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The Effect of Ionization and Recombination on the Resistivity of a Partially Ionized Plasma in a Magnetic Field

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Abstract

The generalized Ohm's law for a partially ionized magnetized plasma composed of ions, electrons and neutral atoms is calculated. The plasma is modelled by a three-fluid treatment, with elastic collisions between all three species, as well as inelastic ionization and recombination collisions being taken into account. Ionization is assumed to be due to electron–atom impacts, and recombination is assumed to be due to three-body electron–electron–atom collisions. The resistivity is calculated, and it is shown that the major effect of ionization and recombination is to reduce the resistivity for currents perpendicular to the magnetic field under typical laboratory conditions. However, this resistivity is still greater than Coulomb resistivity, owing to plasma–neutral gas friction.

1. Introduction

The generalized Ohm's law derived by Cowling (1956) for a partially ionized gas of fixed degree of ionization, consisting of electrons, singly ionized ions and neutral atoms in a magnetic field has been utilized in shock wave studies by Bighel *et al.* (1973, 1977) and Cramer (1975). An important feature of the magnetized partially ionized gas is that the effect of ion slip, when the velocity of ions relative to neutrals is comparable with the velocity of electrons relative to ions, is to enhance the resistivity for currents flowing perpendicularly to the magnetic field, with the extra ohmic dissipation going to ion and neutral heating. The ion and neutral heating observed in ionizing switch-on shock waves by Bighel *et al.* (1973) and in MHD switch-on shock waves by Bighel *et al.* (1977) has been attributed to this process (Cramer 1975). Other experiments by Craig and Paul (1973) and Bighel *et al.* (1974) have resulted in the observation of anomalously high damping of the upstream whistler wave which occurs in MHD switch-on shock waves. This can be explained by the enhanced resistivity due to the low residual concentration of neutrals in such experiments (Cramer 1975).

An alternative explanation of such observed enhanced resistivity is the nonclassical dissipation caused by the ion–acoustic instability (Paul *et al.* 1971). In order to compare the effects on the plasma resistivity of the above two processes, it is important to elucidate all the features of the interaction of the plasma with the neutral gas. To that end, we here examine the effects on the resistivity of two other important processes in such an interaction: ionization and recombination.

2. Generalized Ohm's Law

Consider a partially ionized plasma in electric and magnetic fields $\boldsymbol{E}$ and $\boldsymbol{B}$. The charge and mass of a particle of species s are denoted by e_s and m_s, while the species velocity, number density and pressure are denoted by $\boldsymbol{v}_s$, n_s and P_s respectively. The species density is represented by $\rho_s = n_s m_s$, and the total mass density is denoted by ρ. We consider the case of a fluid composed of singly charged ions of charge $+e$ together with electrons and neutral atoms under conditions of local charge equality ($n_e = n_i = n$), and we neglect any terms of relative order m_e/m_i in the resulting equations, so that we can assume m_a (the mass of an atom) $= m_i$ (the mass of the corresponding ion).

We proceed to establish fluid equations for the three species, which may subsequently be combined to arrive at a generalized Ohm's law. The mass conservation equations for species s may be written (Woods 1975)

$$\partial\rho_s/\partial t + \nabla.(\rho_s \boldsymbol{v}_s) = \sum_t a_{st} - a_{ts}, \tag{1}$$

where a_{st} is the rate per unit volume at which fluid s gains mass from fluid t. We have for ionization and recombination

$$\left.\begin{aligned} a_{ie} &= a_{ei} = 0, \\ a_{ea} &= m_e n/\tau_I, \qquad a_{ae} = m_e n/\tau_R, \\ a_{ia} &= m_i n/\tau_I, \qquad a_{ai} = m_i n/\tau_R, \end{aligned}\right\} \tag{2}$$

where τ_I and τ_R are the ionization and recombination times respectively.

For dense laboratory plasmas, electron impact ionization and three-body recombination would be the dominant processes whereas, for tenuous astrophysical plasmas, photoionization and radiative recombination would dominate. The mechanism of ionization through atom–atom collisions has been invoked in the von Neumann–Z'eldovich model of the ionizing shock wave (Chubb 1968; Hoffert 1968; Molander and Berger 1969). In that model a viscous subshock raises the atom temperature and so initiates ionization by atom–atom collisions, with electron impact ionization occurring in the following relaxation region. However, Bighel *et al.* (1973) assumed that the radiation from the hot back of the ionizing shock photoionizes the gas far upstream until the electron density is large enough for electron impact ionization to dominate, and this model agreed with the experimentally observed electron density profile in the shock.

In order to write down the momentum conservation equations for the fluids, a modification of the term in Wood's (1975) equation describing momentum exchange due to ion–electron recombination is necessary. This is required because recombination is a three-body process, involving the ion and *two* electrons. According to J. J. Thompson's theory of three-body recombination (Mitchner and Kruger 1973), the electron which is to recombine with the ion must suffer an elastic collision with another electron, thus reducing its kinetic energy (relative to the ion) to an extent that it moves in a bound trajectory about the ion. Thus, in the frame of reference of the ion fluid, even though an electron has been lost from the electron fluid, no momentum has been lost from the electron fluid because the momentum was passed on to the second electron in the initial elastic collision. In an arbitrary frame of reference, the momentum loss of the electron fluid per unit time and volume is $a_{ae}\boldsymbol{v}_i$.

The momentum conservation equation for the fluid of species s is

$$\partial(\rho_s \boldsymbol{v}_s)/\partial t + \nabla \cdot (\rho_s \boldsymbol{v}_s \boldsymbol{v}_s) + \nabla P_s - n_s e_s(\boldsymbol{E} + \boldsymbol{v}_s \times \boldsymbol{B}) = \boldsymbol{A}_s, \tag{3}$$

where $\boldsymbol{A}_s$ is the rate of change of momentum of fluid s due to collisions (both elastic and inelastic) with the other species. For the ion fluid, $\boldsymbol{A}_s$ has the form (Woods 1975)

$$\boldsymbol{A}_{\mathrm{i}} = \sum_t \left(\frac{n_{\mathrm{i}} m_{\mathrm{i}} m_t}{m_{\mathrm{i}} + m_t} \frac{\boldsymbol{v}_t - \boldsymbol{v}_{\mathrm{i}}}{\tau_{\mathrm{i}t}} + a_{\mathrm{i}t} \boldsymbol{v}_t - a_{t\mathrm{i}} \boldsymbol{v}_{\mathrm{i}} \right), \tag{4}$$

where $\tau_{\mathrm{i}t}$ is the collision time for elastic momentum transfer from a particle of species t to an ion. (We neglect viscous forces in the momentum equations, and so have assumed scalar forms for the partial pressures.) From the equations (2) we have

$$\boldsymbol{A}_{\mathrm{i}} = \frac{n_{\mathrm{i}} m_{\mathrm{i}} m_{\mathrm{a}}}{m_{\mathrm{i}} + m_{\mathrm{a}}} \frac{\boldsymbol{v}_{\mathrm{a}} - \boldsymbol{v}_{\mathrm{i}}}{\tau_{\mathrm{ia}}} + \frac{n_{\mathrm{i}} m_{\mathrm{i}} m_{\mathrm{e}}}{m_{\mathrm{i}} + m_{\mathrm{e}}} \frac{\boldsymbol{v}_{\mathrm{e}} - \boldsymbol{v}_{\mathrm{i}}}{\tau_{\mathrm{ie}}} + a_{\mathrm{ia}} \boldsymbol{v}_{\mathrm{a}} - a_{\mathrm{ai}} \boldsymbol{v}_{\mathrm{i}}. \tag{5}$$

The electron momentum transfer is

$$\boldsymbol{A}_{\mathrm{e}} = \frac{n_{\mathrm{e}} m_{\mathrm{e}} m_{\mathrm{i}}}{m_{\mathrm{e}} + m_{\mathrm{i}}} \frac{\boldsymbol{v}_{\mathrm{i}} - \boldsymbol{v}_{\mathrm{e}}}{\tau_{\mathrm{ei}}} + \frac{n_{\mathrm{e}} m_{\mathrm{e}} m_{\mathrm{a}}}{m_{\mathrm{e}} + m_{\mathrm{a}}} \frac{\boldsymbol{v}_{\mathrm{a}} - \boldsymbol{v}_{\mathrm{e}}}{\tau_{\mathrm{ea}}} + a_{\mathrm{ea}} \boldsymbol{v}_{\mathrm{a}} - a_{\mathrm{ae}} \boldsymbol{v}_{\mathrm{i}}, \tag{6}$$

where the last term is the momentum loss in recombination discussed above. The atom momentum transfer is

$$\boldsymbol{A}_{\mathrm{a}} = \frac{n_{\mathrm{a}} m_{\mathrm{a}} m_{\mathrm{i}}}{m_{\mathrm{a}} + m_{\mathrm{i}}} \frac{\boldsymbol{v}_{\mathrm{i}} - \boldsymbol{v}_{\mathrm{a}}}{\tau_{\mathrm{ai}}} + \frac{n_{\mathrm{a}} m_{\mathrm{a}} m_{\mathrm{e}}}{m_{\mathrm{a}} + m_{\mathrm{e}}} \frac{\boldsymbol{v}_{\mathrm{e}} - \boldsymbol{v}_{\mathrm{a}}}{\tau_{\mathrm{ae}}} + (a_{\mathrm{ai}} + a_{\mathrm{ae}}) \boldsymbol{v}_{\mathrm{i}} - (a_{\mathrm{ea}} + a_{\mathrm{ia}}) \boldsymbol{v}_{\mathrm{a}}, \tag{7}$$

with the atoms gaining the momentum $a_{\mathrm{ae}} \boldsymbol{v}_{\mathrm{i}}$ lost from the electrons in the recombination process.

Equations (3), (5), (6) and (7) may now be employed to derive the desired generalized Ohm's law. However, before proceeding, it is useful to complete the conservation equations by writing down the equations for energy conservation of the species. The electron energy conservation equation will be shown to be consistent with the above discussion of momentum transfer in recombination. The general equation may be written (Woods 1975):

$$\rho_s \{ \partial u_s / \partial t + (\boldsymbol{v}_s \cdot \nabla) u_s + P_s \partial \rho_s^{-1} / \partial t + P_s (\boldsymbol{v}_s \cdot \nabla) \rho_s^{-1} \} + \nabla \cdot \boldsymbol{Q}_s + \Phi_s = \phi_s + \xi_s + \zeta_s, \tag{8}$$

where u_s is the internal energy per unit mass, $\boldsymbol{Q}_s$ is the heat flux vector, Φ_s is the energy gain of fluid s from the other fluids due to temperature differences, ξ_s is the energy gain of fluid s due to velocity differences, and ζ_s is the energy gain of fluid s due to gain and loss of particles from fluids.

Now for the ion fluid, the last term of equation (8) has the form

$$\begin{aligned} \zeta_{\mathrm{i}} &= \sum_t \{ a_{\mathrm{i}t} (u_t + P_t \rho_t^{-1} + \tfrac{1}{2} v_t^2) - a_{t\mathrm{i}} (u_{\mathrm{i}} + P_{\mathrm{i}} \rho_{\mathrm{i}}^{-1} + \tfrac{1}{2} v_{\mathrm{i}}^2) \} \\ &= a_{\mathrm{ia}} (u_{\mathrm{a}} + P_{\mathrm{a}} \rho_{\mathrm{a}}^{-1} + \tfrac{1}{2} v_{\mathrm{a}}^2) - a_{\mathrm{ai}} (u_{\mathrm{i}} + P_{\mathrm{i}} \rho_{\mathrm{i}}^{-1} + \tfrac{1}{2} v_{\mathrm{i}}^2). \end{aligned} \tag{9}$$

However, for the electron fluid, because of the previously discussed mechanism of recombination, the energy lost per unit mass in a single recombination reaction is equal to that lost by the ion fluid per unit mass. Thus the corresponding term to

(9) for the electron fluid (taking account of the ionization energy E_I lost from the electrons in ionization, and gained by them in recombination) is

$$\zeta_e = a_{ea}(u_a + P_a \rho_a^{-1} + \tfrac{1}{2}v_a^2 + E_I/m_e) - a_{ae}(u_i + P_i \rho_i^{-1} + \tfrac{1}{2}v_i^2 + E_I/m_e). \tag{10}$$

The atom energy transfer term is

$$\zeta_a = (a_{ai} + a_{ae})(u_i + P_i \rho_i^{-1} + \tfrac{1}{2}v_i^2) - (a_{ia} + a_{ea})(u_a + P_a \rho_a^{-1} + \tfrac{1}{2}v_a^2). \tag{11}$$

In the equilibrium state, when ζ_e must vanish and when $a_{ea} = a_{ae}$ holds, we see from equation (10) that the ion temperature T_i and the atom temperature T_a are equal (to within an error of $O(m_e/m_i)$). If the energy lost by the electron fluid were $a_{ae}(u_e + P_e \rho_e^{-1} + \tfrac{1}{2}v_e^2 + E_I/m_e)$ then the vanishing of ζ_e would give $T_e/m_e = T_a/m_a$, whereas for equilibrium we must have $T_e = T_a$. Thus the use of the above recombination model is consistent with the requirements of equilibrium.

Generalizing the notation of Cowling (1956) we write

$$\kappa_i = 1/2\omega_i \tau_{ia}, \quad \kappa_e = 1/\omega_e \tau_{ea}, \quad \kappa = 1/\omega_e \tau_{ei}, \quad \kappa_I = 1/\omega_i \tau_I, \quad \kappa_R = 1/\omega_i \tau_R,$$

where ω_i and ω_e are the ion and electron cyclotron angular frequencies in the field $\boldsymbol{B}$. If we multiply equation (3) by e_s/m_s and sum over all species s we obtain, with the help of equation (1), the generalized Ohm's law

$$\frac{\partial \boldsymbol{J}}{\partial t} + \nabla.(\boldsymbol{J}\boldsymbol{v} + \boldsymbol{v}\boldsymbol{J}) + \nabla\left(\frac{eP_i}{m_i} - \frac{eP_e}{m_e}\right) = \sum_s \left(\frac{n_s e_s^2}{m_s}(\boldsymbol{E} + \boldsymbol{v}_s \times \boldsymbol{B}) + \frac{e_s}{m_s}\boldsymbol{A}_s\right), \tag{12}$$

where

$$\boldsymbol{v} = \sum_s \rho_s \boldsymbol{v}_s \Big/ \sum_s \rho_s \qquad \text{and} \qquad \boldsymbol{J} = \sum_s n_s e_s \boldsymbol{v}_s$$

are the overall fluid velocity and the current density respectively. Neglecting terms of relative order m_e/m_i, and employing equations (4), (5) and (6), equation (12) becomes

$$\begin{aligned}(m_e/e)\{\partial \boldsymbol{J}/\partial t + \nabla.(\boldsymbol{J}\boldsymbol{v} + \boldsymbol{v}\boldsymbol{J})\} - \nabla P_e \\ = ne(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) + \boldsymbol{J}' \times \boldsymbol{B} - \boldsymbol{J} \times \boldsymbol{B} - (\kappa + \kappa_e)B\boldsymbol{J} + \{\kappa_e - (m_e/m_i)\kappa_i\}F^{-1}B\boldsymbol{J}',\end{aligned} \tag{13}$$

where the ion-slip current, i.e. the current carried by the ions relative to the total fluid, is $\boldsymbol{J}' = ne\boldsymbol{V}_i = ne(\boldsymbol{v}_i - \boldsymbol{v})$.

To eliminate the ion-slip current from equation (13), we follow the procedure of Cowling (1956) by making use of the combined ion–electron momentum equation, which may be written

$$\begin{aligned}\partial(\rho_i \boldsymbol{V}_i)/\partial t + \rho_i(\boldsymbol{V}_i.\nabla)\boldsymbol{v} + \nabla.(\rho_i \boldsymbol{v}\boldsymbol{V}_i) + \rho_i\{\partial \boldsymbol{v}/\partial t + (\boldsymbol{v}.\nabla)\boldsymbol{v}\} + \nabla(P_i + P_e) - \boldsymbol{J} \times \boldsymbol{B} \\ = \boldsymbol{A}_e + \boldsymbol{A}_i - (a_{ia} - a_{ai})\boldsymbol{v}.\end{aligned} \tag{14}$$

Electron inertia terms have been neglected here. The first three terms of equation (14) are the rate of change of the 'ion-slip momentum' relative to the overall momentum of the plasma.

In order to apply the results of this analysis to plasmas not in equilibrium, in particular to plasmas with spatial gradients as in a shock wave front, the rate of change of the ion-slip momentum in equation (14) and the convective term in the Ohm's law (equation 13) must in general be retained. However, these terms can be neglected if we assume that the length scale of macroscopic change is much greater than the mean free paths of collision (both elastic and inelastic). For simplicity we also consider only frames of reference in which the plasma quantities are steady, so that we can neglect the time derivative in the Ohm's law (equation 13).

Using the overall plasma momentum equation

$$\rho\{\partial \boldsymbol{v}/\partial t+(\boldsymbol{v}.\nabla)\boldsymbol{v}\}+\nabla P = \boldsymbol{J}\times\boldsymbol{B} \tag{15}$$

to eliminate the fourth term in equation (14), we obtain for the ion-slip current

$$\boldsymbol{J}' = \{F/B(\kappa_{\mathrm{i}}+\kappa_{\mathrm{e}}+\kappa_{\mathrm{I}})\}\{F\boldsymbol{J}\times\boldsymbol{B}+(1-F)\nabla P-\nabla(P_{\mathrm{i}}+P_{\mathrm{e}})+\kappa_{\mathrm{e}}B\boldsymbol{J}\}, \tag{16}$$

where $F = n_{\mathrm{a}}/(n_{\mathrm{a}}+n)$. Inserting equation (16) into (14), the generalized Ohm's law becomes

$$\begin{aligned} ne(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B}) &= \kappa B\boldsymbol{J}+\frac{\kappa_{\mathrm{e}}\kappa_{\mathrm{i}}^*}{\kappa_{\mathrm{e}}+\kappa_{\mathrm{i}}^*}B\boldsymbol{J}+\{1-(\beta+\beta^*)F\}\boldsymbol{J}\times\boldsymbol{B}-[\nabla P_{\mathrm{e}}+\beta^*\{(1-F)\nabla P-\nabla(P_{\mathrm{i}}+P_{\mathrm{e}})\}] \\ &\quad -\frac{F}{B(\kappa_{\mathrm{e}}+\kappa_{\mathrm{i}}^*)}[F(\boldsymbol{J}\times\boldsymbol{B})\times\boldsymbol{B}+\{(1-F)\nabla P-\nabla(P_{\mathrm{i}}+P_{\mathrm{e}})\}\times\boldsymbol{B}], \end{aligned} \tag{17}$$

with

$$\beta = \frac{\kappa_{\mathrm{e}}}{\kappa_{\mathrm{e}}+\kappa_{\mathrm{i}}^*}, \qquad \beta^* = \frac{\kappa_{\mathrm{e}}-(m_{\mathrm{e}}/m_{\mathrm{i}})\kappa_{\mathrm{i}}}{\kappa_{\mathrm{e}}+\kappa_{\mathrm{i}}^*}, \qquad \kappa_{\mathrm{i}}^* = \kappa_{\mathrm{i}}+\kappa_{\mathrm{I}}.$$

Thus the effect of ionization and recombination processes on the Ohm's law of the plasma is to replace κ_{i} by κ_{i}^*. The consequences of this effective increase of the ion–neutral friction for the plasma resistivity are investigated in the following section. We note that our assumption of small spatial gradients implies that $\kappa_{\mathrm{I}} \approx \kappa_{\mathrm{R}}$ from equations (1), that is, that the plasma is in local ionizational equilibrium, so that κ_{R} could replace κ_{I} in κ_{i}^*.

3. Resistivity of Plasma

If we first consider the case that the ion-slip current $\boldsymbol{J}'$ is negligible in the Ohm's law, either because the gas is strongly collision dominated (i.e. the ion–neutral collision frequency is much greater than the ion-cyclotron frequency) or because the gas is nearly fully ionized, then the Ohm's law becomes

$$\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B} = \eta_0\boldsymbol{J}+(\boldsymbol{J}\times\boldsymbol{B}-\nabla P_{\mathrm{e}})/ne, \tag{18}$$

where

$$\eta_0 = (B/ne)(\kappa+\kappa_{\mathrm{e}})$$

is the resistivity of the partially ionized plasma, and is not altered by ionization or recombination. This last result is basically due to the recombination mechanism

discussed in Section 2. In recombination, the electron is first slowed to the ion speed by collision with another electron, so that the net current is not affected. Similarly, ionization produces an electron and ion initially moving with the atom fluid velocity, again not affecting the current.

Taking into account now the ion-slip current, we follow Cowling (1956) to arrive at resistivities for currents parallel and perpendicular to the magnetic field. The pressure gradient terms in equation (17) can usually be neglected (Cowling 1956; Cramer 1975) to give

$$\eta_{\parallel} = \frac{B}{ne}\left(\kappa + \frac{\kappa_e \kappa_i^*}{\kappa_e + \kappa_i^*}\right), \quad \eta_{\perp} = \frac{B}{ne}\left(\kappa + \frac{\kappa_e \kappa_i^* + F^2}{\kappa_e + \kappa_i^*}\right). \tag{19a, b}$$

The parallel resistivity is enhanced by ionization, so that we have

$$\eta_{\parallel}(\kappa_I > 0) > \eta_{\parallel}(\kappa_I = 0). \tag{20}$$

Cowling (1956) showed that plasma–neutral-gas friction increased the perpendicular resistivity (neglecting ionization and recombination), and this result was employed by Bighel *et al.* (1973) and Cramer (1975) to explain features of shock wave structure. The effect of ionization and recombination on the perpendicular resistivity may be assessed in the following way. Since we have

$$\eta_{\perp}(\kappa_I > 0) - \eta_{\perp}(\kappa_I = 0) = \frac{B}{ne}\frac{\kappa_I(\kappa_e^2 - F^2)}{(\kappa_e + \kappa_i^*)(\kappa_e + \kappa_i)}, \tag{21}$$

then, for $\kappa_e > F$, ionization further enhances the perpendicular resistivity whereas, for $\kappa_e < F$, ionization decreases the enhanced perpendicular resistivity. The latter situation is typical of laboratory plasmas with $\kappa_e \ll 1$. We have calculated the enhancement of the perpendicular resistivity, in the form of the ratio $\eta_{\perp}/\eta_{\parallel}$, for conditions typical of the shock wave experiments of Bighel *et al.* (1977). The results are shown in Fig. 1 as functions of T_e for the two cases $T_i = 1 \cdot 5 \times 10^4$ K and $T_i = T_e$.

A result similar to equation (19b) for the perpendicular resistivity has been obtained by Lovberg (1963), who considered the resistivity due only to ionizing collisions in an ionizing current sheet. His argument was based on a consideration of the rate of production of the dipole moment due to the separation of the created ion–electron pair in the magnetic field. Such polarization production constitutes a displacement current. Lovberg found the resistivity for this current to be $B/ne\kappa_I$. The result we obtain from the fluid approach presented here, setting $\kappa = \kappa_e = \kappa_i = 0$ in equation (19b), is $BF^2/ne\kappa_I$. Thus the two approaches agree, if the degree of ionization is low.

4. Energy Dissipation

In the case of negligible ionization and recombination, Cowling (1956) and Cramer (1975) have shown that the enhancement of the perpendicular resistivity due to plasma–neutral-gas interaction is a true enhancement, in the sense that it leads to increased energy dissipation in the plasma, and furthermore they have shown that the extra dissipation goes to heating of the ions and neutrals. The question that

remains is how do the ionization and recombination processes affect the particle heating? In the energy equation (8), the energy dissipations due to velocity differences of the fluids and to ionization and recombination may be written

$$\xi_i+\zeta_i = \tfrac{1}{4}\frac{nm_i}{\tau_{ia}}(\boldsymbol{v}_a-\boldsymbol{v}_i)^2+\tfrac{1}{2}\frac{nm_i}{\tau_I}(\boldsymbol{v}_a-\boldsymbol{v}_i)^2, \tag{22a}$$

$$\xi_a+\zeta_a = \tfrac{1}{2}\frac{nm_i}{\tau_{ia}}(\boldsymbol{v}_a-\boldsymbol{v}_i)^2+\tfrac{1}{2}\frac{nm_i}{\tau_R}(\boldsymbol{v}_a-\boldsymbol{v}_i)^2, \tag{22b}$$

$$\xi_e+\zeta_e = \frac{nm_e}{\tau_{ei}}(\boldsymbol{v}_i-\boldsymbol{v}_e)^2+\frac{nm_e}{\tau_{ea}}(\boldsymbol{v}_a-\boldsymbol{v}_e)^2-nE_I\left(\frac{1}{\tau_I}-\frac{1}{\tau_R}\right). \tag{22c}$$

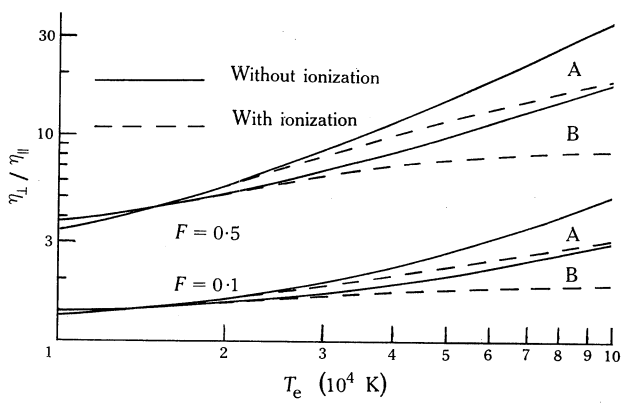

Fig. 1. Plot as a function of the electron temperature T_e of the ratio $\eta_\perp/\eta_\parallel$ of the resistivities for currents perpendicular to and parallel to the magnetic field. The plots are parameterized as $F = 0{\cdot}5$ and $0{\cdot}1$, where $F = n_a/(n_a+n)$ is the mass fraction of neutrals in the plasma. The pair of curves labelled A and B correspond to $T_i = T_a = 1{\cdot}5\times10^4$ K and $T_i = T_a = T_e$ respectively. The magnetic field strength is $0{\cdot}2$ T, the total heavy particle density $n_a+n = 10^{21}\,\mathrm{m}^{-3}$ and the gas is assumed to be hydrogen.

(In the electron energy dissipation term we assume that $E_I \gg kT_e$ and $\frac{1}{2}m_e v_e^2$, a condition that is true for regimes where ionization and recombination are significant.)

From the Ohm's law (equation 13), the ohmic dissipation in the plasma is

$$\boldsymbol{J}.(\boldsymbol{E}+\boldsymbol{v}\times\boldsymbol{B}) = \eta_\parallel J_\parallel^2+\eta_\perp J_\perp^2, \tag{23}$$

where $\eta_\parallel$ and $\eta_\perp$ are defined by the equations (19), and $\boldsymbol{J}_\parallel$ and $\boldsymbol{J}_\perp$ are the parts of the current density $\boldsymbol{J}$ which are respectively parallel and perpendicular to the magnetic field. We now show that the sum of the energy dissipations (22) is equal to the right-hand side of equation (23) plus the inelastic energy change due to ionization and recombination. The velocity differences occurring in the equations (22) are expressed in terms of the current density $\boldsymbol{J}$ by means of the relations

$$\boldsymbol{v}_i-\boldsymbol{v}_e = \boldsymbol{J}/ne, \qquad \boldsymbol{v}_e-\boldsymbol{v}_a = \boldsymbol{V}_i/F-\boldsymbol{J}/ne, \qquad \boldsymbol{v}_i-\boldsymbol{v}_a = \boldsymbol{V}_i/F,$$

together with equation (16) for the ion slip. The result is

$$\xi_i = \tfrac{1}{2}\eta_e J_\parallel^2 + \tfrac{1}{2}(\eta_\perp - \eta_{0\perp}) J_\perp^2, \tag{24a}$$

$$\xi_a = \tfrac{1}{2}(\eta_e - a) J_\parallel^2 + \tfrac{1}{2}(\eta_\perp - \eta\eta_\perp - b) J_\perp^2, \tag{24b}$$

$$\xi_e = (\eta_\parallel - \eta_e) J_\parallel^2 + \eta_{0\perp} J_\perp^2 - nE_I\left(\frac{1}{\tau_I} - \frac{1}{\tau_R}\right), \tag{24c}$$

where

$$\eta_e = \frac{B}{ne}\frac{\kappa_i^* \kappa_e^2}{(\kappa_i^* + \kappa_e)^2} = O\left(\frac{B}{ne}\frac{\kappa_e^2}{\kappa_i^*}\right),$$

$$\eta_{0\perp} = \frac{B}{ne}\left(\kappa + \frac{\kappa_e(\kappa_i^{*2} + F^2)}{(\kappa_i^* + \kappa_e)^2}\right) \approx \frac{B}{ne}\kappa,$$

with the last expression representing the resistivity of the fully ionized plasma of electron density n, while

$$a = \frac{B}{ne}\frac{(\kappa_I - \kappa_R)\kappa_e^2}{(\kappa_i^* + \kappa_e)^2} \ll \eta_\parallel, \qquad b = \frac{B}{ne}\frac{(\kappa_I - \kappa_R)(\kappa_e^2 + F^2)}{(\kappa_i^* + \kappa_e)^2}.$$

Neglecting terms of order $(\kappa_I - \kappa_R)/\kappa_i^*$ and of order κ_e/κ_i^*, we obtain

$$\xi_i + \zeta_i = \xi_a + \zeta_a = \tfrac{1}{2}(\eta_\perp - \eta_{0\perp}) J_\perp^2, \tag{25}$$

$$\xi_e + \zeta_e = \eta_\parallel J_\parallel^2 + \eta_{0\perp} J_\perp^2 - nE_I\left(\frac{1}{\tau_I} - \frac{1}{\tau_R}\right). \tag{26}$$

Thus we find that the sum of dissipation terms for the three species is given by

$$\eta_\parallel J_\parallel^2 + \eta_\perp J_\perp^2 - nE_I\left(\frac{1}{\tau_I} - \frac{1}{\tau_R}\right).$$

Also, we find that the extra ohmic dissipation due to the enhanced perpendicular resistivity, *including* ionization and recombination processes, has resulted in ion and neutral heating, as is seen from equation (25). We note finally that Byrne and Burman (1974) showed that an enhanced resistivity does not necessarily imply a decrease in the energy of the electromagnetic field, a counter instance being when gravitational fields provide the energy source for both ion–neutral frictional dissipation and electromagnetic energy increase. (Byrne and Burman considered an astrophysical example.) In our case, however, the source of energy for both dissipation and ionization is the electromagnetic field, e.g. that driving the shock wave.

5. Conclusions

The major result of this paper is that, if ionization and recombination are taken into account, the enhancement of the resistivity due to ion slip for currents perpendicular to the magnetic field is reduced under conditions typical of laboratory shock wave experiments. It is somewhat surprising that the inclusion of an extra collisional process, namely ionization, should lead to a decrease in the resistivity. However, we interpret this result as a consequence of the reduction of the speed of ions relative to the neutrals caused by ionization (equation 16). The ion slip is reduced, and less plasma–neutral frictional dissipation occurs. The energy dissipated in the plasma

due to the enhanced resistivity has been shown to be dissipated in the ion and neutral fluids, regardless of whether ionization and recombination are taken into account.

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