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Mach's Principle in Nonsymmetric Field Theories

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Abstract

Most field theories allow an isolated particle to possess a rest mass, and thereby violate Mach's principle. The nonsymmetric theories of Schrödinger and of Einstein do not have this defect.

The origin of inertia has been a problem for workers in general relativity since the inception of that theory. Beginning with Einstein (1955), many have tried to attribute the inertia of bodies to interactions with other matter in the universe. None of these attempts has been generally accepted as a satisfactory expression of this idea of the relativity of inertia, usually known as Mach's principle (Mach 1893).

The inertia of elementary particles has also been an important topic in other field theories, in the guise of the self-energy problem. This is also the self-inertia problem, as is shown by Einstein's discovery of the inertia of energy. Discussions of the self-energy have usually concentrated on its divergence, and on ways to adjust a theory to make it finite. But if Mach was right, the mass of a particle in an otherwise empty universe must vanish. There should be no self-energy, and the inertial resistance of particles should be a cosmological effect.

The self-energy of a particle in electrodynamics, for example, comes about because of the interaction between charge and field, but is essentially a *local* effect. In classical electron theory, 99% of the electromagnetic mass of each particle is concentrated within 100 particle radii. (Matters are even worse in quantum theory, where the self-energy diverges logarithmically.) The situation is different in general relativity, because the equivalence principle forbids localization of gravitational energy. Nevertheless, the mass of an isolated, static, uncharged particle in an empty, asymptotically flat space, as calculated with any of the well-known energy–momentum pseudotensors, is the Schwarzschild mass m (Trautman 1962). This is clearly not a result of any interaction with other bodies, for the model contains no other bodies.

Thus conventional field theories, by allowing isolated bodies to possess inertia, violate the basic requirement of Mach's principle. We require a theory in which the energy of a single particle vanishes, and in which inertia arises when other bodies are present. A class of theories satisfying at least the first of these conditions consists of the nonsymmetric generalizations of Einstein's gravitational theory: the purely affine theory of Schrödinger (1963) and the mixed affine–metric theory of Einstein (1955, Appendix II). I shall deal with Schrödinger's theory here. The corresponding results in Einstein's theory may be obtained by simply setting the cosmological constant $\Lambda = 0$.

Schrödinger's Lagrangian is

$$\mathfrak{L} = 2\Lambda^{-1}(-\det R_{\mu\nu})^{\frac{1}{2}},$$

with $R_{\mu\nu}$ the Ricci tensor. A metric tensor is defined by $\Lambda g_{\mu\nu} = R_{\mu\nu}$, so that Λ is the cosmological constant. $\mathfrak{L}$ is a function of the affinity $\Gamma^{\sigma}_{\mu\nu}$ and its first derivatives, and a canonical energy–momentum complex can be constructed. (In gravitation theory, where the metric is the basic quantity, second derivatives of $g_{\mu\nu}$ must be eliminated before this procedure, leading to the Einstein pseudotensor, can be carried out.) This complex is

$$\mathfrak{K}_{\mu}{}^{\nu} = -\delta_{\mu}{}^{\nu}\mathfrak{L} + \Gamma^{\sigma}_{\tau\rho,\mu}\,\partial\mathfrak{L}/\partial\Gamma^{\sigma}_{\tau\rho,\nu} = (-g)^{\frac{1}{2}}(g^{\rho\nu}\Gamma^{\sigma}_{\rho\sigma,\mu} - g^{\rho\sigma}\Gamma^{\nu}_{\rho\sigma,\mu} - 2\Lambda\delta_{\mu}{}^{\nu}).$$

$\partial_{\nu}\mathfrak{K}_{\mu}{}^{\nu} = 0$ when the field equations are satisfied. The part not involving Λ is the corresponding complex for the mixed affine–metric theory, which Einstein (1955, Appendix II) took as the energy–momentum density in his final theory. He noted the essential fact that it gives zero for the energy of any static field.

To proceed, we require a solution of the field equations. The Schwarzschild solution with cosmological term,

$$\mathrm{d}s^2 = A\,\mathrm{d}t^2 - \mathrm{d}r^2/A - r^2\,\mathrm{d}\Omega^2, \qquad \text{with} \qquad A = 1 - 2m/r - \tfrac{1}{3}\Lambda r^2,$$

may be used, though the singularity at $r = 0$ is unsatisfactory from the standpoint of a unified theory. In spite of this defect, we obtain an acceptable result: Denoting by $\mathfrak{K}'_{\mu}{}^{\nu}$ the energy–momentum complex with terms which do not involve m omitted, we find $\mathfrak{K}'_{\theta}{}^{r} = 4m\cos\theta$ to be the only non-vanishing term. The energy density is zero everywhere, and the mass of the 'particle' therefore vanishes. This conclusion is not changed by the fact that r and t exchange roles as space and time coordinates inside $r = 2m$.

The subtraction from $\mathfrak{K}_{\mu}{}^{\nu}$ of terms which do not depend on m is not an omission of self-energy: The omitted terms would give an energy and stresses independent of the particle's existence, and can have nothing to do with determining its mass. The significance of the surviving stress term $\mathfrak{K}'_{\theta}{}^{r}$ is not clear, but it vanishes on integration over a 2-sphere. There are also terms which arise when the particle is in motion, and which represent a type of kinetic energy, though no rest mass can be ascribed to the particle.

The vanishing of the self-energy is merely the *pons asinorum* of Mach's principle. The theory must show how the inertia of a body arises from the presence of other matter. A satisfactory demonstration of this in the nonsymmetric theories must await the discovery of non-empty cosmological models, matter being represented by non-Riemannian features of the geometry. Meanwhile, the fact that the self-energy vanishes in an unforced way seems to be a point in favour of these theories.

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