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Radiation Resistance of a Helical Antenna in a Compressible Electron Plasma

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Abstract

Earlier work on the radiation fields of a travelling-wave helical antenna in a compressible electron plasma has been extended to evolve expressions for the electromagnetic mode component R_e and the electroacoustic or plasma mode component R_p of its radiation resistance. These expressions, which are general in nature but involve complicated integrals, cover the special cases of multiturn loops, circular arcs and linear antennas. Analytical expressions for R_e are then derived for the particular example of a helical antenna of small pitch angle with a fractional or integral number of turns. It is found that R_e increases with pitch angle for some values of plasma frequency. Finally, by imposing the necessary conditions of change of medium, corresponding expressions for the radiation resistance R of a helical antenna in a vacuum are obtained. The value of R is found to increase with the pitch angle for low values of the diameter of the helical cylinder.

Introduction

It is now well known that the total power radiated in the far zone by a source immersed in a compressible electron plasma is the sum of the power radiated in the transverse electromagnetic waves and the longitudinal plasma waves. In most such studies, Hertzian dipoles have been used as radiating sources although a few authors have considered finite sized linear and loop antennas with different forms of current distribution. However, for more than one reason it is desirable to consider a general comprehensive source and from this to deduce the characteristics of other particular sources. With this in view, rigorous expressions for the radiation fields of a travelling-wave helical antenna in a compressible electron plasma were derived by Talekar and Soni (1974*a*) from which corresponding expressions for linear and loop antennas can be obtained as particular cases. In the present paper this work is extended to obtain general expressions for the radiation resistances of helical antennas immersed in a compressible electron plasma, and the effects on the resistance of the pitch angle of the helix and the number of turns in it are investigated.

Radiation Resistance of Helical Antenna

The radiation resistances R_e and R_p of the electromagnetic and electroacoustic modes, referred to the current maximum I_0 (the feed current in the present case), are defined by

$$R_e = \frac{1}{\eta I_0^2} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \{|E_e(\theta)|^2 + |E_e(\phi)|^2\} r^2 \sin\theta \, d\theta \, d\phi \,, \tag{1}$$

$$R_p = \frac{1}{\eta I_0^2} \left(\frac{\omega}{\omega_p}\right)^2 \left(\frac{u}{c}\right) \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} |E_p(r)|^2 r^2 \sin\theta \, d\theta \, d\phi \,, \tag{2}$$

where η is the intrinsic impedance of the plasma medium, ω is the source angular frequency and ω_p the plasma angular frequency, while u is the r.m.s. thermal velocity of the electrons and c is the free-space velocity of light.

Making appropriate substitutions for $E_e(\theta)$, $E_e(\phi)$ and $E_p(r)$ from equations (16) and (18) of Talekar and Soni (1974*a*), and then evaluating the inner integrals over ϕ, we get the following expressions for R_e and R_p:

$$\begin{aligned}R_e = (\eta/4\pi)(\beta_e a)^2 \int_{\theta=0}^{\pi} \sin\theta \Bigg[& \tan^2\psi \sin^2\theta \Bigg\{ \left(\frac{J_0(\beta_e a \sin\theta)}{K_1}\right)^2 \Big(1-\cos(K_1\phi_0)\Big) \\ & + 2\sum_{q=1}^{\infty} \left(\frac{J_q(\beta_e a \sin\theta)}{(K_1^2-q^2)}\right)^2 f_1(q, K_1, \phi_0) \Bigg\} \\ & - \frac{4}{(\beta_e a)} \tan\psi \cos\theta \sum_{q=1}^{\infty} q \left(\frac{J_q(\beta_e a \sin\theta)}{(K_1^2-q^2)}\right)^2 f_2(q, K_1, \phi_0) \\ & + \frac{2\cot^2\theta}{(\beta_e a)^2} \sum_{q=1}^{\infty} q^2 \left(\frac{J_q(\beta_e a \sin\theta)}{(K_1^2-q^2)}\right)^2 f_1(q, K_1, \phi_0) \\ & + \left(\frac{J_1(\beta_e a \sin\theta)}{K_1}\right)^2 \Big(1-\cos(K_1\phi_0)\Big) \\ & + 2\sum_{q=1}^{\infty} \left(\frac{J_q'(\beta_e a \sin\theta)}{(K_1^2-q^2)}\right)^2 f_1(q, K_1, \phi_0) \Bigg] \, d\theta, \end{aligned} \tag{3}$$

$$\begin{aligned}R_p = \frac{(\beta a)^2 \sec^2\psi}{4\pi\varepsilon_0 \beta_p u^2} \left(\frac{\omega_p^2}{\omega}\right) \int_{\theta=0}^{\pi} \sin\theta \Bigg\{ & \left(\frac{J_0(\beta_p a \sin\theta)}{K_2}\right)^2 \Big(1-\cos(K_2\phi_0)\Big) \\ & + 2\sum_{q=1}^{\infty} \left(\frac{J_q(\beta_p a \sin\theta)}{(K_2^2-q^2)}\right)^2 f_3(q, K_2, \phi_0) \Bigg\} \, d\theta, \end{aligned} \tag{4}$$

In the expressions (3) and (4), the propagation constants β_e and β_p are for electromagnetic waves in the plasma and for the plasma mode respectively, the radius and pitch angle of the helix are a and ψ, and the Bessel function of integer order is denoted by J. We also have

$$f_1(q, K_1, \phi_0) = [(K_1^2+q^2)\{1-\cos(q\phi_0)\cos(K_1\phi_0)\} - 2K_1 q \sin(q\phi_0)\sin(K_1\phi_0)], \tag{5}$$

$$f_2(q, K_1, \phi_0) = [(K_1^2+q^2)\sin(q\phi_0)\sin(K_1\phi_0) - 2K_1 q\{1-\cos(q\phi_0)\cos(K_1\phi_0)\}], \tag{6}$$

$$f_3(q, K_2, \phi_0) = [(K_2^2+q^2)\{1-\cos(q\phi_0)\cos(K_2\phi_0)\} - 2K_2 q \sin(q\phi_0)\sin(K_2\phi_0)], \tag{7}$$

$$K_1 = \beta_e a \tan\psi \cos\theta - \beta a \sec\psi, \qquad K_2 = \beta_p a \tan\psi \cos\theta - \beta a \sec\psi, \tag{8}$$

$$\phi_0 = 2\pi n. \tag{9}$$

The comprehensive expressions (3) and (4) hold for an antenna of any size and pitch angle, and for a fractional or integral number n of turns. Corresponding expressions for the radiation resistances pertaining to travelling-wave multi- and single-turn circular loop antennas (TWCLA), travelling-wave circular arc antennas (TWCAA) and nonresonant end-fed linear antennas (NREFLA) are obtained as particular cases. Such results are in agreement with those obtained by Talekar and Soni (1973, 1974*b*) for TWCLAs and by Talekar and Gupta (1967) for NREFLAs.

It is extremely difficult to evaluate the integrals occurring in the expressions for R_e and R_p analytically. However, the integral in equation (3) for R_e can be evaluated analytically for a helix of small pitch angle ($\psi < 10°$) in the case $\beta = \beta_e$, which is suggested by experimental as well as theoretical studies. For this evaluation, the standard formula given by Bailey (1938) may be used along with the procedure given in the Appendix of Talekar and Soni (1973). This yields for R_e:

$$\begin{aligned} R_e = (\eta_0/\pi)(\beta_0 a)\Bigg\{ \sum_{q=0}^{\infty} J_{2q+1}(2\beta_e a) \Bigg(& \tfrac{1}{2} f_4(q+1, \beta_e a, \phi_0) - \tfrac{1}{2} f_4(q, \beta_e a, \phi_0) \\ & + (\beta_e a)^{-2} \sum_{p=0}^{q} (\beta_e^2 a^2 - p^2) f_4(p, \beta_e a, \phi_0) \\ & - \sin^2(\tfrac{1}{2}\beta_e a\phi_0)/(\beta_e a)^2 \Bigg) \Bigg\} \\ + (4\eta/\pi)\psi^2 \Bigg((3!)^{-1} & \sin^2(\tfrac{1}{2}\beta_e a\phi_0)\, {}_2F_3(1/2, 2; 1, 1, 5/2; -\beta_e^2 a^2) \\ & + \sum_{q=1}^{\infty} f_4(q, \beta_e a, \phi_0)(\beta_e a)^{2q+2} \{(q+1)^2/(2q+3)!\} \\ & \times {}_2F_3(q+1/2, q+2; 2q+1, q+1, q+5/2; -\beta_e^2 a^2) \Bigg), \end{aligned} \tag{10}$$

where η_0 is the intrinsic impedence of free space, β_0 is the propagation constant for electromagnetic waves in free space, while

$$\begin{aligned} f_4(q, \beta_e a, \phi_0) = (\beta_e^2 a^2 - q^2)^{-2} [& (\beta_e^2 a^2 + q^2)\{1 - \cos(q\phi_0)\cos(\beta_e a\phi_0)\} \\ & - 2q(\beta_e a)\sin(q\phi_0)\sin(\beta_e a\phi_0)] \end{aligned} \tag{11}$$

and ${}_2F_3$ is the generalized hypergeometric function. Further, for helical antennas having a purely integral number of turns ($n = n_0$), considerable simplification of the expression (10) is possible, enabling it to be reduced to a more compact form. Thus, for $n = n_0$, we have

$$\begin{aligned} R_e = & (\eta_0/\pi)\beta_0 a \sin^2(\beta_e a n_0 \pi) \\ & \times \Bigg\{ \sum_{q=0}^{\infty} J_{2q+1}(2\beta_e a) \Bigg(\frac{\beta_e^2 a^2 + (q+1)^2}{\{\beta_e^2 a^2 - (q+1)^2\}^2} - \frac{\beta_e^2 a^2 + q^2}{(\beta_e^2 a^2 - q^2)^2} - \frac{1}{(\beta_e a)^2} \\ & \qquad + \frac{2}{(\beta_e a)^2} \sum_{p=0}^{q} \frac{\beta_e^2 a^2 + p^2}{\beta_e^2 a^2 - p^2} \Bigg) \Bigg\} \\ & + (4\eta/\pi)\psi^2 \sin^2(\beta_e a n_0 \pi) \\ & \times \Bigg(\frac{1}{3!}\, {}_2F_3(1/2, 2; 1, 1, 5/2; -\beta_e^2 a^2) \\ & + 2 \sum_{q=1}^{\infty} \frac{\beta_e^2 a^2 + q^2}{(\beta_e^2 a^2 - q^2)^2} (\beta_e a)^{2q+2} \frac{(q+1)^2}{(2q+3)!} \\ & \times {}_2F_3(q+1/2, q+2; 2q+1, q+1, q+5/2; -\beta_e^2 a^2) \Bigg). \end{aligned} \tag{12}$$

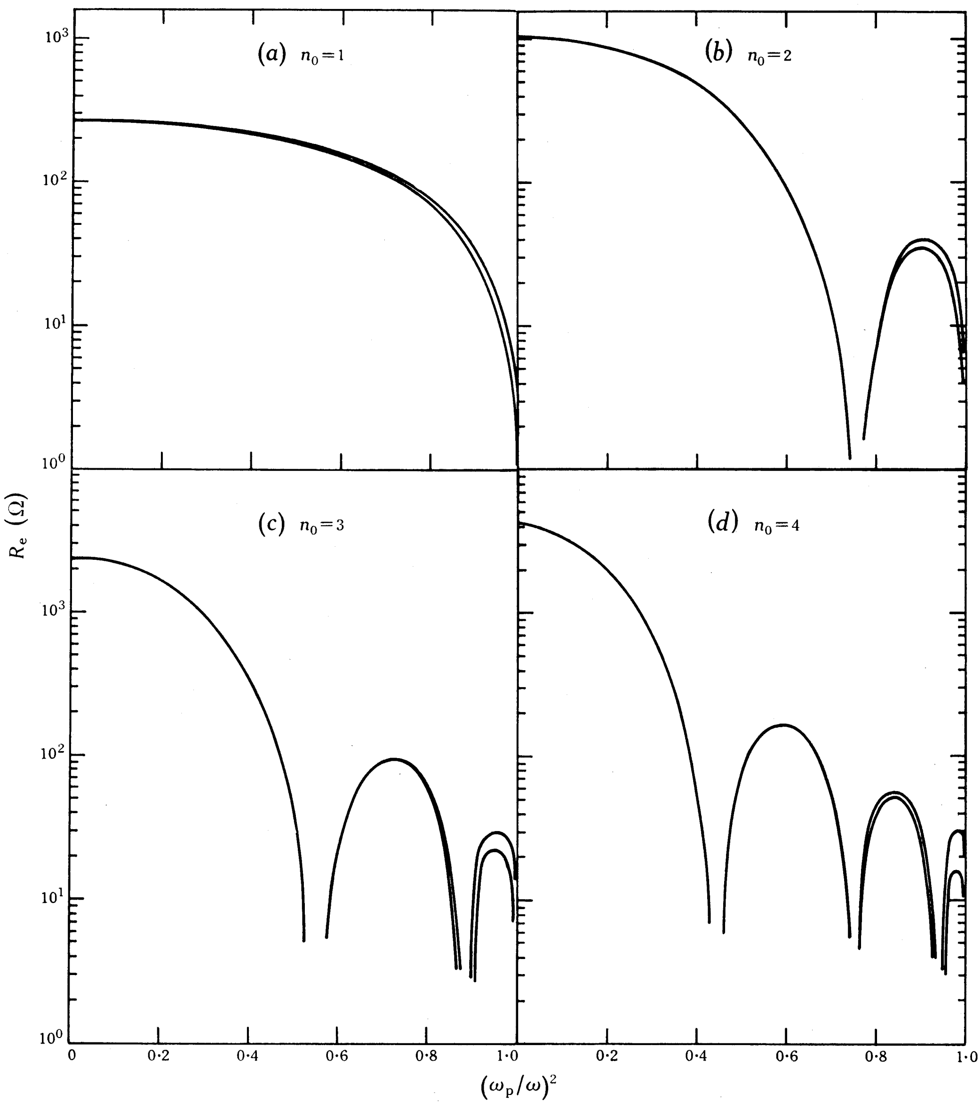

Fig. 1. Calculated dependence of the electromagnetic mode radiation resistance R_e as a function of the square of the normalized plasma frequency $(\omega_p/\omega)^2$ for a travelling-wave multi-turn helical antenna having an integral number of turns n_0 as indicated in (*a*)–(*d*). The calculations were performed for $C_{\lambda_0} = 1 \cdot 0$ and $\psi = 5°$ and $10°$. The $\psi = 5°$ results are those with lower resistance at high values of $(\omega_p/\omega)^2$.

In order to observe the effect of the pitch angle and the number of turns n_0 on R_e, numerical computations were made for two values of ψ (5° and 10°) and for four values of n_0 (1, 2, 3 and 4) for the helix of normalized circumference $C_{\lambda_0} = 2\pi a/\lambda_0$ equal to $1 \cdot 0$. These computed results are shown in Figs 1*a*–1*d*, in which R_e is plotted against the square of the normalized plasma frequency $(\omega_p/\omega)^2$. It can be seen from the graphs that the variation of R_e with $(\omega_p/\omega)^2$ is similar to that for multi-turn loop antennas (Talekar and Soni 1974*b*): in the present case also, R_e passes through maxima and nulls as $(\omega_p/\omega)^2$ is increased from 0 to $1 \cdot 0$.

It can also be seen from Figs 1*a*–1*d* that at lower values of $(\omega_p/\omega)^2$ the increase in R_e with ψ is negligible. However, for higher values of $(\omega_p/\omega)^2$ the increase in R_e with ψ is appreciable. Thus, for source frequencies near to the plasma frequency, it is possible to radiate more power in the electromagnetic mode by increasing the pitch of the helix.

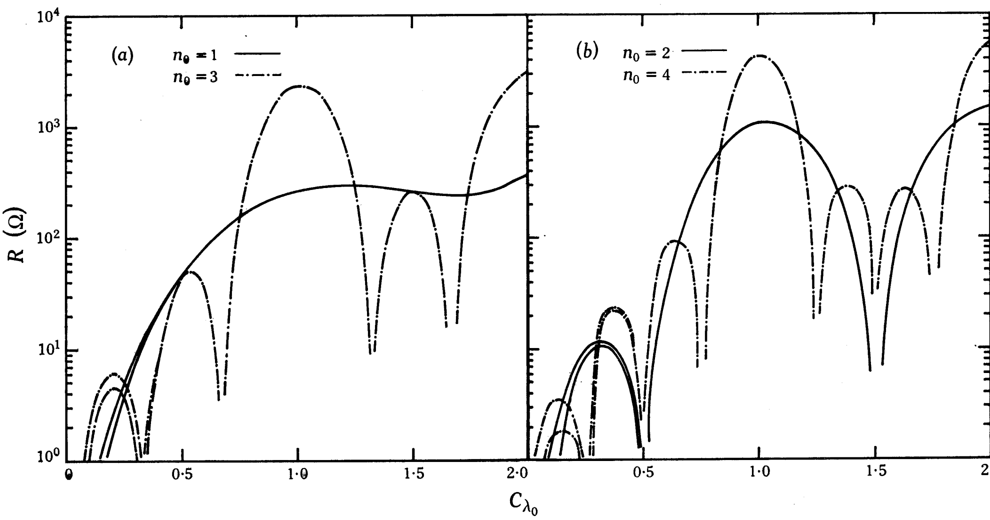

Fig. 2. Calculated dependence of the free-space radiation resistance R as a function of C_{λ_0} for a travelling-wave multi-turn helical antenna having an integral number of turns n_0 as indicated in (*a*) and (*b*). The calculations were performed for $\psi = 5°$ and 10°. The $\psi = 5°$ results are those with lower resistance at low values of C_{λ_0}.

Radiation Resistance of Helical Antenna in Vacuum

If we now consider the medium to be free space instead of a plasma, the only existing mode is electromagnetic. By letting $\omega_p \to 0$, so that $\beta_e = \beta_0$ and $\eta = \eta_0$ in equation (3), we can obtain the general expression for the radiation resistance pertaining to the travelling-wave helical antenna in terms of the general phase propagation constant of the current wave. Further, for $\beta = \beta_0$ and for a helical antenna of a small pitch angle, the radiation resistance is given by

$$\begin{aligned} R = (\eta_0/\pi)\beta_0 a\Bigg\{ \sum_{q=0}^{\infty} \mathrm{J}_{2q+1}(2\beta_0 a)\Bigg(\tfrac{1}{2}f_5(q+1,\beta_0 a,\phi_0) - \tfrac{1}{2}f_5(q,\beta_0 a,\phi_0) \\ +(\beta_0 a)^{-2}\sum_{p=0}^{q}(\beta_0^2 a^2 - p^2) f_5(p,\beta_0 a,\phi_0) \\ -\sin^2(\tfrac{1}{2}\beta_0 a\phi_0)/(\beta_0 a)^2\Bigg)\Bigg\} \\ +4(\eta_0/\pi)\psi^2\Bigg((3!)^{-1}\sin^2(\tfrac{1}{2}\beta_0 a\phi_0)\,{}_2\mathrm{F}_3(1/2,2;1,1,5/2;\,-\beta_0^2 a^2) \\ +\sum_{q=1}^{\infty} f_5(q,\beta_0 a,\phi_0)(\beta_0 a)^{2q+2}(q+1)^2/(2q+3)! \\ \times\,{}_2\mathrm{F}_3(q+1/2,q+2;2q+1,q+1,q+5/2;\,-\beta_0^2 a^2)\Bigg), \end{aligned} \qquad (13)$$

where

$$f_5(q,\beta_0 a,\phi_0) = (\beta_0^2 a^2 - q^2)^{-2}[(\beta_0^2 a^2 + q^2)\{1 - \cos(q\phi_0)\cos(\beta_0 a\phi_0)\} - 2q\beta_0 a \sin(q\phi_0)\sin(\beta_0 a\phi_0)]. \tag{14}$$

For $n = n_0$, the expression (13) reduces to

$$\begin{aligned} R = (\eta_0/\pi)\beta_0 a \sin^2(\beta_0 a n_0 \pi)\Bigg\{ \sum_{q=0}^{\infty} J_{2q+1}(2\beta_0 a)\Bigg(& \frac{\beta_0^2 a^2 + (q+1)^2}{\{\beta_0^2 a^2 - (q+1)^2\}^2} \\ & - \frac{\beta_0^2 a^2 + q^2}{(\beta_0^2 a^2 - q^2)^2} - \frac{1}{(\beta_0 a)^2} \\ & + \frac{2}{(\beta_0^2 a^2)} \sum_{p=0}^{q} \frac{\beta_0^2 a^2 + p^2}{(\beta_0^2 a^2 - p^2)}\Bigg)\Bigg\} \\ + 4(\eta_0/\pi)\psi^2 \sin^2(\beta_0 a n_0 \pi) & \\ \times \Bigg((3!)^{-1}\, {}_2F_3(1/2, 2; 1, 1, 5/2; -\beta_0^2 a^2) & \\ + 2\sum_{q=1}^{\infty} \frac{\beta_0^2 a^2 + q^2}{(\beta_0^2 a^2 - q^2)^2} (\beta_0 a)^{2q+1} \{(q+1)^2/(2q+3)!\} & \\ \times {}_2F_3(q+1/2, q+2; 2q+1, q+1, q+5/2; -\beta_0^2 a^2)\Bigg). & \end{aligned} \tag{15}$$

This expression was used to compute the radiation resistance R of a helical antenna possessing an integral number of turns as a function of C_{λ_0}, for two values of ψ (5° and 10°) and four values of n_0 (1, 2, 3 and 4). These results are shown in Figs 2*a* and 2*b*. It can be seen that, for helical antennas of small pitch angle, the variation of R with C_{λ_0} is similar to that for a travelling-wave multi-turn loop antenna. For low values of C_{λ_0}, R increases with ψ; while for large values of C_{λ_0}, the effect of ψ on R is almost negilgible.

Conclusions

Expressions for the electromagnetic and the electroacoustic mode components of radiation resistance pertaining to a helical antenna in a compressible electron plasma are more generally applicable, and cover the special cases of TWCAAs, TWCLAs and NREFLAs, not only for a plasma medium but also for free space. From the particular results obtained for the electromagnetic mode component of the radiation resistance for small angled helices, we find that the radiation resistance increases with the pitch angle at high values of $(\omega_p/\omega)^2$ but remains almost unaffected at low values. It is therefore concluded that within these limits it is possible to put more power in the electromagnetic mode by increasing the pitch angle of the helix.

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References

Bailey, W. N. (1938). *Q. J. Math. (Oxford)* **9**, 141.
Talekar, V. L., and Gupta, R. K. (1967). *Int. J. Electron.* **23**, 533.
Talekar, V. L., and Soni, K. R. (1973). *Int. J. Electron.* **34**, 497.
Talekar, V. L., and Soni, K. R. (1974*a*). *Int. J. Electron.* **36**, 679.
Talekar, V. L., and Soni, K. R. (1974*b*). *Int. J. Electron.* **36**, 687.

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