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Cell Sizes at the Onset of Thermal Convection in a Fluid Layer Subject to the Influence of Rotation and a Magnetic Field

J. O. Murphy

Department of Mathematics, Monash University, Clayton, Vic. 3168.

Abstract

A comparison is made between the theoretical and experimental cell sizes manifest at the onset of stationary convection in a mercury layer heated from below and subject to the influence of rotation about a vertical axis and a vertical magnetic field. The theoretical cell sizes have been derived on the basis of a variational technique, and the boundary conditions appropriate to the experimental situation have been applied.

Introduction

Astrophysical and geophysical situations are known to exist where both rotation and a magnetic field are present, and convection is cited as the dominant means of energy transport. Hence an appreciation of the conditions prevailing under which convection will become established as an effective mode of heat transfer, subject to the influence of external constraints, is of considerable interest. Bénard's and later experiments illustrated, while Rayleigh's pioneering theory established, that convective heat transport would only take place in a horizontal layer of viscous fluid, heated from below, provided the Rayleigh number exceeded a certain critical value R_c. Moreover, at marginal stability, this R_c is associated with a particular value of the horizontal wave number a_c, which determines the horizontal extent of the convective cell.

A series of experiments conducted by Nakagawa (1957, 1959) determined the values of the critical Rayleigh number and wave number for the onset of cellular convection in a rotating layer of mercury, heated uniformly from below, and subject to an impressed vertical magnetic field. These experimental investigations established that the overall effect of the simultaneous action of a magnetic field and rotation on the convective processes was one of inhibition. In particular, Nakagawa (1959) established the dependence of the critical wave number on the magnetic field strength and, for a constant speed of rotation, showed that a discontinuous change in wave number takes place when the field strength is increased beyond a certain value. At the same time the manner of the instability was observed to change from overstable convection to cellular convection. For the problem under examination, only the role of cellular convection is considered (below).

Here a comparison is made between the theoretical cell sizes, derived on the basis of a variational technique using boundary conditions appropriate to the experimental situation, and the cell sizes observed (Nakagawa 1959) at marginal stability for the same parameter values. In dimensionless terms, the two parameters associated with the magnetic field strength and rate of rotation are

$$Q = \mu_e^2 H_0^2 d^2 \sigma/\rho\nu\,, \quad \text{the Chandrasekhar number},$$

$$\mathscr{T} = 4\Omega^2 d^4/\nu^2\,, \qquad \text{the Taylor number}.$$

The Rayleigh number is defined by

$$R = g\alpha\Delta T d^3/\kappa\nu\,,$$

and the cell size b, measured in centimetres, is related to the dimensionless wave number through (Nakagawa and Frenzen 1955; Nakagawa 1959)

$$b = 4\pi d/\sqrt{3}\,a\,,$$

where d is the depth of the fluid layer. Furthermore (Chandrasekhar 1961), H_0 is the strength of the applied magnetic field, Ω is the angular speed of rotation about a vertical axis, ΔT is the temperature difference across the mercury layer, while g, α, κ, ν, μ_e, ρ and σ denote respectively the acceleration due to gravity, the coefficient of thermal expansion, the coefficient of thermal diffusivity, the kinematic viscosity, the magnetic permeability, the density and the coefficient of electrical conductivity. Throughout these experiments the layer of mercury was of depth 3 cm and was rotated at a constant angular speed of 5 revolutions per minute (or was stationary). Due to differences in the average temperature of the mercury layer, an average value of $\mathscr{T}_1$ was arrived at and given, in the case of the experiments devoted to the determination of the sizes of the convection cells (Nakagawa 1959), by

$$\mathscr{T}_1 = \mathscr{T}/\pi^4 = 7\cdot 30\times 10^5\,,$$

while $Q_1 = Q/\pi^2$ was varied from $9\cdot 46$ to $1\cdot 05\times 10^4$.

The basic differential equations governing thermal instability in a horizontal layer of viscous fluid heated from below and subject to the influence of both rotation about a vertical axis and a vertical magnetic field were derived by Chandrasekhar (1961). In terms of the vertical velocity $W(z)$, temperature fluctuation $F(z)$, vorticity $Z(z)$ and current density $X(z)$ ($0 \leqslant z \leqslant 1$), we have the following system for stationary convection (Chandrasekhar 1961)

$$(\mathrm{D}^2 - a^2)F = -Ra^2 W\,, \tag{1}$$

$$\{(\mathrm{D}^2 - a^2)^2 - Q\mathrm{D}^2\}Z = -(2\Omega d/\nu)\mathrm{D}(\mathrm{D}^2 - a^2)W\,, \tag{2}$$

$$(\mathrm{D}^2 - a^2)X = -(H_0 d/\eta)\mathrm{D}Z\,, \tag{3}$$

$$\{(\mathrm{D}^2 - a^2)^2 - Q\mathrm{D}^2\}W - (2\Omega d^3/\nu)\mathrm{D}Z = F\,, \tag{4}$$

where

$$\mathrm{D} \equiv \mathrm{d}/\mathrm{d}z \qquad \text{and} \qquad \eta = 1/4\pi\mu_e\sigma\,.$$

As a necessary qualification to the above system, it should be added that the onset of stationary convection is independent of the planform of the convective cells and the Prandtl number, which for mercury is rather low, having the value $p = \nu/\kappa = 0\cdot 025$. Furthermore, the variational principle as it applies to this system has also been established by Chandrasekhar (1961), and this characteristic value problem can be solved without reference to the boundary conditions on the magnetic field. Accordingly, the perturbed magnetic field $H(z)$ has been eliminated (without increasing the order) from the differential system given by Chandrasekhar (1961).

When conducting his experiments, Nakagawa (1957) found that the mercury oxidized and formed in effect a rigid-type surface on top, and as a consequence he was unable to observe the cell sizes using fine sand particles as tracers. This difficulty was overcome (Nakagawa 1959) by pouring a thin layer of distilled water on top of the mercury, contained in a bakelite cylinder with a stainless steel base, providing in effect a free surface. Therefore, in the next section, the rigid–free boundary conditions have been applied to $W(z)$ and $Z(z)$, the conducting–nonconducting conditions to $X(z)$, and the conditions for uniform heating to $F(z)$.

Earlier theoretical studies, which involved free–free (Chandrasekhar 1961) and rigid–rigid (Murphy and Steiner 1975) type boundary conditions, established the same qualitative behaviour as observed experimentally. The latter conditions, for reasons given above, are only appropriate to measurements of R_c (Nakagawa 1957). With the mixed boundary conditions now proposed we have a quantitative basis for comparison between theoretical and observed cell sizes at the onset of convective instability.

Variational Solution

In the case under consideration, different boundary conditions have to be satisfied on the two bounding surfaces. For a rigid lower boundary we have

$$W = \mathrm{D}W = F = Z = 0 \qquad \text{at} \qquad z = 0, \tag{5}$$

and for an upper free boundary we have

$$W = \mathrm{D}^2 W = F = \mathrm{D}Z = 0 \qquad \text{at} \qquad z = 1. \tag{6}$$

In view of the general symmetry properties associated with the problem (Roberts 1966), the fluid layer can also be contained over $-\frac{1}{2} \leqslant z \leqslant \frac{1}{2}$ and, in this case, by rigid boundaries with the conditions (5) applying. The variational approach can be utilized by taking odd solutions for $W(z)$ and considering a layer of depth $\frac{1}{2}d\ (-\frac{1}{2} \leqslant z \leqslant 0)$. Under these circumstances the free surface conditions (6) on W will apply at $z = 0$. Coupled with an odd solution for W is an even solution for Z, and hence we have $\mathrm{D}Z = 0$ at $z = 0$ also, and solutions applicable to the rigid–free case follow on dividing a by 2, R and $\mathscr{T}$ by 16, and Q by 4.

At this stage Z can be eliminated by differentiation from equation (4), using (2), to give

$$\{(\mathrm{D}^2-a^2)^2 - Q\mathrm{D}^2\}^2 W + \mathscr{T}\mathrm{D}^2(\mathrm{D}^2-a^2)W = \{(\mathrm{D}^2-a^2)^2 - Q\mathrm{D}^2\}F. \tag{7}$$

Substituting F, expanded as an odd function

$$F = \sum_{m=1} A_m \sin(2m\pi z) \tag{8}$$

into equation (7), the resulting equation to be solved for W then becomes

$$[\{(\mathrm{D}^2-a^2)^2 - Q\mathrm{D}^2\}^2 + \mathscr{T}\mathrm{D}^2(\mathrm{D}^2-a^2)]W = \sum_{m=1} A_m C_{2m} \sin(2m\pi z), \tag{9}$$

with

$$C_{2m} = \{(2m\pi)^2 + a^2\}^2 + Q(2m\pi)^2 . \tag{10}$$

If we have

$$W = \sum_{m=1} A_m W_m \qquad \text{and} \qquad Z = \sum_{m=1} A_m Z_m \tag{11a, b}$$

then W_m and Z_m are the solutions of

$$[\{(\mathrm{D}^2 - a^2)^2 - Q\mathrm{D}^2\}^2 + \mathscr{T}\mathrm{D}^2(\mathrm{D}^2 - a^2)]W_m = C_{2m}\sin(2m\pi z) \tag{12}$$

and

$$\{(\mathrm{D}^2 - a^2)^2 - Q\mathrm{D}^2\}Z_m = -(2\Omega d/\nu)\,\mathrm{D}(\mathrm{D}^2 - a^2)W_m . \tag{13}$$

Now the general solution of equation (12), which is odd, can be written in the form

$$W_m = \sum_{j=1}^{4} B_j^{(m)} \sinh(q_j z) + C_{2m}\gamma_{2m}\sin(2m\pi z), \tag{14}$$

with

$$1/\gamma_{2m} = C_{2m}^2 + \mathscr{T}(2m\pi)^2\{(2m\pi)^2 + a^2\} . \tag{15}$$

Here the $B_j^{(m)}$ ($j = 1, 2, 3, 4;\; m = 1, 2, \ldots$) are constants of integration and the $\pm q_j$ are the roots of the eighth degree polynomial equation

$$\{(q^2 - a^2)^2 - Qq^2\}^2 + \mathscr{T}q^2(q^2 - a^2) = 0 . \tag{16}$$

A solution for Z_m is now given by

$$Z_m = -\left(\frac{2\Omega d}{\nu}\right)\left(\sum_{j=1}^{4} \frac{B_j^{(m)} q_j(q_j^2 - a^2)}{(q_j^2 - a^2)^2 - Qq_j^2}\cosh(q_j z) - \gamma_{2m}(2m\pi)\{2m\pi^2 + a^2\}\cos(2m\pi z)\right). \tag{17}$$

It is also convenient to expand $X(z)$ in the form

$$X = \sum_{m=1} A_m X_m ,$$

and an appropriate odd solution of equation (3) is then

$$X_m = \left(\frac{2\Omega H_0 d^2}{\nu\eta}\right)\left(\sum_{j=1}^{4} \frac{B_j^{(m)} q_j^2}{x_j(q_j^2 - a^2)}\sinh(q_j z) - \gamma_{2m}(2m\pi)^2\sin(2m\pi z)\right), \tag{18}$$

where

$$x_j = \{(q_j^2 - a^2)^2 - Qq_j^2\}/(q_j^2 - a^2) .$$

When the rigid boundary conditions (5), together with the condition $\mathrm{D}X = 0$ for a conducting boundary, are taken into account at $z = \pm\frac{1}{2}$, the following equations for the constants $B_j^{(m)}$ in the sum (14) are obtained:

For $W_m = 0$,

$$\sum_{j=1}^{4} B_j^{(m)} \sinh(\tfrac{1}{2}q_j) = 0 . \tag{19}$$

For $DW_m = 0$,

$$\sum_{j=1}^{4} q_j B_j^{(m)} \cosh(\tfrac{1}{2}q_j) = (-1)^{m+1} C_{2m} \gamma_{2m} 2m\pi\,. \tag{20}$$

For $Z_m = 0$,

$$\sum_{j=1}^{4} \frac{q_j(q_j^2-a^2)B_j^{(m)}}{(q_j^2-a^2)^2 - Qq_j^2} \cosh(\tfrac{1}{2}q_j) = (-1)^m \gamma_{2m} 2m\pi\{(2m\pi)^2+a^2\}\,. \tag{21}$$

For $DX_m = 0$,

$$\sum_{j=1}^{4} \frac{B_j^{(m)} q_j^3}{x_j(q_j^2-a^2)} \cosh(\tfrac{1}{2}q_j) = (-1)^m \gamma_{2m} (2m\pi)^3\,. \tag{22}$$

The solution of

$$\begin{bmatrix} \sinh(\tfrac{1}{2}q_1) & \sinh(\tfrac{1}{2}q_2) & \sinh(\tfrac{1}{2}q_3) & \sinh(\tfrac{1}{2}q_4) \\ q_1\cosh(\tfrac{1}{2}q_1) & q_2\cosh(\tfrac{1}{2}q_2) & q_3\cosh(\tfrac{1}{2}q_3) & q_4\cosh(\tfrac{1}{2}q_4) \\ \frac{q_1}{x_1}\cosh(\tfrac{1}{2}q_1) & \frac{q_2}{x_2}\cosh(\tfrac{1}{2}q_2) & \frac{q_3}{x_3}\cosh(\tfrac{1}{2}q_3) & \frac{q_4}{x_4}\cosh(\tfrac{1}{2}q_4) \\ \frac{q_1^3\cosh(\frac{1}{2}q_1)}{x_1(q_1^2-a^2)} & \frac{q_2^3\cosh(\frac{1}{2}q_2)}{x_2(q_2^2-a^2)} & \frac{q_3^3\cosh(\frac{1}{2}q_3)}{x_3(q_3^2-a^2)} & \frac{q_4^3\cosh(\frac{1}{2}q_4)}{x_4(q_4^2-a^2)} \end{bmatrix} \begin{bmatrix} B_1^{(m)} \\ B_2^{(m)} \\ B_3^{(m)} \\ B_4^{(m)} \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ (-1)^{m+1} C_{2m}\gamma_{2m} 2m\pi \\ (-1)^m \gamma_{2m} 2m\pi\{(2m\pi)^2+a^2\} \\ (-1)^m \gamma_{2m}(2m\pi)^3 \end{bmatrix} \tag{23}$$

is required for $m = 1, 2, 3, \ldots$, where the q_j are complex and given by equation (16).

In addition, these solutions for W, F and Z clearly satisfy the free boundary conditions (6) and the condition $X = 0$ for a nonconducting upper boundary at $z = 0$. On this basis we can now proceed to obtain the value of R_c for the onset of stationary convection in a fluid layer of depth $\frac{1}{2}d$, with one rigid and one free boundary. Substituting the expansions for F and W in accordance with equations (8) and (11a) into (1), establishes

$$\sum_m A_m\{(2m\pi)^2+a^2\}\sin(2m\pi z) = Ra^2 \sum_m A_m W_m$$

$$= Ra^2 \sum_m A_m\left(C_{2m}\gamma_{2m}\sin(2m\pi z) + \sum_{j=1}^{4} B_j^{(m)}\sinh(q_j z)\right). \tag{24}$$

Next multiplying both sides of equation (24) by $\sin(2n\pi z)$ ($n = 1, 2, \ldots$) and integrating over the range of z from $-\frac{1}{2}$ to $\frac{1}{2}$ leads to a set of linear homogeneous equations for the coefficients A_m. Now the determinant of this system of equations must vanish, and the following characteristic equation for R is established:

$$\left\| \left(\frac{(2n\pi)^2 + a^2}{Ra^2} - C_{2n}\gamma_{2n} \right) \tfrac{1}{2}\delta_{mn} - \sum_{m=1} \left(\frac{n}{m} \right) \right\| = 0\,, \tag{25}$$

where, in Chandrasekhar's (1961) notation,

$$\left(\frac{n}{m} \right) = \sum_{j=1}^{4} B_j^{(m)} \int_{-\frac{1}{2}}^{\frac{1}{2}} \sin\kappa(q_j z) \sin(2n\pi z)\, \mathrm{d}z = (-1)^{n+1}\, 4\pi n \sum_{j=1}^{4} \frac{B_j^{(m)} \sinh(\frac{1}{2}q_j)}{q_j^2 + (2n\pi)^2}\,. \tag{26}$$

Numerical Methods and Results

The first stage in the numerical procedures utilized here involved the determination of the four roots q_j^2 $(j = 1,2,3,4)$ of the polynomial equation (16), which has coefficients dependent on a, $\mathscr{T}$ and Q. This reduced quartic equation allows two real roots and two complex conjugate roots. Computer evaluation of the matrix equation (23) for the constants of integration $B_j^{(m)}$ now clearly includes calculations with complex elements, and it was found expedient to employ the FORTRAN 'type complex' option. The $B_j^{(m)}$ were also found to be complex. However, the elements (n/m) of equation (25), when evaluated from (26), were found to have zero imaginary part. This is in accordance with the physical expectations of R taking only a real value.

The required characteristic values of R as a function of a, $\mathscr{T}$ and Q were evaluated with increasing precision by including successively more rows and columns in the determinantal equation (25). The expression (26) for the determinant elements (n/m) is symmetric in n and m, and an initial approximation for R to initiate any calculations was obtained from the first element when $m = n = 1$. Thereafter a half-interval search procedure was employed on an iterative basis to solve for R. Sufficient precision was always obtained for this value from a 3×3 determinant. To calculate the theoretical cell size that could be expected at the onset of stationary convection for a particular speed of rotation and magnetic field strength, it was necessary to obtain the $a = a_c$ corresponding to minimum $R = R_c$ for these values of $\mathscr{T}$ and Q. Overall, this approach provides a solution for rigid–free boundaries applicable to a cell depth $\frac{1}{2}d$, horizontal wave number $\frac{1}{2}a$, Rayleigh number $\frac{1}{16}R$, Chandrasekhar number $\frac{1}{4}Q$ and Taylor number $\frac{1}{16}\mathscr{T}$.

Solutions obtained from a completely independent approach, using an initial value method, were found to be in agreement with the variational method of solution already described. Under these circumstances, the Runge–Kutta technique was applied to the 10th-order linear differential system represented by equations (1)–(4), together with the lower and upper boundary conditions (5) and (6) supplemented by $\mathrm{D}X = 0$ at $z = 0$ and $X = 0$ at $z = 1$. For specified a, $\mathscr{T}$ and Q, the role of R is that of an eigenvalue. The generalized Newton–Raphson method was also incorporated in the numerical routine to improve on successive iterations the initial estimates of the required variables, including R, not already determined by the boundary conditions at $z = 0$. In this manner the numerical procedures automatically established convergence for the system and therefore gave the required value of R. However, at large values of the parameters some convergence problems were experienced.

The general features of the solutions pertaining to this problem with a conducting lower rigid boundary and a nonconducting upper free boundary are also representative of the associated free–free case (Chandrasekhar 1961) and rigid–rigid case (Murphy and Steiner 1975). For example, the $\log_{10} R$ versus a dependence for $\mathscr{T} = 10^8$ and varying Q given in Fig. 1 is representative of all three cases. For Q small, a_c is large; minimum R_c occurs for $Q \approx \mathscr{T}^{\frac{1}{2}}$ with $a_c \approx O(1)$, and any further increase in Q increases both minimum R_c and a_c, with the magnetic forces ultimately dominating. Application of a magnetic field to this rotating system clearly facilitates the onset of cellular convection as established experimentally by Nakagawa (1957). Eltayeb (1972) reached similar conclusions in the case of the double limit $\mathscr{T} \to \infty$ and $Q \to \infty$ for nonmixed boundaries.

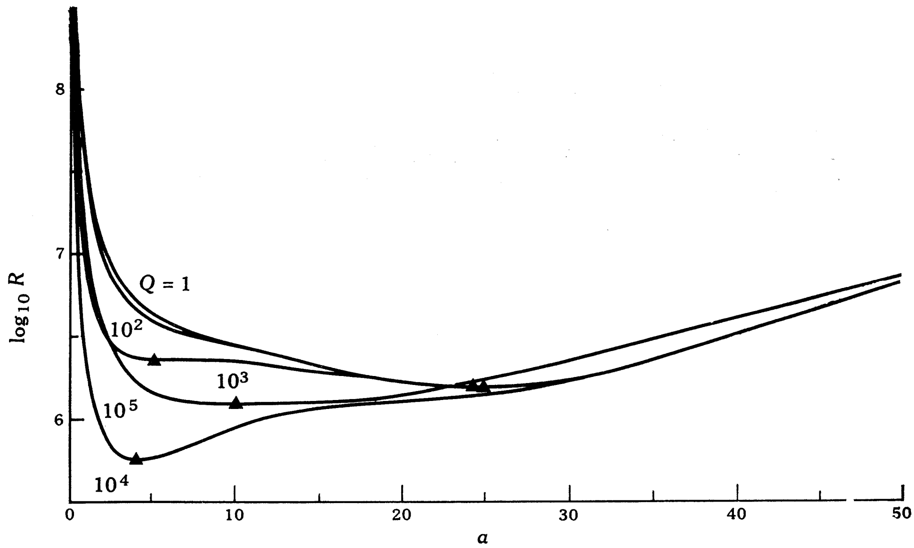
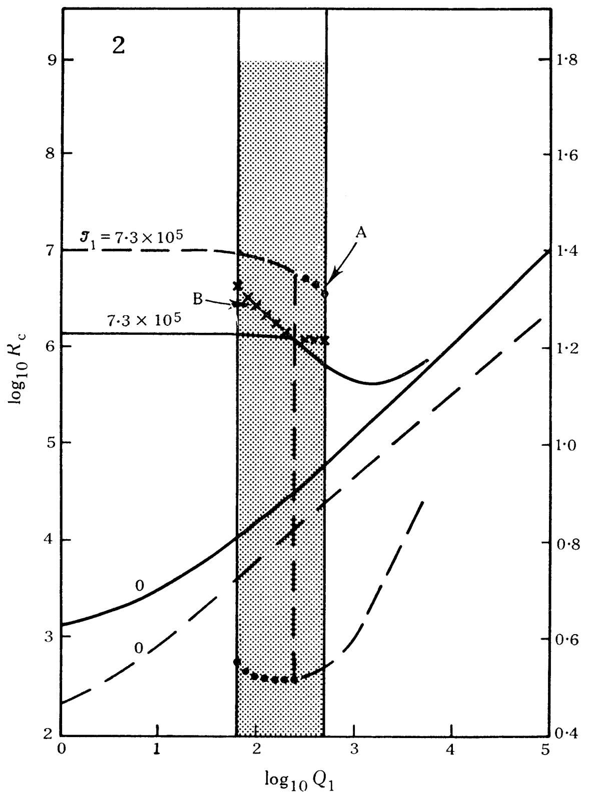


Fig. 1. Dependence of the Rayleigh number R on the horizontal wave number a for the indicated values of the Chandrasekhar number Q when the Taylor number $\mathscr{T} = 10^8$. The critical values of a_c and R_c for the onset of stationary convection are indicated by triangles.

The theoretical dependence of R_c and a_c on Q_1 (where $Q_1 = Q/\pi^2$), for Nakagawa's (1959) parameter values, is given in Fig. 2 and, for comparison, the $\mathscr{T}_1 = 0$ (where $\mathscr{T}_1 = \mathscr{T}/\pi^4$) curves for rigid–free boundaries have also been included. These values were calculated from the characteristic determinant given by Chandrasekhar (1961). Separate ranges of Q_1 values in this figure illustrate the evident dominance on the convective processes of either the magnetic or rotational forces involved. The shaded region, $1{\cdot}8 \leqslant \log_{10} Q_1 \leqslant 2{\cdot}7$, indicates the range of magnetic field strengths where two minima exist on the $R-a$ curve for $\mathscr{T}_1 = 7{\cdot}3 \times 10^5$. The discontinuous change in the wave number corresponds to stationary convection always being established at the absolute R_c minimum. In fact, only for rotation rates where $\mathscr{T} \gtrsim 10^6$ can we expect to observe any dramatic change in cell size with increasing Q—the rate of 5 revolutions per minute in Nakagawa's experiments gives a Taylor number well in excess of this value.

In keeping with the stated aims of this study, our main interest and conclusions are centred on Fig. 3 where the theoretical and observed cell sizes at marginal stability are compared in relation to the field strength Q_1. To focus attention on the role played by rotation in this situation, the results for $\mathscr{T}_1 = 0$ as well as for $\mathscr{T}_1 = 7{\cdot}3 \times 10^5$ have been included. The experimental results for the cell sizes b, together



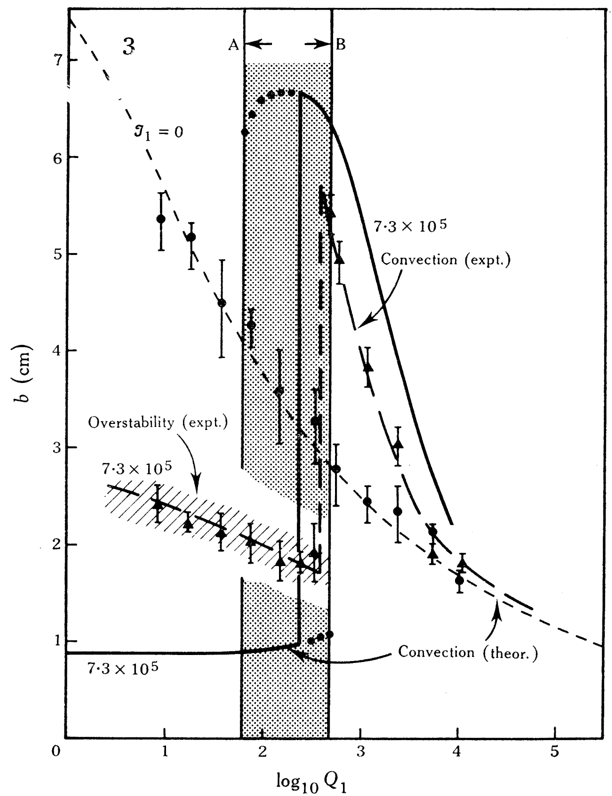


Fig. 2. Numerical results for the variation of R_c and a_c with the magnetic field strength Q_1 when $\mathcal{T}_1 = 0$ and $7 \cdot 3 \times 10^5$ (as indicated) in the case of a lower rigid conducting boundary and an upper free nonconducting boundary. The shaded region represents the range of Q_1 values where the $R-a$ curve has two minima. Points A and B denote secondary minima of a and R respectively. Full and dashed curves are for R_c and a_c respectively.

Fig. 3. Comparison of experimental and theoretical results for the dependence of the convective cell size b on the magnetic field strength Q_1 at marginal stability. Nakagawa's (1959) experimental results are: triangles, $\mathcal{T}_1 = 7 \cdot 3 \times 10^5$; circles, $\mathcal{T}_1 = 0$. Theoretical numerical results are: solid curve, fluid layer contained by a lower rigid conducting boundary and an upper free nonconducting boundary: short dashed curve, fluid layer contained by rigid–free boundaries. The stippled region designates the range of values of Q_1 where two minimum values exist on the theoretical $R-a$ curve for $\mathcal{T}_1 = 7 \cdot 3 \times 10^5$, while the cross-hatched region indicates that overstability was observed during Nakagawa's experiments. Points A and B denote the range of Q_1 values associated with two possible cell sizes.

with the error bars are from Nakagawa's (1959) work. Clearly the agreement with the rigid–free theoretical values for $\mathscr{T}_1 = 0$ is acceptable. Although the horizontal branch of the cellular convection curve at low values of Q_1 for $\mathscr{T}_1 = 7 \cdot 3 \times 10^5$ cannot be directly compared with the corresponding experimental curve because of the different mode of instability observed, it can be said that in the presence of rotation the magnetic field has little initial influence on the resultant cell size for stationary convection. Again, the shading specifies the range of Q_1, calculated in the case of rigid–free boundaries for $\mathscr{T}_1 = 7 \cdot 3 \times 10^5$, where two cell sizes can occur at the onset of cellular convection, and it is seen that the observed experimental transition takes place within this region. It should also be pointed out that the calculated discontinuous cell size change does not correspond to the same value of Q_1 as for minimum theoretical R_c which, as can be noted from Fig. 2, is achieved at a higher value of Q_1, but instead takes place at the largest possible cell size—i.e. minimum a_c. This feature of the rigid–free results compares very favourably with the experimental situation, whereas calculations for the rigid–rigid case show that minimum a_c is attained, for Q_1 increasing, after the discontinuous change in cell size.

Table 1. Variation in cell size at onset of cellular convection

Cell size b at onset of cellular convection is listed as a function of the magnetic field strength Q_1 at Taylor number $\mathscr{T}_1 = 7 \cdot 3 \times 10^5$ for upper and lower boundaries nonconducting. Also listed are the minimum values of the horizontal wavenumber a and the Rayleigh number R

$\log_{10} Q_1$	a_{min}		$10^{-5} R_{min}$		b (cm)
0·0	25·13		13·775		0·87
0·2	25·13		13·774		0·87
0·4	25·13		13·773		0·87
0·6	25·13		13·771		0·87
0·8	25·12		13·767		0·87
1·0	25·10		13·761		0·87
1·2	25·08		13·752		0·87
1·4	25·05		13·737		0·87
1·6	25·01		13·714		0·87
1·7	24·97		13·698		0·87
1·8	24·91	3·99	13·677		0·88
1·9	24·85	3·66	13·651		0·88
2·0	24·78	3·51	13·618	26·550	0·88
2·1	24·69	3·43	13·577	21·561	0·88
2·2	24·56	3·37	13·524	17·477	0·89
2·3	24·39	3·35	13·457	14·174	0·89
2·4	24·18	3·33	13·372	11·531	6·55
2·5	23·90	3·35	13·262	9·433	6·51
2·6	23·51	3·38	13·121	7·861	6·45
2·7	22·95	3·45	12·937	6·576	6·32
2·8		3·56		5·659	6·12
2·9		3·72		5·014	5·86
3·0		3·96		4·602	5·51
3·2		4·62		4·363	4·72
3·4		5·58		4·816	3·91

In Table 1 the critical Rayleigh and wave numbers, and the calculated cell size at the onset of stationary convection in a fluid layer of depth 3 cm are given as functions of Q_1 for $\mathscr{T}_1 = 7 \cdot 3 \times 10^5$ when both the rigid lower boundary and free upper boundary are taken to be nonconducting. The condition $X = 0$ now applies on both boundaries, and to undertake these calculations necessitated appropriate modification of equations (22) and (23).

To complete the theoretical analysis of the experimental findings, the equivalent problem of overstability (which requires specification of the Prandtl number) in a mercury layer with rigid–free boundaries should be considered. Additional experimental results in this field would further aid our understanding of the interaction of rotation and a magnetic field on the convective processes.

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