation in the solar wind velocity is small. None of the computed power spectra from the Molonglo observations showed any signs of cosinusoidal modulation which, if it existed, was masked by the noise. It is, however, theoretically possible to enhance the Fresnel structure by performing a Bessel transform on the data (Bourgeois 1972). Bessel transforms were computed by the method outlined by Lovell *et al.* Unfortunately, all the transforms proved to be very unstable, in that the power occasionally went negative through the frequency range. These might have been interpreted as Fresnel minima, but inspection of many transforms showed that the negatively going power occurred quite randomly. The instability was shown using artificial data to be due to the poor signal-to-noise ratio of $\sim 3:1$ (the statistical uncertainty in the power being $\sim 30\%$).

6. Conclusions

The present IPS results, together with all the spacecraft and other IPS data, indicate that the best overall description of the interplanetary plasma is that the three-dimensional spatial power spectrum $P_{ns}(K)$ is described by a power law with an approximately constant index $\beta \approx 3.5$ over a range of scale sizes from ~ 6 to $\sim 10^6 \text{ km}$ for heliocentric distances larger than $\sim 0.2 \text{ A.U.}$ It is possible that the irregularity spectrum extends below $\sim 6 \text{ km}$ (i.e. beyond 10 Hz) since all observed IPS spectra (and the spectra of Unti *et al.* 1973) show no signs of tapering off at high frequencies. The spectra all meet the noise before any such effect is observed. Gaussian spectra were shown definitely to be unsuitable for all the present observations. There is possibly some flattening of the spectra around 0.5 Hz (scale size $\sim 100 \text{ km}$) and further spacecraft observations will be needed to clarify this point.

For heliocentric distances $p < 0.2 \text{ A.U.}$, the IPS spectrum at 408 MHz broadens out but still appears to be best described by a power law. As the scattering gets stronger the measured index n decreases rapidly. It is quite possible that the strong scattering spectra could have reached an asymptotic slope of ~ 2.4 (similar to that

in weak scattering) if the spectrum could have been measured over three or more decades before it reached the noise level, as is predicted by the strong scattering theory of Rumsey (1975). Unfortunately the noise level was too high to show this.

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