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Effect of Voigt–Palacios–Gordon Transformations on Thermodynamic Quantities

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Abstract

The effect of transformations of Voigt–Palacios–Gordon type on thermodynamic quantities is considered. It is found that, while most quantities behave as they do under a Lorentz transformation, the pressure is, surprisingly, not invariant.

The logical character of special relativity and its ever-expanding basis for experimental verification has fostered a growing confidence in the validity of this theory. Soon after its proposal Planck (1907) and Einstein (1907) applied the theory to the thermodynamics of moving systems, and thus through Lorentz transformations derived expressions relating the experimental findings of an observer stationary with respect to a particular thermodynamic system to those of another observer moving with respect to that system. Planck used a variational principle while Einstein showed that the transformation formulae could be derived directly and the variational principle followed from them. In recent years there has been considerable controversial discussion concerning the transformation of quantities like the temperature T and change in heat content $\mathrm{d}Q$. No one proposal has yet found universal acceptance and the physical significance of the problem remains in some doubt.

Our aim in this note is to follow the earlier approach adopted by Tolman (1934) and to analyse the problem with respect to asymmetric linear transformations of the type considered by Voigt (1887), Palacios (1960) and Gordon (1962), referred to as VPG transformations. Consider a frame K' which is moving uniformly with velocity u with respect to an inertial frame K, the motion being along their common x axis. With the primed quantities referring to the K' system, the VPG transformations are

$$x' = bk(x-ut), \qquad y' = ky, \qquad z' = kz, \qquad t' = bk(t-ux/c^2), \tag{1}$$

where

$$k^{-1} = (1-u^2/c^2)^{n+\frac{1}{2}}, \qquad b = (1-u^2/c^2)^{-\frac{1}{2}}, \qquad -\tfrac{1}{2} \leqslant n < 0.$$

The inverse transformations of (1) are

$$x = bk'(x'+ut'), \qquad y = k'y', \qquad z = k'z', \qquad t = bk'(t'+ux'/c^2), \tag{2}$$

with $kk' = 1$. These transformations, which admit the transversal contraction of length, do not form a group but tend to the Lorentz formulae as n approaches $-\frac{1}{2}$.

Podlaha (1974) has established the invariance of Maxwell's equations with respect to VPG transformations. It is interesting to note that the Einstein transformation formulae for velocities are also valid for the nonrelativistic theories of Palacios and Gordon.

Consider now the transformation of thermodynamic quantities from the frames K' to K under the VPG formulae, i.e. the findings of an observer with respect to whom the thermodynamic system is in motion. The main results of the calculations may be summarized as follows.

(i) *Volume V.* From the contraction of length we obtain

$$V = \delta x\,\delta y\,\delta z = V'/bk^3\,. \tag{3}$$

(ii) *Momentum* $\boldsymbol{G}$. Since it is found that the VPG transformations for mass m and velocity $\boldsymbol{v}$ do not involve the factor k, the formulae for momentum components are the same as under a Lorentz transformation. Thus we have, with E' the total energy of the system in K',

$$G_x = b(G'_x + uE'/c^2)\,, \qquad G_y = G'_y\,, \qquad G_z = G'_z\,. \tag{4}$$

(iii) *Force f.* We have $f_x = \mathrm{d}G_x/\mathrm{d}t = (\mathrm{d}G_x/\mathrm{d}t')/(\mathrm{d}t/\mathrm{d}t')$. Since the scalar product of the vectors $\boldsymbol{f}'.\boldsymbol{v}'$ is equal to $\mathrm{d}E'/\mathrm{d}t'$, from the formulae (2) and the expression for G_x in equations (4) we find $f_x = kf'_x$. Similarly we can obtain the other components. The final result is

$$f_x = kf'_x\,, \qquad f_y = (k/b)f'_y\,, \qquad f_z = (k/b)f'_z\,. \tag{5}$$

(iv) *Pressure* $\boldsymbol{P}$. With $\boldsymbol{P}$ defined as the perpendicular force per unit area A, that is, $P_x = f_x/A_x$, from equations (5) and the results $A_x = A'_x/k^2$, $A_y = A'_y/bk^2$ and $A_z = A'_z/bk^2$ we have

$$P_x = k^3P'_x\,, \quad P_y = k^3P'_y\,, \quad P_z = k^3P'_z \qquad \text{or} \qquad \boldsymbol{P} = k^3\boldsymbol{P}'\,. \tag{6}$$

(v) *Energy E.* Following Tolman (1934), from the expression

$$\mathrm{d}E/\mathrm{d}t = \boldsymbol{f}.\boldsymbol{u} - P\,\mathrm{d}V/\mathrm{d}t\,,$$

where the external force $\boldsymbol{f}$ is given by

$$\boldsymbol{f} = \mathrm{d}\boldsymbol{G}/\mathrm{d}t = \mathrm{d}\{(E+PV)\boldsymbol{u}/c^2\}/\mathrm{d}t\,,$$

we obtain the result

$$E+PV = b(E'+P'V')\,,$$

or, since $PV = P'V'/b$,

$$E = b(E' + P'V'u^2/c^2)\,. \tag{7}$$

(vi) *Work W.* The work done in producing a change in the internal state of the system (keeping the velocity constant) is given by

$$\mathrm{d}W = P\,\mathrm{d}V - \boldsymbol{u}.\mathrm{d}\boldsymbol{G}\,.$$

From equation (7) we then obtain

$$\mathrm{d}W = b^{-1}\,\mathrm{d}W' - (bu^2/c^2)\,\mathrm{d}(E'+P'V')\,. \tag{8}$$

(vii) *Heat Q*. The first law of thermodynamics may be written

$$\mathrm{d}Q = \mathrm{d}E + \mathrm{d}W\,.$$

From equations (7) and (8) it then follows that

$$\mathrm{d}Q = b^{-1}\,\mathrm{d}Q'\,. \tag{9}$$

(viii) *Temperature T*. It is assumed that the entropy S of the system is unaltered by a reversible adiabatic change in velocity, i.e. we have $S = S'$. Then we also have from the second law of thermodynamics $\mathrm{d}S = \mathrm{d}Q/T$ and $\mathrm{d}S' = \mathrm{d}Q'/T'$. From equation (9) it clearly follows that

$$T = T'/b\,. \tag{10}$$

From this study of the effects of a VPG transformation, it is interesting to note that, although the formulae (1) and (2) differ from the Lorentz equations, the transformations of the thermodynamic quantities $\mathrm{d}Q$, S and T are the same as those given by Tolman (1934) who employed a Lorentz transformation. However, whereas the mechanical quantities energy and work also transform in a similar way, the pressure is, surprisingly, not invariant.

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