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# Direct Reaction Inelastic Proton Scattering from $^{20}$Ne

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*Abstract*

The transition amplitudes for direct reaction inelastic proton scattering from nuclei are formulated using target spectroscopy given by angular momentum projection from axially symmetric minimal energy Hartree–Fock intrinsic states of $N = Z$ nuclei. Applications to the scatterings from $^{20}$Ne are made, from which the importance of virtual excitations of giant resonances is evident.

## Introduction

Analyses of direct reaction inelastic scattering of nucleons leading to discrete final states in target nuclei are of current interest not only because they are sensitive to giant resonance properties of the target (Geramb *et al.* 1975) but also because they test details of microscopic models of spectroscopy (Morrison *et al.* 1975; Nesci *et al.* 1975). When spin-dependent data (polarizations, asymmetries and spin-flip probabilities) as well as the differential cross sections are measured then the reaction analyses are tests of spectroscopy not only complementary to but also more stringent than those provided by the $B(E\lambda)$ estimates for $\gamma$ ray decay between the nuclear states (Smith and Amos 1973, 1975). To be so, however, the reaction analyses must include full antisymmetry (Geramb and Amos 1971; Satchler 1973) and all essential components in the reaction mechanism (Geramb *et al.* 1975).

These requirements necessitate lengthy calculations, and consequently most analyses to date have been restricted to using spherical shell model spectroscopy (Satchler 1973, and references cited therein; Morrison *et al.* 1975; Nesci *et al.* 1975). This restriction prohibits adequate applications to reactions from deformed nuclei, such as most of the s–d shell. For these nuclei a target spectroscopy, such as given by the projected Hartree–Fock (PHF) approach, is more pertinent. Indeed, analyses of ground state band reaction data from $^{20}$Ne, $^{24}$Mg, $^{28}$Si and $^{32}$S using PHF spectroscopy (Braley and Ford 1969; Ford *et al.* 1971; Asciutto *et al.* 1972) have demonstrated this. But those analyses were made without antisymmetry and essential reaction mechanism components. Consequently we have begun a re-analysis of data from s–d shell targets, in particular of transitions in the ground state bands of $N = Z$ nuclei using PHF spectroscopy in the antisymmetrized distorted wave approximation.

In this paper, we derive the scattering amplitudes in a form that is not only compatible with the spherical spectroscopy derivations (Geramb and Amos 1971; Geramb *et al.* 1975) but also complementary to the work of Braley and Ford (1969). As an application, we have analysed data from the inelastic scattering of protons leading to the $2^+$ (1·63 MeV) and $4^+$ (4·25 MeV) states in $^{20}$Ne using a simple s–d

shell expansion basis in the PHF theory (Ripka 1968; Braley and Ford). These reactions are of additional interest since the transitions can also be analysed using spectroscopy from both a spherical and a deformed shell model (I. Morrison, personal communication).

## Theory

In the distorted wave approximation (DWA), the measurables associated with direct reaction inelastic proton scattering from nuclei leading to discrete final states are all related to transition amplitudes that have the form (Geramb and Amos 1971):

$$\begin{aligned} T_{fi} &= A\langle \chi_f^{(-)}(0)\, \Psi_{J_f M_f}(1 \ldots A) \,|\, t(0,1) \,|\, \mathscr{A}_{01}\{\chi_i^{(+)}(0)\, \Psi_{J_i M_i}(1 \ldots A)\}\rangle \\ &= \sum_{j_1 j_2 I} S(j_1 j_2; J_i J_f; I)\, T_{\mathrm{sp}}(j_1 j_2; J_i J_f; I), \end{aligned} \tag{1}$$

where the spectroscopic amplitude is defined by

$$S(j_1 j_2; J_i J_f; I) = \langle \Psi_{J_f} \,||\, [a_{j_2}^{+} \times a_{j_1}]^I \,||\, \Psi_{J_i}\rangle \tag{2}$$

and the 'single-particle' scattering amplitudes are

$$\begin{aligned} &T_{\mathrm{sp}}(j_1 j_2; J_i J_f; I) \\ &\quad = \sum_{m_1 m_2 N} (-)^{j_1 - m_1} \langle j_1\, j_2\, m_1 - m_2 \,|\, I - N\rangle (2J_f + 1)^{-\frac{1}{2}} \\ &\qquad \times \langle J_i I M_i N \,|\, J_f M_f\rangle \langle \chi_f^{(-)}(0)\, \phi_{j_2 m_2}(1) \,|\, t(0,1) \,|\, \mathscr{A}_{01}\{\chi_i^{(+)}(0)\, \phi_{j_1 m_1}(1)\}\rangle . \end{aligned} \tag{3}$$

All details of the derivations of these equations, of the notation, and of the terminology used when (spherical) shell model states represent the target spectroscopy are given in earlier publications (Geramb and Amos 1971; Geramb *et al.* 1975).

To obtain the above results, the many-body nuclear states must be expanded in cofactors. For a spherical model of spectroscopy, this expansion is

$$\Psi_{JM}(1 \ldots A) = A^{-\frac{1}{2}} \sum_{jm} \phi_{jm}(1)\{a_{jm}\, \Psi_{JM}(1 \ldots A)\}, \tag{4}$$

where $a_{jm}$ are the appropriate particle annihilation operators. With this expansion, the separation of the two-body (projectile and a single bound-state particle) and many-body (spectroscopic amplitude) attributes of the transition amplitudes is achieved. For deformed nuclei, similar expansions and derivations of the inelastic scattering transition amplitudes can be made (Braley and Ford 1969), and in the present paper we present such expansions and derive the form of the scattering amplitudes for inelastic proton scattering to members of the ground state rotational bands of $N = Z$ nuclei, but in a form that can be directly compared with the expression given in equation (1).

We suppose that the ground state band of a deformed nucleus is described by states of good angular momentum projected out from the minimal energy single-determinant solution of the appropriate axially symmetric Hartree–Fock (HF) equations. Thus, with $K$ being the projection quantum number along the symmetry

axis, the physical states $\Psi_{JMK}$ for $N = Z$ nuclei are given by

$$\Psi_{JMK}(1 \ldots A) = N_{JMK} P^J_{MK} \Phi_{(K)}(1 \ldots A), \tag{5}$$

where $N_{JMK}$ is a normalization factor and $K = 0$ for the ground state band. The minimal energy HF solution $\Phi_{(K)}$ can be expressed as

$$\Phi_{(K)}(1 \ldots A) = (A!)^{-\frac{1}{2}} \det[\psi_\lambda], \qquad \text{where} \qquad K = \sum \lambda. \tag{6a, b}$$

The $\psi_\lambda$ are intrinsic single-nucleon deformed states which, expanded in a complete spherical basis, are given by

$$\psi_\lambda(i) = \sum_{jm} C^{(\lambda)}_{jm} \phi_{jm}(i), \tag{7}$$

where, if axial symmetry is imposed, the $m$ summation reduces to a single term having the $k$ value ($\lambda$) of the particular deformed orbit. The expansion coefficients in equation (7), which are determined by the variational procedure in the HF calculations, are real and satisfy a time-reversal symmetry, namely,

$$C_{j-m} = (-)^{j-m} C_{jm}. \tag{8}$$

It should be noted that more than one deformed orbit can have the same projection quantum number $k$.

The operators $P^J_{MK}$ in equation (5) project states of good angular momenta from the intrinsic wavefunction $\Phi_K$ (Peierls and Yoccoz 1957). Their properties are given in the Appendix as is their use in determining that the normalizations $N_{JMK}$ are:

$$N_{JMK} = \left( (2J+1)^{\frac{1}{2}} \int_0^\pi \mathrm{d}\beta \, \sin(\beta) \, d^J_{KK}(\beta) \det[B(\beta)] \right)^{-\frac{1}{2}}, \tag{9}$$

where $B$ is an $A \times A$ matrix with elements

$$(B)_{k'k} = \sum_j C_{jk} C_{jk'} d^j_{k'k}(\beta) \delta_{\tau'\tau}, \tag{10}$$

with $\tau$ being the isospin quantum number of the particle. The dimensionality of this matrix can be reduced by truncation of the (spherical) expansion basis whence, for the s–d shell expansions for the $^{20}$Ne application considered in this paper (2 protons and 2 neutrons in the $k^\pi = \frac{1}{2}^+$ deformed orbital), we need only deal with a $4 \times 4$ matrix. Because of the implicit isospin constraint in equation (10), this can be partitioned into two $2 \times 2$ matrices and two null matrices. But it is the partition property of $P^J_{MK}$ given in equations (A5) and (A6) of the Appendix that is central in the development of the transition amplitudes, since it enables us to express the PHF states of equation (5) as

$$\Psi_{JMK} = N_{JMK} (A)^{-\frac{1}{2}} \sum_{jm} \phi_{jm}(1) \sum_{J_c k M_c M'_c} C_{jk} \langle J_c j M_c m \,|\, JM \rangle \langle J_c j M'_c k \,|\, JK \rangle \, \xi(2 \ldots A),$$

where

$$\xi(2 \ldots A) = P^{J_c}_{M_c M'_c}(2 \ldots A) \alpha_k \Phi_{(K)}(1 \ldots A). \tag{11}$$

In this equation, $\alpha_k$ is a single-particle deformed state destruction operator, which can be expanded in terms of spherical operators as

$$\alpha_k = \sum_j C_{jk} a_{jk}. \tag{12}$$

Then, using equation (11) in the transition amplitude (1), the spectroscopic amplitude $S$ is given by

$$\begin{aligned}
&\langle J_i I M_i N \,|\, J_f M_f\rangle\, S(j_1 j_2; J_i J_f; I) \\
&\quad = (N_{J_i M_i K_i} N_{J_f M_f K_f})(2J_f+1)^{\frac{1}{2}} \sum_{J_c m_1 m_2 k_1 k_2 M_c K_c K'_c} (-)^{j_1 - m_1} \langle j_1 j_2 m_1 m_2 \,|\, I-N\rangle \\
&\qquad \times C_{j_2 k_2} C_{j_1 k_1} \langle j_1 J_c m_1 M_c \,|\, J_i M_i\rangle \langle j_2 J_c m_2 M_c \,|\, J_f M_f\rangle \langle j_1 J_c k_1 K_c \,|\, J_i K_i\rangle \\
&\qquad \times \langle j_2 J_c k_2 K'_c \,|\, J_f K_f\rangle \langle \Phi_{(K_f)} \,|\, \alpha^\dagger_{k_2} P^{J_c}_{K'_c, K_c}(2 \ldots A)\, \alpha_{k_1} \,|\, \Phi_{(K_i)}\rangle .
\end{aligned} \tag{13}$$

Standard angular momentum coupling enables this expression to be simplified, whence for ground state band transitions in $N = Z$ axially symmetric cases ($K_i = K_f = 0$) we have

$$\begin{aligned}
&S(j_1 j_2; J_i J_f; I) \\
&\quad \equiv (N_{J_i M_i} N_{J_f M_f})[(2J_f+1)(2I+1)(2J_i+1)]^{\frac{1}{2}} \\
&\qquad \times \sum_{J_c} (-)^{J_i + I + J_c + j_2} \begin{pmatrix} I & j_1 & j_2 \\ J_c & J_f & J_i \end{pmatrix} \\
&\qquad \times \sum_{k_1 k_2} C_{j_2 k_2} C_{j_1 k_1} \langle j_1 J_c k_1 - k_1 \,|\, J_i 0\rangle \langle j_2 J_c k_2 - k_2 \,|\, J_f 0\rangle\, G(J_c k_1 k_2),
\end{aligned} \tag{14}$$

where

$$G(J_c k_1 k_2) = \langle \Phi_0 \,|\, \alpha^\dagger_{k_2} P^{J_c}_{-k_2 - k_1} \alpha_{k_1} \,|\, \Phi_0\rangle . \tag{15}$$

The quantity $G(J_c k_1 k_2)$ is an $A-1$ particle determinant which, by using the integral representation of the operators $P^J_{MK}$, reduces to

$$G(J_c k_1 k_2) = (2J_c+1)^{\frac{1}{2}} \int_0^\pi \mathrm{d}\beta \sin(\beta)\, d^{J_c}_{-k_2 - k_1}(\beta) \langle \Phi_0 \,|\, \alpha^\dagger_{k_2} \exp(-\mathrm{i}\beta J_y)\, \alpha_{k_1} \,|\, \Phi_0\rangle . \tag{16}$$

The determinant function in this integral is the $k_2 k_1$ cofactor of the determinant $B(\beta)$ in equations (9) and (10), and can be evaluated by using a cofactor theorem, namely,

$$(\text{cofactor}[A])_{ij} = \det[A]\,(A^{-1})_{ji}. \tag{17}$$

Thus we obtain

$$G(J_c k_1 k_2) = (2J_c+1)^{\frac{1}{2}} \int_0^\pi \mathrm{d}\beta \sin(\beta)\, d^{J_c}_{-k_2 - k_1}(\beta)\, (B^{-1}(\beta))_{k_1 k_2} \det[B(\beta)]. \tag{18}$$

Hence, once the expansion coefficients $C_{jk}$ for the minimal energy HF state are determined, so also are the spectroscopic amplitudes. Then, from equation (1) the inelastic scattering transition matrix elements can be determined as can the $B(\mathrm{E}I)$ values for $\gamma$ ray transitions between the band states, since the latter depend upon the

same spectroscopic amplitudes (Morrison *et al.* 1975) via

$$B(\mathrm{E}I; J_i \to J_f) = [(2J_i+1)(2I+1)]^{-1} \times \left( \sum_{j_1 j_2} S(j_1 j_2; J_i J_f; I) \langle \phi_{j_2} \| e_{\mathrm{eff}} r^I Y_I(\Omega) \| \phi_{j_1} \rangle \right)^2 . \quad (19)$$

**Application**

We have applied the preceding formalism to analyse the differential cross sections and asymmetries from the inelastic scattering of 24·5 MeV protons leading to the $2^+$ (1·63 MeV) and $4^+$ (4·25 MeV) states in $^{20}$Ne (de Swiniarski *et al.* 1969, 1972). These reactions are particularly useful and interesting in many respects. The inelastic scattering data include spin-dependent asymmetries, and the $B$(E2) values for $\gamma$ ray transitions are well known (Skorka *et al.* 1966). The projectile energy is such that two-step processes, in which giant resonances of the target act as doorway states (Geramb *et al.* 1975), can be significant. Most interesting, however, is the availability of pertinent spectroscopic information from a variety of structure models. Specifically, properties of the ground state band of $^{20}$Ne have been predicted from PHF calculations using small (1s–0d shell) and large (0s through 0g shells) expansion bases (Ripka 1968; Ford *et al.* 1971), and from (1s–0d shell) spherical and deformed shell model calculations (I. Morrison, personal communication).

**Table 1. Spectroscopic amplitudes for $^{20}$Ne transitions**

| Transition | | $S(j_1 j_2; 02; 2)$ | | | $S(j_1 j_2; 04; 4)$ |
|---|---|---|---|---|---|
| $j_1$ | $j_2$ | PHF | SM | DSM | PHF |
| $1s_{1/2}$ | $0d_{3/2}$ | 0·457 | 0·361 | 0·381 | — |
| $1s_{1/2}$ | $0d_{5/2}$ | −0·880 | −0·653 | −0·730 | — |
| $0d_{3/2}$ | $1s_{1/2}$ | −0·251 | −0·276 | −0·259 | — |
| $0d_{3/2}$ | $0d_{3/2}$ | −0·117 | −0·195 | −0·155 | — |
| $0d_{3/2}$ | $0d_{5/2}$ | −0·120 | −0·171 | −0·151 | 0·443 |
| $0d_{5/2}$ | $1s_{1/2}$ | −0·642 | −0·557 | −0·571 | — |
| $0d_{5/2}$ | $0d_{3/2}$ | 0·160 | 0·197 | 0·184 | −0·590 |
| $0d_{5/2}$ | $0d_{5/2}$ | −0·503 | −0·604 | −0·584 | 0·655 |

In this paper only the 1s–0d shell wavefunctions are used, not only to enable us to compare the shell model and PHF spectroscopy but also because the spectroscopic amplitudes and reaction transition amplitudes given by the preceding formalism are most easily evaluated. Specifically, the low $J$ members of the ground state band ($0^+$, $2^+$, $4^+$) are determined by four nucleons (2 protons and 2 neutrons) in the deformed orbital ($k^\pi = \frac{1}{2}^+$) and, from a variational calculation (Braley and Ford 1969), this orbit is specified by expansion coefficients $C_{1/2,1/2} = -0{\cdot}5729$, $C_{3/2,1/2} = -0{\cdot}378$ and $C_{5/2,1/2} = 0{\cdot}7273$. With these coefficients the normalization factors obtained from equation (9) are $N_{000} = 2{\cdot}928$, $N_{200} = 1{\cdot}55$ and $N_{400} = 1{\cdot}737$. The PHF spectroscopic amplitudes then result from calculations using equation (14). These are compared with the spherical model (SM) and deformed shell model (DSM) values (I. Morrison, personal communication) in Table 1. The three model results are very similar, as can be expected from the severe basis restriction, and should lead to reaction results that are essentially identical, especially since the $B$(E2, 0 → 2) values determined from use of equation (19) match the experimental

number (Lamme and Boeker 1969) of $286\pm40\ e^2\ \mathrm{fm}^4$ when polarization charges of $0{\cdot}38\pm0{\cdot}06e$, $0{\cdot}32\pm0{\cdot}06e$ and $0{\cdot}32\pm0{\cdot}06e$ are used in the PHF, spherical shell and deformed shell model calculations respectively.

## Results

The differential cross sections and asymmetries for the inelastic scattering of 24·5 MeV protons from $^{20}$Ne were calculated in the antisymmetrized distorted wave approximation using the spectroscopic amplitudes given in Table 1. In addition the distorted waves were calculated using an optical model potential with parameter values (de Swiniarski *et al.* 1969, 1972) determined from a fit to elastic scattering data, and all single-particle bound states were described by wavefunctions of a harmonic oscillator having an oscillator energy of 11·22 MeV. For these analyses, a three-component reaction mechanism is appropriate (Geramb *et al.* 1975).

The first component of the reaction mechanism, the valence interaction component, yields all transition amplitudes in which scattering occurs via an explicit interaction between the projectile and any valence bound nucleon. Previous studies of natural parity transitions (Morrison *et al.* 1975) have shown that the effective interaction

$$V_{\mathrm{eff}}(01) = -25\exp(-0{\cdot}275\,r_{01}^2)\,\delta_{S0}\,\delta_{T1} - 47\exp(-0{\cdot}3373\,r_{01}^2)\,\delta_{S1}\,\delta_{T0} \qquad (20)$$

is an adequate description for this part of the reaction mechanism, and such was used in our calculations. The second component, which yields the core polarization amplitudes, is a direct scattering correction required whenever the target spectroscopy is seriously impaired because of restrictions on the basis states. This is most essential in analyses of natural parity transitions since the associated spectroscopy usually must use effective charges to match the observed $B(\mathrm{E}\lambda)$ values. From these effective charges, and using a collective model representation of this reaction mechanism, the core polarization transition amplitudes can be determined (Morrison *et al.* 1975). The third and last component of the reaction mechanism is that by which giant resonances of the target act as doorway states mediating the inelastic scattering. The associated scattering amplitudes result from the capture of the projectile into a bound orbit by exciting high-lying particle–hole states in the target, which subsequently decay by nucleon emission (Geramb *et al.* 1975). Such transition amplitudes are appreciable only if the capture and decay widths are large, and this is the case if the intermediate particle–hole states constitute giant resonances. Numerical evaluation of these resonance amplitudes is facilitated by use of a collective model representation (Geramb *et al.*) in which the gross properties of these giant resonance doorway states (multipolarity $L$ and isospin $T$) are carried in energy-dependent complex coupling coefficients $y_L^T(\bar{Q})$, the energy $\bar{Q}$ being the sum of the projectile kinetic energy and the binding energy of the capture state. In terms of properties of the resonances, these coupling coefficients can be expressed as (Geramb *et al.*)

$$y_L^T(\bar{Q}) = -(\beta_L^T)^2\,(2L+1)^{-1}\,(\bar{Q}-\hbar\omega_L+\tfrac{1}{2}\mathrm{i}\Gamma_L)^{-1}\,. \qquad (21)$$

Obviously, data from projectile energy variation, and therefore of $\bar{Q}$, are required in small energy steps to unfold resonance structure details from the $y_L^T(\bar{Q})$. Consequently, as data on the reaction analysed herein are sparse (in energy), only an indication of the resonance effects can be obtained.

The results of our calculations are compared with the experimental data of de Swiniarski *et al.* (1969, 1972) in Figs 1 and 2. In Figs 1*a* and 2*a* respectively, the differential cross section from inelastic scattering to the $2^+$ (1·63 MeV) and to the $4^+$ (4·25 MeV) state are given, and in Figs 1*b* and 2*b* are given the corresponding asymmetries. In all of the results shown, the PHF states were used. The valence results (short-dash curves) were obtained from fully antisymmetrized calculations omitting any core polarization correction and any resonance contribution. When the valence amplitudes are supplemented by a core polarization correction, predictions depicted by the long-dash curves result. The solid curves then are the results of adding resonance contributions to the valence and core polarization amplitudes.

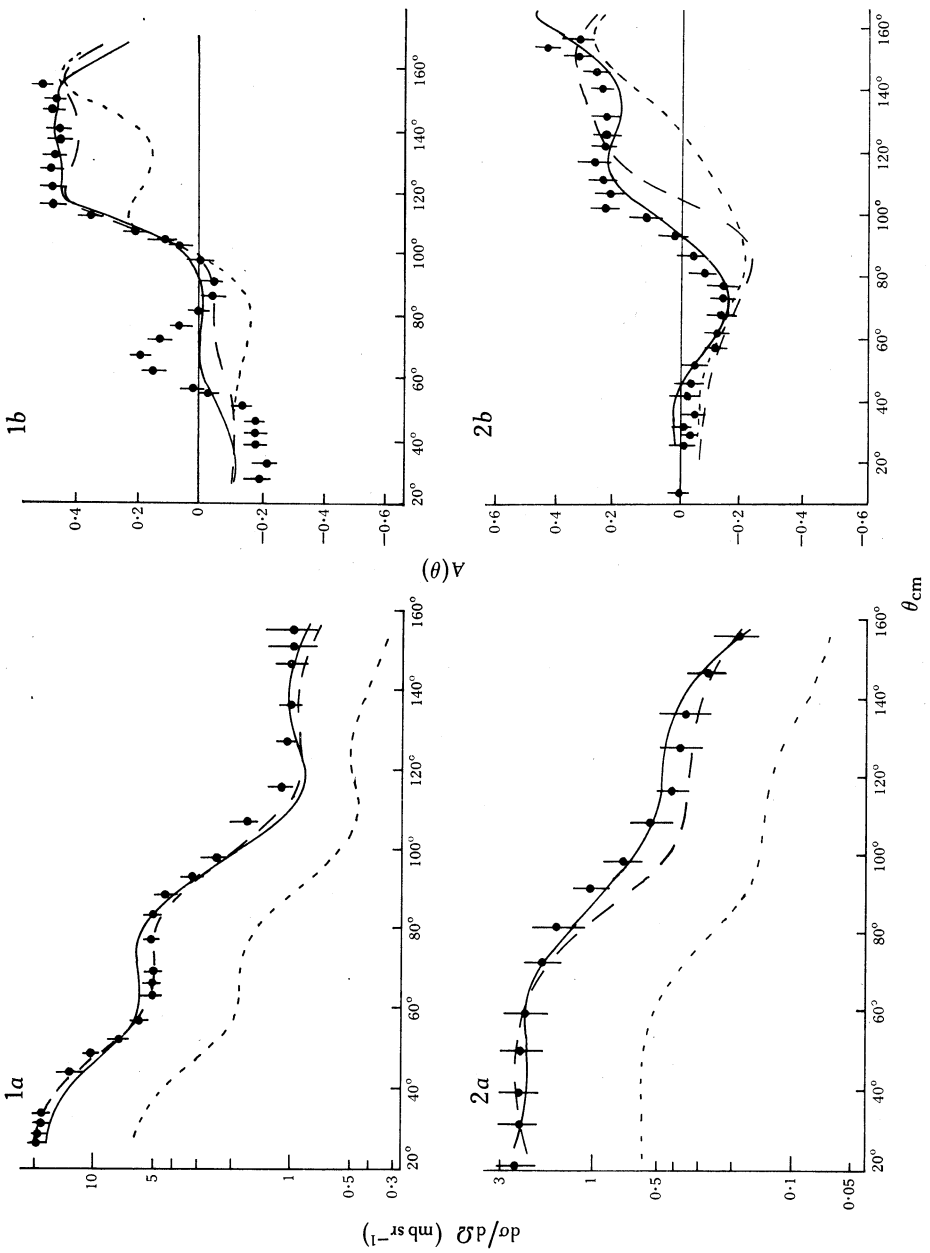
The valence results for differential cross sections are a factor of four smaller than and slightly different in structure from the data, as is shown in Figs 1*a* and 2*a* for the $2^+$ and $4^+$ transitions respectively. These calculated results are significantly larger than the equivalent ones reported earlier (Braley and Ford 1969); the discrepancy being due to the inclusion in our calculations of exchange amplitudes. The extent to which exchange amplitudes influence these specific reactions has been studied previously (Satchler 1973).

As stated above, the valence results were obtained by projecting states of good angular momentum from a minimal-energy HF state in an s–d shell basis expansion. Calculations were also made using both a spherical and a deformed shell model spectroscopy, i.e. using the other spectroscopic amplitudes given in Table 1. The resulting differential cross sections are as given by the short-dash curves of Figs 1*a* and 2*a* to within a few per cent. Small differences do occur in the predictions of the asymmetries but they are not displayed since they are swamped by the core polarization correction required when any of the three spectroscopy models are used.

In making the core polarization correction to the valence transition amplitudes, the collective model in which the full Thomas spin–orbit (Morrison *et al.* 1975) potential is deformed was used and the overall strength of the core polarization amplitudes was chosen to give a fit to the forward-angle $2^+$ differential cross sections. The long-dash curves in all four figures are the results. The required core polarization strength corresponds to an effective charge of $1{\cdot}46\,e$ which compares well with that required by the spectroscopy to match the observed $B$(E2), namely, $1{\cdot}38 \pm 0{\cdot}06\,e$. The predicted differential cross sections agree quite well with the data save that the $4^+$ transition prediction is low in the region of 80°–120°. The predicted asymmetries are less satisfactory, showing a major discrepancy with the $2^+$ data around 70° and only reproducing the general trend of the $4^+$ data. Without the full Thomas deformation, the forward angle asymmetry predictions are even less satisfactory.

Some of the discrepancies described above may be due to resonance processes since, with a projectile energy of 24·5 MeV, the corresponding $\bar{Q}$ in equation (21) is about 30 MeV—an excitation energy in $^{20}$Ne that should be appropriate for E2 or E3 resonances (Geramb *et al.* 1975). Hence, with the valence and core polarization correction amplitudes fixed, E2 and E3 resonance amplitudes were included and their complex coupling coefficients adjusted to best fit the data. The full curves in Figs 1 and 2 are the results which were obtained using coupling coefficients having magnitudes [phases] of: for the $2^+$ transition

$$y_2(\bar{Q}) = 0{\cdot}0361\ \text{MeV}^{-1}\ \ [17°], \qquad y_3(\bar{Q}) = 0{\cdot}0355\ \text{MeV}^{-1}\ \ [52°];$$




**Figs 1 and 2.** Angular dependence of (*a*) differential cross sections $d\sigma/d\Omega$ and (*b*) asymmetries $A$ for the inelastic scattering of 24·5 MeV protons leading to the $2^+$ (1·63 MeV) state (Fig. 1) and to the $4^+$ (4·25 MeV) state (Fig. 2). Individual curves are identified in the text.

for the $4^+$ transition

$$y_2(\bar{Q}) = 0{\cdot}0336\ \text{MeV}^{-1}\ \ [17°], \qquad y_3(\bar{Q}) = 0{\cdot}0897\ \text{MeV}^{-1}\ \ [52°].$$

As seen in Figs 1*a* and 2*a*, the $2^+$ cross section remains well fitted, while the discrepancy in the $4^+$ cross section analysis has been removed, resulting in an excellent fit to the data. The fit to the $2^+$ asymmetry data has been somewhat improved, albeit that the marked 70° discrepancy remains. However, it is the change in the analyses of the $4^+$ asymmetry that is most striking. Inclusion of the resonance effects not only gave an excellent fit to the cross section (Fig. 2*a*) but also to the asymmetry (Fig. 2*b*).

## Conclusions

Nuclear spectroscopy from the PHF theory and from the spherical or deformed shell model studies can be tested by analyses of inelastic scattering. When comparable bases are used (the s–d shell in the studies reported herein), the analyses with all three spectroscopic models are equivalent and consistent with the effective charges required when those same spectroscopic models are used to match observed $B$(E2) values. However, as the PHF calculations can be made (for the ground state band in $N = Z$ nuclei) using a very large expansion basis, and thus requiring no effective charges, analysis of inelastic scattering data will be an extremely good test of details of this spectroscopy, especially if the very sensitive asymmetry data can be obtained. Nevertheless, to do so requires a proper reaction mechanism description and, as indicated by our analyses, this must include two-step processes in which giant resonances of the target act as doorway states.

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## Appendix

The 'projection' operators $P^J_{MK}$, when acting upon axial symmetric solutions of the HF equations, project out states of good quantum number $J$ and $K$ and then transmute $K$ into $M$. They can be defined by (Braley and Ford 1969; Corbett 1971; Irvine 1972)

$$P^J_{MK} = \sum_\alpha |JM\alpha\rangle\langle JK\alpha|, \tag{A1}$$

and thereby they satisfy the relations

$$P^J_{MK} P^{J'}_{K'M'} = P^J_{MM'} \delta_{JJ'} \delta_{KK'} \qquad \text{and} \qquad (P^J_{MK})^\dagger = P^J_{KM}. \tag{A2a, b}$$

Also these operators can be expressed in an integral representation, namely,

$$P^J_{MK} \Phi_K = \{(2J+1)/8\pi^2\} \int \mathrm{d}\Omega \, \mathscr{D}^J_{MK}(\Omega) \exp(-\mathrm{i}\boldsymbol{\Omega} \cdot \boldsymbol{J}/\hbar) \Phi_K \tag{A3}$$

which, for an axial symmetry constraint upon $\Phi_K$, reduces to

$$P^J_{MK} \Phi_K = (2J+1)\tfrac{1}{2} \int_0^\pi \mathrm{d}\beta \sin(\beta) \, d^J_{MK}(\beta) \exp(-\mathrm{i}\beta J_y) \Phi_K. \tag{A4}$$

Further, these operators can be partitioned (Irvine 1972) as

$$P^J_{MK} = \sum_{J_1 J_2 M_1 M_2 K_1 K_2} \langle J_1 J_2 M_1 M_2 | JM\rangle \langle J_1 J_2 K_1 K_2 | JK\rangle P^{J_2}_{M_2 K_2} P^{J_1}_{M_1 K_1} \tag{A5}$$

whence, acting upon an eigenfunction of angular momentum, e.g. a spherical basis single-particle state, we obtain

$$P^J_{MK}(1 \ldots A) \phi_{jm}(1) = \sum_{m'} \phi_{jm'}(1) \sum_{J'\mu'M'} \langle J' j \mu' m' | JM\rangle \langle J' j M' m | JK\rangle P^{J'}_{\mu' M'}(2 \ldots A). \tag{A6}$$

The normalization of the axially symmetric PHF states $N_{JMK}$ of equation (5) follow from the above properties as

$$\begin{aligned} (N_{JMK})^{-2} &= \langle \Phi_K | (P^J_{MK})^\dagger P^J_{MK} | \Phi_K \rangle \\ &= \langle \Phi_K | P^J_{KK} | \Phi_K \rangle \\ &= (2J+1)\tfrac{1}{2} \int_0^\pi \mathrm{d}\beta \sin(\beta) \, d^J_{KK}(\beta) \langle \Phi_K | \exp(-\mathrm{i}\beta J_y) | \Phi_K \rangle. \end{aligned} \tag{A7}$$

The quantity

$$\det[B(\beta)] = \langle \Phi_K | \exp(-\mathrm{i}\beta J_y) | \Phi_K \rangle \tag{A8}$$

is the determinant of an $A \times A$ matrix, the elements of which are

$$(B)_{k_2 k_1} = \langle \psi_{k_2} | \exp(-i\beta J_y) | \psi_{k_1} \rangle , \tag{A9}$$

where

$$\psi_K = \sum_j C_{jK} \phi_{jK} . \tag{A10}$$

Thus, as the $C_{jK}$ coefficients are real, the matrix elements are

$$(B)_{R_2 R_1} = \sum_j C_{jR_1} C_{jR_2} d^j_{R_2 R_1}(\beta) , \tag{A11}$$

with an implicit constraint on the charge of the particle.

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