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Ion Currents, Ion–Neutral Collisions and Plasma Transport Phenomena

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Abstract

It is pointed out that, in the commonly encountered plasma configuration of perpendicular electric and magnetic fields, the Pedersen current (in the $\boldsymbol{E}$ direction) is carried by the ions, the Hall current (in the $\boldsymbol{E} \times \boldsymbol{B}$ direction) is carried by both ions and electrons in nearly equal and mutually cancelling parts, and the Birkeland current (in the $\boldsymbol{B}$ direction) is carried by the electrons. One of the consequences of the ion current's existence is that the massive ions are accelerated directly by the fields and not, as in a metallic conductor, through momentum transfer from the electrons. Another is that in the presence of a third neutral component of a partially ionized plasma, again a commonly encountered situation, the electron component of the plasma may be in effect collisionless, the ion component collision-dominated, and the plasma resistivity may then be expected to differ markedly from the resistivity calculated on the basis of Coulomb collisions between electrons and ions. It is shown, for example, that the current structure of an ionizing shock may be adequately explained in terms of an ion-carried current subject to a resistivity, determined solely by ion–neutral collisions, of value two orders of magnitude greater than the Coulomb-collision resistivity. In this, as in the other similarly affected plasma transport phenomenon of diffusion, there is no anomaly nor is there need to invoke plasma instability or fluctuation theory for further explanation.

Introduction

It is frequently assumed, and sometimes stated, that in the plasma transport phenomenon of electrical conduction only the light mobile electrons need be considered and not the ponderous ions. This may be justified for a fully ionized plasma subject to an electric field alone, or to parallel $\boldsymbol{E}$ and $\boldsymbol{B}$ fields, and in which charge neutrality is preserved. In that case, the ion-carried current j_i is *smaller* than the electron-carried current j_e by the factor m_e/m_i. The statement cannot be justified where there is also a nonparallel magnetic field, the difference between the two cases arising because of the additional velocity-dependent Lorentz force in the equations of motion:

for ions

$$m_i \, \mathrm{d}\boldsymbol{v}_i/\mathrm{d}t = q_i(\boldsymbol{E} + \boldsymbol{v}_i \times \boldsymbol{B}), \tag{1a}$$

for electrons

$$m_e \, \mathrm{d}\boldsymbol{v}_e/\mathrm{d}t = q_e(\boldsymbol{E} + \boldsymbol{v}_e \times \boldsymbol{B}). \tag{1b}$$

Indeed, in the case of orthogonal $\boldsymbol{E}$ and $\boldsymbol{B}$ fields, a configuration common to many (if not all) of the plasma phenomena usually interpreted in terms of a continuum-fluid magnetohydrodynamic model (and often referred to in consequence as MHD phenomena), the switch-on of an E field slowly relative to an ion-cyclotron period gives rise to an ion-carried Pedersen current $j_i = (n_i q_i^2/m_i \omega_{ci}^2)\partial E/\partial t$, which is *greater*

than the electron-carried current by the factor m_i/m_e (Millar 1975*a*), where ω_{ci} is the ion-cyclotron frequency $(q_i B/m_i)$. For a more rapidly rising E field, changing, for example, as a step function from 0 to E_0, i.e. in a time short compared with the ion-cyclotron period, it is easy to show (see equation (2) below) that the Pedersen current will also be carried by ions and given by the expression $j_i \approx (n_i q_i^2/m_i)tE_0$ during an initial time interval $\omega_{ce}^{-1} < t < \omega_{ci}^{-1}$. Such collisionless ion currents are of importance in Alfvén wave phenomena (Millar 1975*a*, 1975*b*) and no doubt also in true collisionless plasma shocks. They are also of course the source of the $\boldsymbol{j}_i \times \boldsymbol{B}$ force which accelerates the massive part of the plasma, the ion component, to its final steady drift velocity (equal to that of the electron component) in the $\boldsymbol{E} \times \boldsymbol{B}$, or Hall, direction:

$$\boldsymbol{v}_H = \boldsymbol{E}_0 \times \boldsymbol{B}/B^2 = \rho_i^{-1} \int_0^t \boldsymbol{j}_i \times \boldsymbol{B}\, dt, \qquad \text{where} \qquad t \gg \omega_{ci}^{-1}.$$

Similarly, the drift kinetic energy of the ion component (the *only* form of energy acquired from the field in the slow switch-on collisionless case) is given by

$$\tfrac{1}{2}\rho_i v_H^2 = \int_0^t \boldsymbol{j}_i . \boldsymbol{E}\, dt, \qquad t \gg \omega_{ci}^{-1},$$

with a similar expression for the electron component:

$$\tfrac{1}{2}\rho_e v_H^2 = \int_0^t \boldsymbol{j}_e . \boldsymbol{E}\, dt, \qquad t \gg \omega_{ce}^{-1}.$$

It is this ion drift-velocity which makes the major contribution to, and imparts such limited physical significance as is possessed by, the centre of mass velocity $\boldsymbol{v} = (\rho_i \boldsymbol{v}_i + \rho_e \boldsymbol{v}_e)/(\rho_i + \rho_e)$ which appears in the single-fluid treatment of magnetohydrodynamics. The other variable in such a treatment, the total current in the plasma, which is usually taken as $\boldsymbol{j} = \boldsymbol{j}_i + \boldsymbol{j}_e = n_i q_i \boldsymbol{v}_i + n_e q_e \boldsymbol{v}_e$, replaces along with $\boldsymbol{v}$ the mathematically equivalent pair of variables $\boldsymbol{v}_i$ and $\boldsymbol{v}_e$ of the equations (1). Of course $\boldsymbol{j}$ does have greater significance, since it can be measured experimentally through its accompanying $\boldsymbol{B}$ field.

Because it is the ion component which in the collisionless case acquires from the crossed $\boldsymbol{E}$ and $\boldsymbol{B}$ fields momentum and energy very much greater than the electron component, it is of interest to look at the effects of collisions upon the ion current, and this is done here. In particular it is shown that, in the presence of a neutral component of the same chemical species as the ions, the very large cross section for the ion–neutral resonance charge transfer process may profoundly affect the plasma behaviour. Processes, which because of the high temperature and low density of the electrons may be considered to be collisionless if electron–ion Coulomb collisions only are considered, may be seen to be in fact collision-dominated, and it is then no longer necessary to invoke the supposed effects of plasma instabilities or fluctuations to explain otherwise anomalous observations of plasma transport phenomena.

Ion–Neutral Collisions

In a collisionless plasma, ions and electrons interact only through space-charge produced electric fields, which may arise from local departures from the condition

of charge neutrality or through magnetic fields induced by the total current, of density $\boldsymbol{j} = \boldsymbol{j}_0 + \boldsymbol{j}_i + \boldsymbol{j}_e$, within the plasma. The vacuum displacement current $\boldsymbol{j}_0 = \varepsilon_0 \, \partial \boldsymbol{E}/\partial t$ may be ignored at low frequencies as in an Alfvén wave, or may be dominant at high frequencies well above any of the characteristic frequencies of the plasma. The fields produced by the plasma itself, whether they be steady or fluctuating, as well as those imposed by an external source, must then be included in the fields $\boldsymbol{E}$ and $\boldsymbol{B}$ of the Lorentz equations (1) above. The rapid, random and highly localized fluctuations of the $\boldsymbol{E}$ field arising from particle–particle interactions are usually considered separately from the smoothed out $\boldsymbol{E}$ and $\boldsymbol{B}$ fields of the equations (1), and the effect of these fluctuating fields is incorporated in a separate collision term in that equation as in (1′a) below. These collisions provide a further interaction mechanism between ions and electrons and, if there is a third component present as in a partially ionized plasma, between all three components. A measure of the strength of this interaction is the collision frequency for momentum interchange between pairs of components, or its inverse, the collision time τ.

One of these collision processes which may dominate all others in a partially ionized plasma is that between an ion and a neutral atom of the same species—the resonance charge transfer process—and, as is justified for the particular experimental situations discussed below, a reasonable simplification for such a plasma in crossed electric and magnetic fields is to assume collisionless electrons and collision-dominated ions.

The resonance charge transfer process has several features which are noteworthy, particularly in view of the current interest in the possibility of the heating of fusion plasmas by injecting into them beams of high energy neutral particles and also in view of the diagnostic technique whereby the ion temperature T_i is deduced from measurements of the energies of neutral particles emitted by the plasma. Firstly, the cross section is large: e.g. for helium and hydrogen it exceeds the Coulomb cross section for ion–ion collisions for all ion energies $\gtrsim 10$ eV (Banks 1966; Duman and Smirnov 1974) up to fusion energies (Riviere 1971). Indeed, at a deuteron energy of 10 keV the charge-exchange cross section is a factor of $\gtrsim 10^5$ larger than the Coulomb ion–ion cross section. Secondly, the process may be regarded as one in which a fast ion passes a stationary neutral at relatively large impact parameter (in view of the foregoing), captures an electron from the neutral atom and becomes itself converted into a fast neutral. The target atom is left behind as a stationary ion. The process, on this extreme model, is therefore a pure momentum-transfer collision, like a head-on or zero impact-parameter collision between identical elastic spheres. However, the usual factor of 2 relating momentum-exchange and total collision frequency for collisions between such spheres is no longer appropriate. The collision is also relatively ineffective as far as ion–neutral heating is concerned, a point which was ignored by Bighel *et al.* (1973). On the other hand, Sherman (1974) has pointed out that a large difference may develop between 'temperatures' of the ions normal and parallel to the $\boldsymbol{B}$ field because of this feature of the ion–neutral collision process. Here one may note that it is the trochoidal trajectory of the ion between collisions which provides the randomness in direction, without which heating and temperature are questionable concepts. The ion component gaining momentum from the $\boldsymbol{E}$ and $\boldsymbol{B}$ fields between collisions imparts this momentum to the neutral component in these collisions. Without the effect of the $\boldsymbol{B}$ field on the ion's direction of motion,

the ion component would play a role akin to that of the piston in a normal gas compression: subsequent collisions generating transverse momentum are necessary to convert the directed kinetic energy acquired from the piston into random thermal energy. Lastly, from the plotted values of the charge-exchange cross section Q_{i0} as given by e.g. Riviere (1971), it may be deduced that, up to fusion energies, Q_{i0} decreases with ion velocity as $v_i^{-\nu}$, with $\nu \approx 0{\cdot}3$ rather than 4 as for a Coulomb collision. Nevertheless we adopt the assumption of a collision time $\tau_{i0} = 1/n_0\langle Q_{i0} v_i\rangle$ independent of v_i, without assuming unjustifiably (and integrating over) a Boltzmann velocity distribution characterized by a temperature T_i.

If now we consider a partially ionized plasma in a cylindrical geometry subject to steady fields $\boldsymbol{E} = (E_r, 0, 0)$ and $\boldsymbol{B} = (0, B_\theta, B_z)$, the mean displacement of the ion between collisions with neutrals is given by

$$\boldsymbol{\lambda}_{i0} = \int_0^\infty \exp(-t/\tau_{i0})\left(\int_0^t \boldsymbol{v}\,\mathrm{d}t\right)\mathrm{d}t/\tau_{i0}\,.$$

Here $\boldsymbol{v}$ is the solution of the Lorentz equation (1a) such that $\boldsymbol{v} = \boldsymbol{0}$ at $t = 0$ (Millar 1975*a*):

$$\boldsymbol{v}_i = \left((E/B)\sin(\omega_{ci}t),\quad -(EB_z/B^2)\{1-\cos(\omega_{ci}t)\},\quad (EB_\theta/B^2)\{1-\cos(\omega_{ci}t)\}\right), \tag{2}$$

so that the ion's mean free path between collisions is given by

$$\boldsymbol{\lambda}_{i0} = \frac{1}{\omega_{ci}}\left(\frac{E}{B}\frac{(\omega\tau)_i^2}{1+(\omega\tau)_i^2},\quad -\frac{EB_z}{B^2}\frac{(\omega\tau)_i^3}{1+(\omega\tau)_i^2},\quad \frac{EB_\theta}{B^2}\frac{(\omega\tau)_i^3}{1+(\omega\tau)_i^2}\right). \tag{3}$$

In this last result, the Hall parameter $\omega_{ci}\tau_{i0}$ has been written as $(\omega\tau)_i$. The well-known expression for the average drift velocity may be similarly (or directly by dividing equation (3) by τ_{i0}) deduced as

$$\bar{\boldsymbol{v}}_i = \left(\frac{E}{B}\frac{(\omega\tau)_i}{1+(\omega\tau)_i^2},\quad -\frac{EB_z}{B^2}\frac{(\omega\tau)_i^2}{1+(\omega\tau)_i^2},\quad \frac{EB_\theta}{B^2}\frac{(\omega\tau)_i^2}{1+(\omega\tau)_i^2}\right). \tag{4a}$$

This relation is precisely the same as is obtained by solving the steady state Lorentz equation (1a) for the ions with a Lorentz-model momentum-loss collision term $m_i\boldsymbol{v}_i/\tau_{i0}$ added to the right-hand side, that is, equation (1a) becomes

$$\mathrm{d}\boldsymbol{v}_i/\mathrm{d}t = 0 = \omega_{ci}(\boldsymbol{E}/B + \boldsymbol{v}_i \times \boldsymbol{B}/B) - \boldsymbol{v}_i/\tau_{i0}\,. \tag{1'a}$$

For the collisionless electrons, on the other hand, the steady state solution of equation (1b) is $\bar{\boldsymbol{v}}_e = \boldsymbol{E}\times\boldsymbol{B}/B^2$, or

$$\bar{\boldsymbol{v}}_e = (0,\quad -EB_z/B^2,\quad EB_\theta/B^2), \tag{4b}$$

so that the Pedersen current j_r is carried entirely by the ions and is given by

$$j_r = n_i q_i v_{ri} = \frac{n_i q_i}{B}\frac{(\omega\tau)_i}{1+(\omega\tau)_i^2}E_r = \frac{\sigma_0}{1+(\omega\tau)_i^2}E_r = \sigma_0\{E_r + (\boldsymbol{v}_i\times\boldsymbol{B})_r\}\,,$$

with $\sigma_0 = n_i q_i^2\tau_{i0}/m_i$ rather than the conductivity of a fully ionized plasma (Spitzer

1962) $\sigma_S = n_e q_e^2 \tau_e/m_e$. Note that we may write $\omega_{ci}\tau_{i0} = B\sigma_0/n_i q_i$ so that, while in the collision-dominated regime with $(\omega\tau)_i \ll 1$ we get $j_r = \sigma_0 E_r$, the familiar form of Ohm's law, in the more usual plasma situation with Hall parameter $(\omega\tau) \gg 1$ we get $j_r = (n_i q_i/B)^2(E_r/\sigma_0)$, so that j_r varies *inversely* as the conductivity σ_0 of the plasma, and as B^{-2}.

In this B dependence, the phenomenon of plasma electrical conduction is similar to that of other transport properties (e.g. diffusion and thermal conduction) in a direction transverse to the magnetic field, as predicted using a Lorentz model, the coefficients varying as B^0 for $(\omega\tau)^2 \ll 1$, as $\sim B^{-1}$ for $\omega\tau \approx 1$ and as B^{-2} for $(\omega\tau)^2 \gg 1$. Indeed the foregoing discussion may be readily extended to include these other transport properties by adding a second collision term $-\nabla p_i/\rho_i$ to the right-hand side of equation (1′a). If the ion partial-pressure gradient has a readial component only, so that $\nabla p_i = (\partial p_i/\partial r, 0, 0)$, then the expression (4a) is modified by replacing E_r where it occurs in that expression by $E_r - (n_i q_i)^{-1}\,\partial p_i/\partial r$. Introducing thermodynamics into an otherwise cold plasma model by putting $p_i = n_i kT_i$, one may then readily derive the Lorentz-model expression for the thermal conductivity

$$K_\perp = \frac{n_i k\varepsilon_i}{q_i B}\frac{(\omega\tau)_i}{1+(\omega\tau)_i^2},$$

with $\varepsilon_i = \frac{1}{2}kT_i$ the average random ion energy. Braginskii (1958) has given a fuller version involving higher-order terms in $(\omega\tau)_i$ arising from a Boltzmann-equation treatment. Braginskii's expression for $K_\perp$ is greater than this simpler one by a factor ~ 3 for $(\omega\tau)_i < 1$ and by a factor $\sim 1\cdot 5$ for $(\omega\tau)_i \gtrsim 1$. Similarly the diffusion coefficient may be derived:

$$D_\perp = \frac{kT_i}{q_i B}\frac{(\omega\tau)_i}{1+(\omega\tau)_i^2}.$$

This is reminiscent of the form of the Bohm diffusion coefficient $D_B = kT_e/16eB$ (Glasstone and Lovberg 1960) with which experimental observations of 'anomalous' diffusion are frequently compared. Indeed, if one considers that an experimental observation of a plasma containment time of α times the Bohm containment time may be caused by ion collisions then one may write $D_\perp = \alpha^{-1} D_B$ and hence derive the result

$$(\omega\tau)_i = 8\alpha(T_i/T_e)\left[1 \pm \{1-(T_e/8\alpha T_i)^2\}^{\frac{1}{2}}\right]. \tag{5}$$

For the original experiments (Guthrie and Wakerling 1949) which gave rise to the Bohm diffusion coefficient, the value of α was 1. More recently, the Princeton Stellarator observations, as reviewed by Young (1974), gave values of α in the range 3–5. The presence of neutrals in these last experiments, and at least some of the consequences of this presence, have been frequently noted (see e.g. Brown *et al.* 1969*a*, 1969*b*).

The paradoxical result that the Pedersen current is inversely proportional to the electrical conductivity σ, for the condition $\omega\tau \gg 1$, is qualitatively to be expected. Whatever the model for the electrical conduction process in a plasma one expects that in the collisionless limit of infinite conductivity no steady Pedersen current j_{ri} or j_{re} will flow. Only a transient current arising from a time-varying electric field is

possible. In this collisionless limit the steady azimuthal, or Hall, current $j_\theta = j_{\theta i} + j_{\theta e}$ will also be zero—almost; but now because $j_{\theta i}$ and $j_{\theta e}$ are almost equal in magnitude and of opposite sign. The equality is exact in a cartesian geometry but not in a cylindrical rotating plasma since for rotational stability

$$j_{\theta i} B_z = -n_i m_i \bar{v}_{\theta i}^2/r - n_i q_i E_r \qquad \text{and} \qquad j_{\theta e} B_z = -n_e m_e \bar{v}_{\theta e}^2/r - n_e q_e E_r$$

hold on the macroscopic scale (Brennan *et al.* 1963). On the microscopic scale it can be readily shown that the difference between $\bar{v}_{\theta i}$ and $\bar{v}_{\theta e}$ occurs because of the r^{-1} dependence of the electric field: ions at radius r have been accelerated in a slightly higher (lower) magnitude E field for E_r positive (negative) than have the electrons at that same radius. The two explanations lead to the same result, namely,

$$\bar{v}_{\theta i} \approx -(E/B)(1 + E/B\omega_{ci} r), \qquad \bar{v}_{\theta e} \approx -(E/B)(1 + E/B\omega_{ce} r),$$

where the correction terms $E/B\omega_c r$ have magnitude $\ll 1$. The resulting diamagnetic Hall current j_θ (which will be reduced and may even be reversed if ion–neutral collisions occur) should give rise to a wave magnetic field b_z in a collisionless torsional Alfvén wave of twice the frequency of the wave field b_θ. Brennan *et al.* (1975) have made a similar prediction.

Ionizing Shock

The present model of ions affected by ion–neutral collisions and of collisionless electrons is now applied to the case of an ionizing shock, such as has been studied extensively in the Wills Plasma Physics laboratory (Cross *et al.* 1969; Bighel and Watson-Munro 1972; Bighel *et al.* 1974). In particular the results of this model are compared with experimental observations (Bighel *et al.* 1973) for a shock propagating into neutral helium at a pressure of 0·1 torr and parallel to a B_z field of 1 T. The

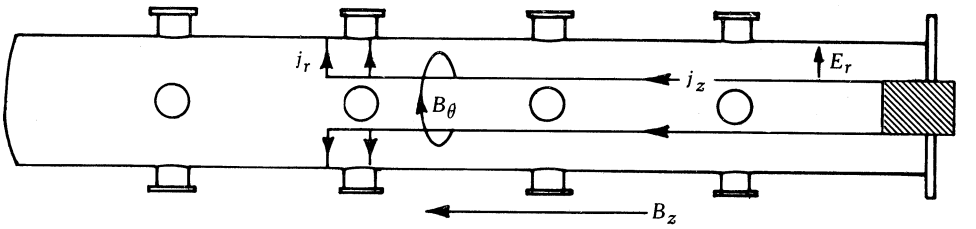

Fig. 1. Schematic diagram of SUPPER II showing an ionizing shock propagating from right to left into a neutral gas, leaving behind a fully ionized rotating plasma.

shocks were studied in SUPPER II (Fig. 1), a cylindrical stainless steel vessel of diameter 0·21 m and length 1·7 m in a uniform B_z field parallel to the vessel axis. A capacitor bank is discharged between a central end-electrode, of length and diameter 75 mm, and the concentric cylindrical vessel. The discharge produces a radial breakdown current through the neutral helium gas at the electrode end of the machine and initiates an ionizing shock which travels along the vessel with a well-defined velocity, converting the neutral helium ahead of the shock into a fully ionized rotating plasma behind it, as indicated in Fig. 1. Behind the shock there exists a switched-on B_θ field produced by the plasma current j_z feeding the shock front, and an electric

field E_r arising from the potential difference between the concentric electrodes. Experimental measurements using 90° scattered ruby laser light have been made of n_e and T_e at a radial position of 70 mm as the shock front sweeps past the observation port 0·8 m along the machine. Probe measurements at that same radius have been made of the E_r field and of the B_θ field. From these last has been deduced the experimental distribution of current j_r within the current front, as shown by the crosses in Fig. 2. Also shown by the full curve in Fig. 2 is the current

$$j_r = n_i e \frac{E_r}{B} \frac{(\omega\tau)_i}{1+(\omega\tau)_i^2},$$

as predicted by the present model. In this expression, the ion density n_i is assumed to equal the measured values of n_e approximated by the dashed curve in Fig. 2.

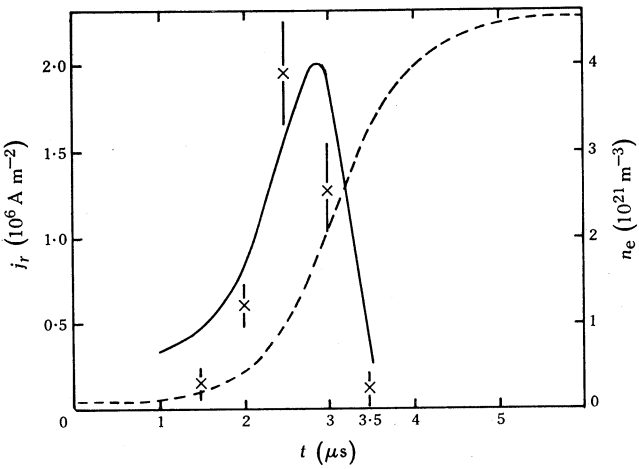

Fig. 2. Predicted radial current density j_r (full curve) as a function of time t (initial time arbitrary) assuming ion–neutral collisions only and $n_i = n_e$, with n_e the measured electron density approximated to by the dashed curve. The experimental points (Bighel *et al.* 1973) are values of j_r deduced from magnetic-probe measurements for a shock propagating from right to left at $1\cdot1\times10^5$ m s^{-1} into neutral helium gas at 0·1 torr, with $B_z = 1$ T, switched-on $B_\theta = 0\cdot3$ T and $E_r = 3\times10^4$ V m^{-1}.

Table 1. Ion and electron Hall parameters and conductivities

Quantity	Value at t =	1	2	3	3·5	4 μs
$(\omega\tau)_i$		1	2	3·3	100	
σ_0 (Ω^{-1} m^{-1})		16	100	800	$4\cdot8\times10^4$	
σ_s (Ω^{-1} m^{-1})		$1\cdot3\times10^4$	$3\cdot2\times10^4$	$7\cdot9\times10^4$		12×10^4
$(\omega\tau)_e$		550	530	220		190

We also have $\tau_{i0}^{-1} = n_0\langle Qv\rangle_{i0}$, with n_0 as a function of time given by the relation $n_0 = N_0 - n_i$, where $N_0 = 3\cdot3\times10^{21}$ m^{-3} is the initial helium filling density. Also $\langle Qv\rangle_{i0}$ has been taken as 4×10^{-15} m^3 s^{-1} to normalize the predicted peak current of 2×10^6 A m^{-2} to the observed peak current. This is equal to the value which may be deduced from the data of Banks (1966) for 10 eV helium ions. The ion temperature

T_i was measured from Doppler broadening of emission lines as having a maximum value of ~ 40 eV. The agreement with the experimental j_r distribution is reasonable, given the simplicity of the model and in particular the neglect of the acceleration of the neutral component and of ion–electron and ion–ion collisions, this last a questionable contribution in the light of the conclusion of Simon (1955*b*) and of Longmire and Rosenbluth (1956).

The physics of the process depicted in Fig. 2 is thus as follows. Ahead of the ionizing current front (on its left in Fig. 2) no detectable radial current flows despite the existence of an electric field E_r there. The large mobility of the electrons in the Birkeland direction ensures that the B-field lines are equipotentials, as in the mechanism proposed by Simon (1955*a*)—in that case for shorting out any ambipolar E_r field which might otherwise be expected in the volume of plasma between plane conducting end-plates in a linear discharge. In the present case the E_r field imposed between electrodes at one end of the cylinder (Fig. 1) is by the same mechanism established throughout the length of the cylinder. Ahead of the front the radial conduction is low because the density n_i of charge carriers in the neutral gas ahead of the front is low, even though for these few ions $(\omega\tau)_i$ is low (~ 1). Also, behind the current front little radial current flows because of the high conductivity of the plasma. We have $1 \ll -\omega_{ce}\tau_{ei}$ ($= \omega_{ci}\tau_{ie}$, under conditions of questionable validity) and $\omega_{ci}\tau_{i0} \to \infty$ because $n_0 \to 0$. Only within the ionizing current front, where $\omega_{ci}\tau_{i0} \approx 1$ and an appreciable n_i is produced, does a measurable current flow. That current is carried by ions and arises because of ion–neutral collisions.

In Table 1 are given, for the times indicated on the horizontal scale of Fig. 2, the values of $\omega_{ci}\tau_{i0}$ and of the ion–neutral conductivity σ_0. For comparison is shown the corresponding value at these times of the Spitzer conductivity taken as $\sigma_S = T_e^{3/2}/65\cdot 3 \ln \Lambda\ \Omega^{-1}\,\mathrm{m}^{-1}$, with $\ln \Lambda = 10$ and T_e the experimentally measured electron temperature as given by Bighel *et al.* (1973). From these values the effective Hall parameter for electrons has been obtained $\omega_{ce}\tau_e = B\sigma_S/n_e e$. It will be observed that we have $\omega_{ce}\tau_e \gg 1$ whereas $\omega_{ci}\tau_{i0} \approx 1$, thereby confirming the validity of the model of collisionless electrons and collision-dominated ions. (Compare expressions (4a) and (4b) above.) It will also be noted that the resistivity σ_0^{-1} predicted by the ion–neutral collisional model is considerably larger, by a factor of 100 at maximum current density, than the electron–ion Spitzer resistivity σ_S^{-1}.

Discussion

Ionizing Shocks

A three-fluid model involving Cowling-type interaction terms, but assuming with Cowling (1956) the elastic-collision relation between momentum exchange and total cross sections, has been applied by Bighel *et al.* (1973) to account satisfactorily for the experimentally observed heating of ions and neutrals, and by Cramer (1975) to account for shock structure under conditions where $-\omega_{ce}\tau_{ei} \gg 1$ did not hold and where a constant ionization density could be assumed. Ohmic heating of ions, but arising because of collisions with impurity ions, had been proposed earlier by Ware and Wesson (1961). Lovberg (1963) has already noted the dominant role played by the positive-ion displacement current in a collisionless ionizing shock and Millar (1975*a*) has drawn attention to this phenomenon in crossed E and B fields as an ion-energizing process with possible fusion application. It may be pointed out that in this

last phenomenon it is the randomness in time of the ionizing events which ensures the random phase and the consequent possibility of ion–ion collisions (Coulomb or fusion) interrupting the subsequent gyrational motion of the ions in the drifting plasma. Collective motions of ions, on the other hand, such as occur in Alfvén waves and such as may be predicted by plasma instability theory do not imply such interparticle collisions. As Buneman (1958) pointed out, coarse-grained turbulence (rather than microrandomness of particle motion) does not constitute true thermal motion. Nor, it may be added, does it imply as a consequence the ion–ion collisions necessary for heating or for nuclear fusion to occur.

The experiment of Bighel *et al.* (1973) discussed above has been referred to as an ionizing shock or, alternatively, as an ionizing current front. There is no piston driving a clearly separated slug of hot gas ahead of it in the present model; however, the front does travel with supersonic velocity into the neutral cold gas ahead. The current is observed to rise in a time of the order of 1 μs over a distance of 0·1 m (the shock velocity is $1\cdot1\times10^5\ \mathrm{m\,s^{-1}}$). This is much longer than the mean free path for neutral helium atoms at this density ($\sim$3 mm) or λ_{zi0} ($\sim$1 mm) as given by the expression (3) above for the experimental conditions: $E_r = +3\times10^4\ \mathrm{V\,m^{-1}}$, $B_z = 1$ T, $B_\theta = 0\cdot3$ T and $(\omega\tau)_{i0} = 3$. The scale length is not then determined by either of these mean free paths but rather by the following more complex sequence of processes: ohmic ion heating, energy transfer from ions to electrons as well as direct ohmic heating of electrons, and subsequent ionization of neutrals by these electrons as proposed by Bighel *et al.* The word shock may therefore be queried. In the case of a shock propagating into a fully ionized plasma, on the other hand, one might expect the scale length to be determined by the relation (2), or by (3) and (4), taking into account the fact that B_θ is then no longer a minor perturbation on the steady field B_z ($B^2 = B_\theta^2 + B_z^2$) and that B_θ is itself switched on by the current j_r flowing in the shock. In the collisionless case it would appear that such a nonlinear purely electromagnetic model has not as yet been developed. Most, but not all (see Cairns and Sherwell 1973), treatments of such collisionless shocks invoke anomalous transport properties ascribed to the effects of turbulence or instabilities in order to account for the experimentally observed shock width, which is too small to be explained in terms of Coulomb collisions. More fundamentally it may be argued that some such pseudocollisional effects must be invoked if the concepts of thermodynamic equilibrium are to be retained (such as temperature, adiabatic compression involving the ratio of the specific heats γ), as they have been in most models so far considered. It should be noted that in the present ion–neutral collisional model the concept of temperature has not proved necessary and the model therefore involves no thermodynamics. The ion velocity between collisions, of order E/B, is the important velocity rather than the thermal velocity. The usual assumption, valid for nonrunaway electrons as current carriers, that the field-induced drift velocity is always much smaller than the thermal velocity is neither necessary nor true in this case.

Lastly, in connection with this experiment, it may be noted that the outward radial flux of charge-carrying ions, together with an accompanying flux of neutrals, will lead to the build-up of a radial pressure gradient of both components towards the outer wall of the vessel, as in the plasma centrifuge (James and Simpson 1976). This also has been ignored in the present treatment.

Anomalous Transport Phenomena

The contribution of ion–neutral collisions to anomalous resistivity or diffusion (anomalous, that is, when compared with prediction based on a consideration of electron–ion collisions alone) appears not to have been pursued further since Simon (1955*a*, 1959) pointed out, using admittedly somewhat uncertain a cross section, that this process together with the shorting-out of the ambipolar field E_r satisfactorily accounted for the magnitude of the originally observed Bohm diffusion rate. Indeed for these experimental conditions (argon gas, 10% ionized at pressure 1 mTorr, $B_z \approx 0{\cdot}4$ T and $T_e \approx 2$ eV) and taking the argon ion–neutral cross section as given by Duman and Smirnov (1974) one may readily deduce that $\omega_{ci}\tau_{i0} = 75$, 30 or 12 for ion energies respectively of 0·1, 1 and 10 eV, leading to values of $D_\perp$ of 3×10^{-3}, 0·1 and 2 $m^2\,s^{-1}$. This is to be compared with the observed (Bohm) value of 0·3 $m^2\,s^{-1}$ which, according to Glasstone and Lovberg (1960), is larger by a factor of 600 than the Coulomb-collision value. If in these experiments T_i was equal to T_e (according to equation (5) above $(\omega\tau)_i = 16$) then it will be seen that the ion–neutral collision estimate is in good agreement with observation. If, on the other hand, T_i was appreciably smaller than T_e in these experiments then it could be argued that the additional effects of $\partial T_i/\partial r$-driven diffusion would suffice to reconcile the ion–neutral collisional model with experimental observation. As indicated above the general expression for the radial diffusion velocity is

$$v_r = B^{-1}\{E_r - (n_i q_i)^{-1}\,\partial p_i/\partial r\}(\omega\tau)_i/\{1+(\omega\tau)_i^2\},$$

with $p_i = n_i k T_i$. The field E_r may of course be imposed externally upon the plasma or it may be the ambipolar electric field shorted out by the Simon (1955*a*) mechanism.

Similarly some at least of the experimental observations of anomalous resistivity may be explained, as in the ionizing shock experiment discussed here, by Pedersen ion currents affected by ion–neutral resistivity. The 8 m long θ-pinch of Bodin *et al.* (1969*a*, 1969*b*), which showed that Coulomb-collision diffusion satisfactorily accounted for the behaviour of the hot current-free fully ionized plasma during most of its life, also showed that during the first 2 μs of the θ-pinch it did not, and 'anomalous' values of diffusion and resistivity were required to fit the experimental data. It is of course during this initial time-interval that the neutral deuterium density would change from an initial value at $t = 0$, estimated at less than 50% of the filling density, to 0% at $t \approx 2\,\mu$s. If we assume that at 1 μs after switch-on (i.e. half-way through this anomalous initial period) the neutral density was 3×10^{20} m^{-3} (25% of the initial filling density) and that $B_z(\sim 2{\cdot}5\sin(2\pi t/2\times10^{-5})\,\mathrm{T})$ was 0·8 T (and hence $E_\theta = -\frac{1}{2}r\,\partial B_z/\partial t \approx r.3{\cdot}7\times10^5$ V m^{-1}) then we have $\omega_{ci}\tau_{i0} \approx 18$, 7 and 3 for deuteron energies of 2, 20 and 200 eV respectively. On the other hand, if we take $T_e \approx 50$ eV at 1 μs (about half of the value reached at 2 μs) and $n_e = 1\times10^{21}$ (75% of the initial filling density) then we obtain $\omega_{ce}\tau_e \approx 3300$. The ratio of the ion–neutral to the electron–ion resistivity will then be $(\omega\tau)_e/(\omega\tau)_i \approx 200$, 500 or 1000 for the above deuteron energies. As in the ionizing shock considered above, the Pedersen current, j_θ in this geometry, is again carried entirely by the ions. The Hall current $j_r \approx (\omega\tau)_i^{-1} j_\theta$ arises because ions and electrons are drifting inward with radial velocities $[(\omega\tau)_i^2/\{1+(\omega\tau)_i^2\}]E_\theta/B_z$ and $\sim E_\theta/B_z$ respectively. The well-known

effects of the presence of a static reverse-bias B_z field upon the thickness of a θ-pinch produced shock, and also the existence of azimuthal motion for the ion component and hence for the plasma as a whole are also, qualitatively at least, readily understood on the above model.

In a truly collisionless situation, on the other hand, of collisionless ions as well as electrons, the Pedersen ion current may still be expected to play a dominant role with pronounced effect upon the relationship between E, B and j, a relationship which, as indicated above, should not be expected to conform to that of a linear Ohm's law type.

Even in the case of the Birkeland electron-carried current it has previously been noted (Millar 1974) that the self-field B_θ produced by the current j_z will have a pronounced effect upon the relation between j_z and E_z when $(B_\theta/B_z)^2 \ll 1$ does *not* hold, leading to effective filamentation of the current, quite apart from any pinch-effect or minimum-energy considerations (Taylor 1974). Such filaments, contorted into helical paths, have been noted by Millar and Watson-Munro (1966) in ionizing shock experiments of the type discussed here. Such a current geometry will also explain the reversal of the B_z field as observed for example by Butt *et al.* (1975). Rotation or more irregular motion of such a system of current filaments will lead to fluctuations of probe-measured values of $\boldsymbol{B}$, of $\partial B/\partial t$ and hence of $\boldsymbol{E}$ in outer regions of the plasma. Such turbulence is a common concomitant to observations of anomalous resistivity (Hirose *et al.* 1970) and such turbulent E and B fields will impart drift velocity, momentum and energy to the outer regions of the plasma. The plasma will then offer an impedance to the external driving circuit greater than may be explained by electron–ion resistivity alone, even assuming the geometry of the current paths to be correctly taken into account when deducing this resistivity from measured values of E_z and of the total plasma current I_z.

Electric Field and Ion Current

Finally, this discussion ends with a plea that greater consideration be given to the importance of the electric field as well as the ion current in MHD phenomena. ($\boldsymbol{E}$ is usually the first variable to be eliminated from the set of MHD equations.) It may of course be argued that since both the $\boldsymbol{E}$ field and $\boldsymbol{j}_i$ can be transformed away by choosing as reference frame one which moves with the centre of mass velocity, neither is important. Such a view obscures the fact that it is in the laboratory frame that the experimentalist switches on his electric field, either directly between electrodes, or indirectly through a switched-on time-varying magnetic field. In MHD phenomena (such as θ and z pinches, and normal and transverse shocks) this $\boldsymbol{E}$ field is associated with a perpendicular $\boldsymbol{B}$ field, so that it is the ions, not the electrons, which are directly energized by the switch-on process. In this, fortunately for the prospects of achieving controlled nuclear fusion, a plasma differs from a metal conductor, liquid or solid.

Conclusions

The point has been made that in perpendicular $\boldsymbol{E}$ and $\boldsymbol{B}$ fields the Pedersen current is carried by ions. In a partially ionized plasma, because of ion–neutral collisions both the plasma resistivity and the diffusion coefficient transverse to the magnetic field may then be considerably larger than is predicted on the assumption of

electron–ion collisions alone. Satisfactory agreement has been obtained between a model based on this concept and experimental observations of anomalous diffusion and resistivity.

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