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The Molonglo Deep Sky Survey of Radio Sources. III* Source Counts

J. G. Robertson

School of Physics, University of Sydney, N.S.W. 2006.

Abstract

Number–flux density counts of radio sources are given for the first two deep surveys made with the Molonglo Mills Cross at 408 MHz, extending to lower limits of 84 and 88 mJy. Practical techniques are developed for the calculation of corrections to the counts when confusion errors are significant. The resulting corrections due to noise, confusion and other effects are given. Counts are also given for the MC2 and MC3 catalogues in the range 2–10 Jy to reduce the statistical uncertainties there. Results are shown on a composite plot, using relevant Molonglo surveys and an all-sky catalogue of strong sources. The existence of a convergence in the number of weak sources is confirmed.

1. Introduction

A preliminary analysis of the counts of radio sources at 408 MHz has been made by Mills *et al.* (1973) (hereafter referred to as MDR) using the results of the Molonglo MC1 survey (Davies *et al.* 1973) and the all-sky catalogue of Robertson (1973). Two deep surveys have been made at Molonglo to extend the flux density coverage below the lower limit of the MC1 survey: the catalogue for the declination zone $-20°$ is given in Part I (Robertson 1977*a*, present issue pp. 209–30) and that for $-62°$ in Part II (Robertson 1977*b*, present issue pp. 231–9). Counts from both zones are given in the present paper. In addition, the publication of the MC2 and MC3 catalogues (Sutton *et al.* 1974) allows improvement of the count statistics in the region 2–10 Jy, which was not well covered by MC1 alone.

Count corrections for the deep surveys are developed in Section 2 and applied in Section 3. The combined counts are given in Section 4 and are compared with results from the Cambridge 5C surveys. The flux density scale due to Wyllie (1969) has been used throughout. A discussion of the cosmological significance of the results has been deferred to a later paper.

2. Evaluation of Count Corrections in Presence of Confusion Errors

Murdoch *et al.* (1973) (hereafter referred to as MCJ) have shown the importance of a thorough analysis of the effects of flux density errors on source counts, and that corrections to the counts may be required even at quite high ratios of flux density to error. Count corrections are easily made if the errors in flux density are due to gaussian noise alone. However, if there are significant errors due to confusion, as

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in the present deep surveys, then it is necessary to use the Monte Carlo method to find the error distributions. This method also responds to the effects of noise, obscuration of weak sources by stronger sources, and any bias in the observed flux densities. The general approach to the treatment of confusion errors has been discussed by MCJ, but there are a number of practical problems which had to be solved in the detailed application of the method. The background is given in the present Section, while the details are given in the Appendix.

In the case where the error distributions have a significant tail due to confusion errors, a large number of Monte Carlo sources are required to obtain even moderate statistical accuracy in the tails. For example, in the present work over 1000 Monte Carlo sources were inserted and analysed at each declination zone (the resulting error distributions are given in Parts I and II). Following the terminology of MCJ and Part I, let S be the true flux density of a source and F be its observed flux density. These are related by the error distribution $P(F|S)$, which can be obtained (with some statistical uncertainty) from the distribution of flux densities that are fitted to the Monte Carlo sources of true flux density S. Let $P(S)$ and $P(F)$ be respectively the true and observed probability distributions (or differential counts).* If finite intervals are used for differential counts, the ratio of the number of sources expected in the ith flux density interval in the presence of errors to that in the absence of errors is shown by MDR to be given by x_i/y_i, where

$$x_i = \int_{F_{i1}}^{F_{iu}} P(F)\,\mathrm{d}F = \int_{F_{i1}}^{F_{iu}} \int_0^{\infty} P(F\,|\,S)\,P(S)\,\mathrm{d}S\,\mathrm{d}F\,, \tag{1a}$$

$$y_i = \int_{F_{i1}}^{F_{iu}} P(S)\,\mathrm{d}S\,. \tag{1b}$$

The limits F_{i1} and F_{iu} are the lower and upper boundaries respectively of the ith interval. If x_i and y_i can be found for each interval then the observed counts can be corrected by multiplying by y_i/x_i. Grouped counts are used here instead of the ungrouped model-fitting approach adopted by MCJ, because the latter does not allow us to display and examine the counts over the entire flux density range.

The distribution $P(S)$ is of course unknown, since it is the function we wish to obtain. If the corrections are small, however, it is quite adequate to assume a form for $P(S)$, derived from the uncorrected counts. For example, a power law

$$P(S)\,\mathrm{d}S = KS^{-(\gamma+1)}\,\mathrm{d}S \tag{2}$$

is often a good fit over the required range of flux density. The sensitivity of the count corrections to reasonable variations in the form assumed for $P(S)$ should be tested. The first step in calculation of the corrections is to choose a number of values of F for which the inner integral of equation (1a) is to be evaluated. For each value of F, the integrand $P(F|S)P(S)$ is then plotted as a function of S/F. These are the ϕ curves of MCJ. As an example, Fig. 1 shows the ϕ curve for $F = 125$ mJy for the $-20°$ deep survey. Basically, this curve gives the relative probabilities of various

* Note that in reality $P(S)$ and $P(F)$ are two different functions of the same variable, which is simply flux density. However, this is the conventional notation and will be used until it is necessary to replace it in the Appendix.

values of true flux density being observed as $F = 125$ mJy. Note that large overestimations of the flux density correspond to small values of S/F.

Given the practical restriction on the total number of Monte Carlo sources, MCJ recommended that a large number of such sources be inserted at each of a few chosen values of true flux density. The result is a reasonably accurate determination of $P(F|S)$ as a function of F, but for only a few fixed values of S. Thus interpolation of $P(F|S)$ with respect to S is required in constructing the ϕ curves. The form of $P(F|S)$ is usually such that the probability of obtaining a given error E (that is, $F = S+E$) varies little with S; that is, the value of $P(F = S+E|S)$ is not a rapidly varying function of S, at least in the vicinity of the peak of the distributions. This allows interpolation of $P(F|S)$ to be carried out on a graph of $P(F = S+E|S)$ as a function of S, for a given value of E. The Appendix gives details of the methods developed for evaluation of the count corrections.

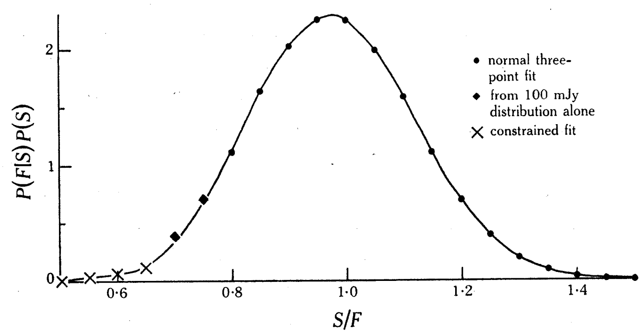

Fig. 1. Integrand $P(F|S)P(S)$ of equation (1a) plotted as a function of the ratio S/F of the true to the observed flux density for $F = 125$ mJy, for the $-20°$ deep survey. The three symbols distinguish different methods of fitting to plots of $P(F = S+E|S)$ as a function of S (see text).

3. Count Corrections for the Deep Surveys

(*a*) *Evaluation*

The methods described in Section 2 and in the Appendix were used to calculate the count corrections for the deep surveys. These corrections are small, but an effort has still been made to use the best possible methods to obtain them. In addition to noise and confusion, there were also significant effects due to obscuration of weak sources by stronger sources, and a small bias in the flux densities. All four effects are allowed for by the Monte Carlo analysis. The forms assumed for $P(S)$ were power laws (equation 2) which were adequate over the relevant flux density range of about 80–400 mJy. The exponents γ used were $1 \cdot 079$ at $-20°$ and $0 \cdot 939$ at $-62°$.

The Monte Carlo sources were inserted with $S = 100$, 200 and 300 mJy at $-20°$, and $S = 100$, 150 and 200 mJy at $-62°$. Thus there were only three data points on the plots of $P(F = S+E|S)$ as a function of S. A least squares fit to a straight line was made to these points (except for some cases of extrapolation in which the $S = 100$ mJy distribution alone was used—see the Appendix). Where the constraint described in the Appendix was applicable, the straight line was forced to pass through the appropriate point. In Fig. 1 different symbols are used to distinguish points resulting from different fitting procedures.

Some discussion of the small bias of flux densities has been given in Part I where, in calculating the $P(F|S)$ distributions, an average flux density bias was used at each declination zone. Further details of all aspects of the count corrections are given by Robertson (1976). The factors x_i/y_i are given in column 3 of Table 1 (below), which summarizes the source counts for several surveys. The flux density intervals for the deep survey catalogues are logarithmic, and correspond to a ratio of $\sqrt{2}$, except for the lowest interval, which for both surveys corresponds to a ratio of $2^{\frac{1}{4}}$. Although the lower limit of both catalogues is at or above five times the r.m.s. error, the source counts in these lowest intervals are regarded as tentative, for safety.

(*b*) *Discussion*

The values of x_i/y_i for the deep surveys, given in Table 1, are less than unity, showing that the counts in the presence of errors were underestimated with respect to the error-free counts. This contrasts with the overestimation obtained due to the population-law effect when noise alone is significant (see e.g. MCJ), and is explained by the effects of obscuration and the small flux density bias. Obscuration has a relatively large (but well-defined) effect due to the safety criterion adopted to decide whether a source was obscured or resolved (as described in Part I). In addition, the overestimation due to the population-law effect is not large, because of the flattening of the source counts and the high ratio of flux density to error.

The synthetic sources used in the Monte Carlo analysis were point sources, and 'point source' flux densities were used for all catalogued sources except those with quite significant broadening; see the catalogue papers (Parts I and II). This could introduce errors into the counts due to partial resolution of slightly extended sources. However, a check on the distribution of a width parameter given by the source fitting program for the catalogued sources showed no significant population of slightly extended sources, and hence there should be no significant effect on the counts. Another effect not allowed for by the Monte Carlo analysis is that due to random calibration errors, which were shown in Part I to have an r.m.s. value of $\sim 4\%$. The resulting error in the counts is quite negligible (Robertson 1973).

The uncertainty of the count correction factors, allowing for statistical errors and the interpolations and integrations performed, was estimated to be $\sim 3\%$. The corrections were found to be quite insensitive to reasonable alterations in the form of the assumed source count $P(S)$, and thus no significant uncertainty has been added by the assumption of a particular form.

4. Source Counts

(*a*) *Results*

The differential counts are given in Table 1, in which the data are grouped according to their origin. Groups *a* and *b* contain the counts from the two deep surveys, with corrections calculated as described in the previous sections. Group *c* contains the counts from the lower flux density ranges of the MC1 catalogue (Davies *et al.* 1973; MDR). They are repeated here because the scale for the integrated flux densities used in the MC1 counts has been found to be overestimated by 5·5% compared with the 'point source' flux densities (B. Y. Mills, personal communication).

The factors x_i/y_i have been adjusted to allow for this error. To make this adjustment (without altering the flux density intervals used) it was necessary to assume a value for the source count slope over the intervals—any additional uncertainty introduced in this way is no greater than $\sim 1\%$. The adjustments were increases of x_i/y_i by a factor 1·06 for the intervals up to 0·526 Jy, 1·08 at 0·743 Jy and 1·09 for all higher ranges. The MC1 count in the interval 0·22–0·31 Jy has been omitted because it involves sources with a ratio of flux density to r.m.s. error of less than five.

Table 1. Source counts at 408 MHz

(1) s_i (Jy)	(2) $\Delta N_{obs,i}$	(3) x_i/y_i	(4) $\Delta N_{corr,i}/\Omega$	(5) $\Delta N_{corr,i}/\Delta N_0$
	(a) Deep survey $-62°$ *zone* ($\Omega = 5 \cdot 51 \times 10^{-3}$ sr)			
0·092	12	0·95	2302	0·272
0·119	26	0·94	5044	0·436
0·168	16	0·95	3046	0·443
0·238	20	0·99	3666	0·897
0·337	8		1452	0·597
0·476	6		1089	0·753
0·673	6		1089	1·268
	(b) Deep survey $-20°$ *zone* ($\Omega = 0 \cdot 0201$ sr)			
0·096	59	0·94	3102	0·391
0·125	96	0·93	5100	0·473
0·176	75	0·94	3959	0·616
0·249	57	0·97	2909	0·762
0·352	37		1837	0·809
0·498	21		1043	0·772
0·704	21		1043	1·298
	(c) MC1 catalogue corrected (*see text*) ($\Omega = 0 \cdot 160$ sr)			
0·372	244	1·08	1457	0·696
0·526	186	1·16	1028	0·826
0·743	155	1·14	848	1·145
1·051	72	1·12	401	0·912
1·487	53	1·11	298	1·140
	(d) MC1, MC2 and MC3 catalogues (*see text*) ($\Omega = 0 \cdot 400$ sr)			
2·102	88		215	1·380
2·973	43		104	1·124
4·205	20		48	0·876
5·946	13		31	0·956
8·409	9		21	1·104

The counts in the range 2–10 Jy for the MC1 catalogue were subject to large statistical uncertainties due to the small number of sources, but the publication of the MC2 and MC3 catalogues (Sutton *et al.* 1974) has extended the solid angle covered from 0·16 to 0·40 sr, with a consequent reduction in the uncertainties. Count corrections were not available for the MC2 and MC3 surveys, and hence their results were used only above 1·77 Jy, where the corrections would be negligible. The right ascension ranges used were $11^h 28^m$–$18^h 20^m$ and $20^h 00^m$–$01^h 23^m$ for MC2, and $13^h 31^m$–$18^h 28^m$ and $20^h 12^m$–$04^h 13^m$ for MC3. ‘Point source’ flux densities have been used for the counts from the MC2 and MC3 catalogues; but the consequent errors in the counts due to resolution of a few sources are considerably less than the statistical uncertainties. The portion of the count due to MC1 has been corrected for the scale error, while no corresponding corrections were applied for MC2 and MC3.* The combined counts are given in group *d*. Murdoch (1976) has compared the source densities above 0·97 Jy for *part* of the MC2 and MC3 surveys with that for MC1, and found that the count for MC2 and MC3 is higher by a factor of $\sim 1 \cdot 4$. This is only marginally significant and could not invalidate the incorporation of these counts in the present paper, because the difference in density is confined to flux densities less than ~ 3 Jy. Only the point at 2·97 Jy has been raised by the addition of the MC2 and MC3 count.

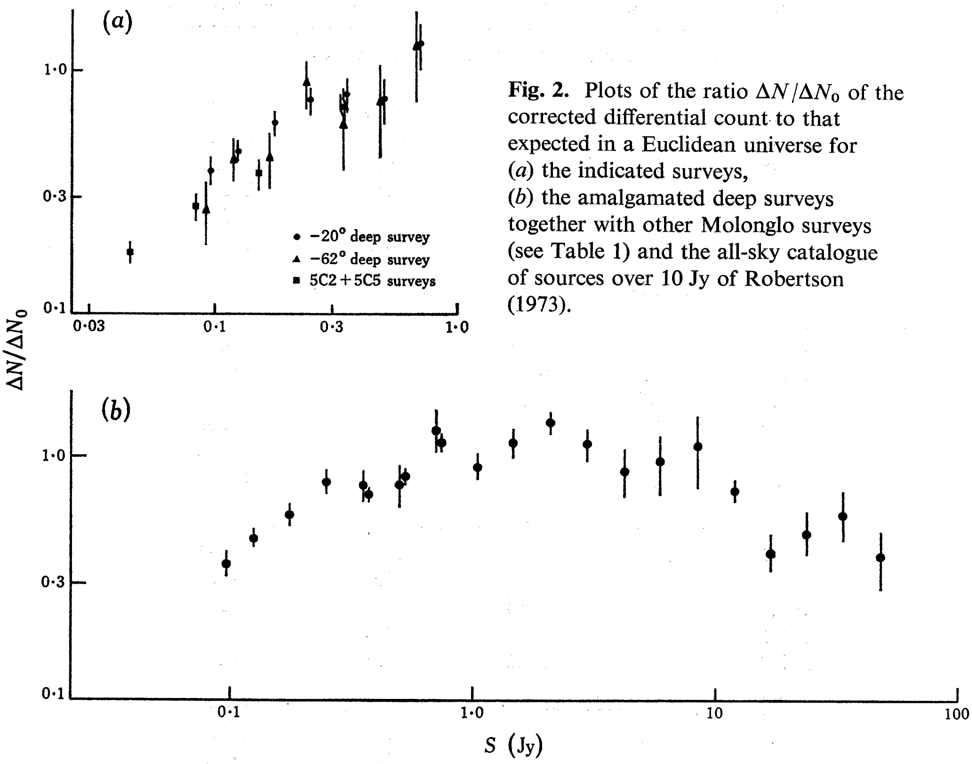

Fig. 2. Plots of the ratio $\Delta N/\Delta N_0$ of the corrected differential count to that expected in a Euclidean universe for (*a*) the indicated surveys, (*b*) the amalgamated deep surveys together with other Molonglo surveys (see Table 1) and the all-sky catalogue of sources over 10 Jy of Robertson (1973).

In Table 1 the solid angle Ω covered is shown for each group of counts. Column 1 gives s_i, the logarithmic centre of the flux density range (where the symbol s is introduced in the Appendix). Column 2 gives the actual number of sources catalogued in the appropriate interval. Column 3 gives the factor x_i/y_i, as discussed above. Where no value is given, the correction is not significant (except for group *d*, as already mentioned). Column 4 gives the corrected number of sources per steradian, while column 5 gives $\Delta N/\Delta N_0$, the ratio of the corrected count to the count expected in a static Euclidean model, in which $P(S)\,\mathrm{d}S = KS^{-2\cdot 5}\,\mathrm{d}S$. The normalization for the latter was $K = 1354$ sources sr^{-1} (for S given in Jy) as in MDR.

* Murdoch (1976) has shown that the point source flux density scale for the MC2 and MC3 surveys may be too high by $3\% \pm 2\%$. This is not statistically significant however, and does not warrant a correction.

The counts $\Delta N/\Delta N_0$ for the two deep survey zones are graphed in Fig. 2*a*. The differential counts from the combined Cambridge 5C2 and 5C5 aperture synthesis surveys (Pearson 1975) are also shown, normalized to the same Euclidean law as the other counts, and adjusted to the Wyllie scale of flux densities. Fig. 2*a* shows that the tentative points at the lower flux density limits of both deep survey catalogues clearly continue the trend of the counts in the range 0·1–0·7 Jy, showing (as expected) that there is no significant incompleteness near the limit of these catalogues. The counts from the two deep surveys show good agreement, and thus it was possible to amalgamate them in order to present the results as clearly as possible. This was done by using each point of the $-62°$ count to predict a count at the abscissa of the nearby $-20°$ point. It was necessary to assume a value for the source count slope between each pair of points, but the difference in flux density between them was so small that no significant uncertainties were introduced. The composite count was then found for each point. Fig. 2*b* shows the amalgamated counts of the deep surveys, as well as the Molonglo results for higher flux densities (i.e. groups *c* and *d* of Table 1). The results for flux densities greater than 10 Jy have been derived from the all-sky catalogue of Robertson (1973); the counts are given by MDR. Note that the count in the highest flux density interval is an 'equivalent number', representing all sources of flux density greater than 40 Jy, and assuming a Euclidean source count slope (see MDR). This point carries information about the total number of very strong sources, but no information about the slope.

(*b*) *Discussion*

The deep survey counts show a slope very similar to the Cambridge results in the region of overlap (Fig. 2*a*), thus confirming the reality of the so-called convergence of counts at very low flux densities. There is, however, a difference in the absolute numbers, with the Cambridge counts being lower. By fitting power law models of the same slope (the mean of the individual values) to both the Cambridge results and amalgamated deep surveys, it was found that the counts differed by a factor of 1·29. This is significant at about the 1% level, allowing only for statistical uncertainties. However, the result can be regarded as only marginally significant in view of possible *systematic* differences due to causes other than true anisotropy of counts. For example, Condon and Jauncey (1973) have raised doubts regarding some of the Cambridge surveys (including 5C2). Further, the survey areas for both 5C2 and 5C1 (which partly overlaps 5C5) were preselected to contain no intense sources, and Jauncey (1975) believes that they are not necessarily representative areas of weak sources. It is thus too early to form any conclusion about a possible anisotropy in the counts.

The agreement between the counts from the $-62°$ deep survey, $-20°$ deep survey and MC1 is good. Thus no anisotropy is evident in these results from Molonglo. The addition of the MC2 and MC3 catalogues has moderated the sharp transition near 10 Jy (as compared with Fig. 3 of MDR), but has not changed the overall picture of a plateau at intermediate flux densities, with a sharp drop at the higher flux density end. Further discussion and analysis of the counts will be made in a later paper.

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Appendix

Practical Techniques for Evaluating Count Corrections in Presence of Confusion Errors

One of the chief problems brought about by the presence of confusion errors is the poor statistical accuracy obtained in the tails of the empirical error distributions, compared with that in the peak regions. This problem was mentioned by MCJ but no further discussion was given. In fact, when *multiple* blending is negligible, it is possible to apply a useful constraint to the interpolation of $P(F|S)$ for the values of E at which the tails are dominant. We consider large positive values of E where $F = S+E$ is almost equal to $2S$. Usually the only way such large errors can arise is through the confusion tails and, as explained in Part I and MCJ, overestimations of more than S cannot arise in blends of two sources—only higher-order blending can cause this. Such multiple blending is in fact negligible in the Molonglo deep surveys. The basis of the constraint is that the error distributions carry almost no information about multiple blending, yet apart from noise, it is the only way overestimations of $F = 2S$ can be obtained. Thus for any overestimation error that cannot be obtained by noise (i.e. if the confusion tails dominate) we impose the constraint $P(F = 2S|S) = 0$. This removes much of the statistical uncertainty in making the interpolations of $P(F = S+E|S)$ as a function of S, by providing another point, with zero formal error, in the relation. This is particularly helpful because the constraint is applicable in the region of low true flux density, where any uncertainties are magnified because $P(F|S)$ is multiplied by large values of $P(S)$ (equations 1a and 2). When this constraint is applied, the ϕ curves will tend to a limit of zero at $S/F = 0\cdot5$, as in Fig. 1.

In constructing the ϕ curves, some extrapolation of $P(F|S)$ to low values of S will usually be required also, because it is not practical to insert and analyse Monte Carlo sources at the lowest values of S which are able to contribute to the ϕ curves. If any of the contributing points is due to the tail region of its distribution, then it is very uncertain, and extrapolation increases this uncertainty. In the present work this has been dealt with by using the value of $P(F|S)$ (for the appropriate value of E) from the distribution with the lowest value of S available. The resulting modifications to the ϕ curves are minor. Further details are given by Robertson (1976).

A slight clarification of the notation is necessary before proceeding: in making count corrections it is necessary to compare $P(S)$ and $P(F)$ over the same ranges of flux density, and thus it is best to write them as $P_S(s)$ and $P_F(s)$ to emphasize that they are different functions of the same variable, simply flux density. (Note that s does not represent the same quantity as in MCJ.) The result of integrating each ϕ curve is a value for P_F. It is convenient to plot $P_F(s)/P_S(s)$ as a function of $\log_{10} s$. Interpolation can be carried out by fitting a smooth curve to this plot. The next step is to perform the outer integration of equation (1a). The precision required in making this integration can be greatly relaxed by introducing

$$\varepsilon(s) = 1 - P_F(s)/P_S(s)\,. \tag{A1}$$

It then follows that

$$x_i/y_i = 1 - \ln 10 \left(\int_{\log_{10} s_{i1}}^{\log_{10} s_{iu}} s\,\varepsilon(s)\,P_S(s)\,\mathrm{d}q \right) \Big/ \int_{s_{i1}}^{s_{iu}} P_S(s)\,\mathrm{d}s\,, \tag{A2}$$

where logarithmic integration with $q = \log_{10} s$ has been used in the numerator for convenience. The ratio of the integrals represents the fractional error in the counts, and thus moderate accuracy in its evaluation is sufficient. Numerical integration will be required for the numerator (planimetry was used in the present work) while analytic integration may be used for the denominator.

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