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Extensive Air Showers and Radio Frequency Electromagnetic Fields

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Abstract

An estimate is made of the electric field expected from the ionization electrons produced by an extensive air shower moving in the geoelectric field for frequencies from 10 kHz to 10 MHz. The calculations are for a geoelectric production mechanism, and they invoke quite reasonable assumptions regarding the shower development. The calculated fields are found to be comparable with those produced by the geomagnetic mechanism, and fall short of the high values observed in this frequency range. Higher fields cannot be obtained from the present shower mechanism under normal weather conditions, but would require exceptionally large values for the geoelectric field (1 $\mathrm{MV\,m^{-1}}$) or a model for electron diffusion that is radically different from that assumed here.

Introduction

Radiation from extensive air showers (EAS) at a frequency of 60 MHz was first observed by Jelley *et al.* (1965) and has since been studied by many groups at frequencies ranging from 100 kHz to several hundred megahertz. Except for frequencies below 10 MHz, fairly good agreement has been found between the experimental observations and the theoretical predictions. A comprehensive review of the early work, including theoretical attempts to understand the radiation mechanism, has been given by Allan (1971). Owing to the rather flat lateral distribution of the radiation from the showers and the relatively fewer and less-complicated detectors needed, it was thought that radio studies would prove to be the best means of investigating cosmic rays at energies exceeding 100 PeV. Detailed calculations by Allan *et al.* (1973) showed that the lateral distribution of the radiation was a sensitive function of the longitudinal development of the shower. However, later studies (Allan *et al.* 1975) seemed to indicate that radio investigations fail to yield unambiguous conclusions because of observational difficulties.

The general mechanism for radio emission from EAS has been established to be the movement of the shower disc in the geomagnetic field: the radiation arises either from the transverse current consequent upon charge separation or from the moving dipoles formed by the separated charge pairs (Prescott *et al.* 1971). Kahn and Lerche (1966) gave the first detailed model for radio emission from a shower disc, in which the radiation arose from the transverse displacement of the shower electrons in the geomagnetic field. Owing to the relatively high energy of the electrons, these displacements are short lived, and the disc radiates coherently only at frequencies between

10–100 MHz. Coherence is lost outside this frequency domain, where the radiation field falls off rapidly. However, the fields observed at frequencies <20 MHz are all in excess of those predicted by the geomagnetic mechanism (Allan *et al.* 1970, 1975; Hough *et al.* 1971; Stubbs 1971; Felgate and Stubbs 1972; Atrashkevich *et al.* 1973; Clay *et al.* 1973, 1975). Even the inclusion of very low energy shower electrons (Clay 1972), neglected in earlier calculations, did not improve the fit between theory and observation. This led Clay *et al.* (1975) to suspect the presence of another mechanism, valid at frequencies <20 MHz, which involved the vertical *geoelectric* field.

The suggestion that the passage of a shower through the atmosphere might cause a detectable change in the geoelectric field, due to the presence of a large number of ionization electrons, was first made by Wilson (1957). An attempt to measure this change, as reflected by changes in the atmospheric conductivity, was made by Curry *et al.* (1974), who observed significant correlation between the arrival of an EAS and the DC field near the shower core. Earlier, Charman and Jelley (1968) had estimated the field in the 10 MHz domain to be expected from a near-horizontal shower. The movement of the shower electrons in the geoelectric field yielded radiation fields that were negligible compared with those from the geomagnetic effect, while a rough estimate of the field due to the drifting of ionization electrons in the geoelectric field yielded values that were comparable with the latter.

In the present work an attempt is made to estimate the electric field produced by the ionization electrons left in the wake of a shower as they drift in a vertical geoelectric field of $\sim 100\ \mathrm{V\,m^{-1}}$. The frequency range of interest is from a few kilohertz to several megahertz.

Drift of Ionization Electrons

R. W. Crompton (personal communication)* obtained results pertaining to the drift of electron swarms in air at standard temperature and pressure and for an electric field of $1\ \mathrm{kV\,m^{-1}}$. In the present work, his results (for dry air) have been scaled linearly to apply to a field of $100\ \mathrm{V\,m^{-1}}$. These data show that the ionization electrons are thermalized from their initial energy of ~ 30 eV in a time of ~ 100 ps, and thereafter the swarm will drift in the electric field. The drift velocity rises for 10 ns, attaining a terminal value of $150\ \mathrm{m\,s^{-1}}$. As the swarm drifts, it is continuously being depleted (mainly by attachment to oxygen molecules) with an exponential time dependence. The time constant for this loss is 10 ns, while loss due to recombination occurs on a time scale of seconds. For computational facility, the time dependence of the drift velocity is assumed to be as shown in Fig. 1. Such a linear variation is a good approximation to the curve obtained by Crompton.

Field Calculation

The field at different distances from the axis of a vertically incident shower of 10^6 particles was calculated in the present work by superposition of the fields produced by the electron swarms left in the wake of the shower disc. A cylindrical coordinate system was used to exploit the cylindrical symmetry of the shower. For

* Some of these data were given in an unpublished paper by R. W. Crompton on 'The history of free electrons produced by cosmic rays at sea level' presented at the Aust. Inst. Phys. Natl Congr., Adelaide, 1974.

computational convenience the following assumptions have been made regarding the characteristics of the shower and the electron swarm.

(i) The shower is approximated by a disc of infinitesimal thickness moving vertically downwards with the velocity of light.

(ii) The shower electrons are assumed to have a lateral distribution described by the Nishimura–Kamata–Greisen function with $s = 1\cdot 2$, independent of the height at which the shower is present, The shower disc is assumed to have a diameter of 200 m, all electrons beyond 100 m from the axis being neglected.

(iii) The shower starts at a height of 5 km and courses downwards unattenuated.

(iv) The electric field in which the ions and electrons drift is vertical and has a strength of $100\ \mathrm{V\,m^{-1}}$ up to 5 km.

(v) The density of the atmosphere is constant up to 5 km, being $1\cdot 293\ \mathrm{kg\,m^{-3}}$. (This assumption, together with assumption (iv), implies that the characteristics of the electron swarm are independent of its position in the atmosphere.)

(vi) A uniform ionization loss of $220\ \mathrm{keV\,kg^{-1}\,m^2}$ is suffered by the shower electrons, losing 30 eV per ionization.

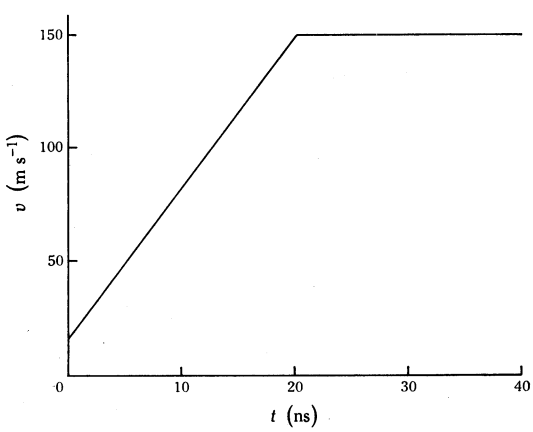

Fig. 1. Assumed time dependence of the drift velocity v of an electron swarm in air with an electric field of $100\ \mathrm{V\,m^{-1}}$.

The first assumption is justified since, in the frequency range of interest, a disc thickness of 2–3 m leads to a time uncertainty of only 10 ns. Deviations from all the above assumptions constitute an error of only 10% in the calculated field, owing to the inverse distance dependence of the field; even this error is for the field at 200 m, the error at closer locations being considerably less.

Consider the shower disc at a height z above the observation level (see Fig. 2). The shower electrons produce an electron swarm with n_e electrons in the infinitesimal volume element around Q:

$$n_e = 7\cdot 33\times 10^6\,\rho\Delta_e(r)\,r\,\mathrm{d}r\,\mathrm{d}z\,\mathrm{d}\phi\,, \tag{1}$$

where

$$\Delta_e = 69\cdot 63\,(r/80)^{-0\cdot 8}\,(1+r/80)^{-3\cdot 3} \tag{2}$$

and $\rho = 1\cdot 293\ \mathrm{kg\,m^{-3}}$ is the density of air. At the observation point $P(r_0, 0, 0)$, this swarm produces an electric field given by (Cowan 1968)

$$\boldsymbol{E} = (n_e e/4\pi\varepsilon_0)\{s^{-3}(\boldsymbol{r}' - c^{-1}r'\boldsymbol{v})(1 - v^2c^{-2} + c^{-2}\boldsymbol{r}'.\dot{\boldsymbol{v}}) - \dot{\boldsymbol{v}}r'/c^2s^2\}\,, \tag{3}$$

where c is the velocity of light,

$$s = r'(1 - \boldsymbol{v}.\boldsymbol{r}'/|c\boldsymbol{r}'|) = r'\{1 + v\cos(\theta/c)\}, \tag{4}$$

and, from the geometry of Fig. 2,

$$r' = (r^2 + r_0^2 + z^2 - 2rr_0\cos\theta)^{\frac{1}{2}}. \tag{5}$$

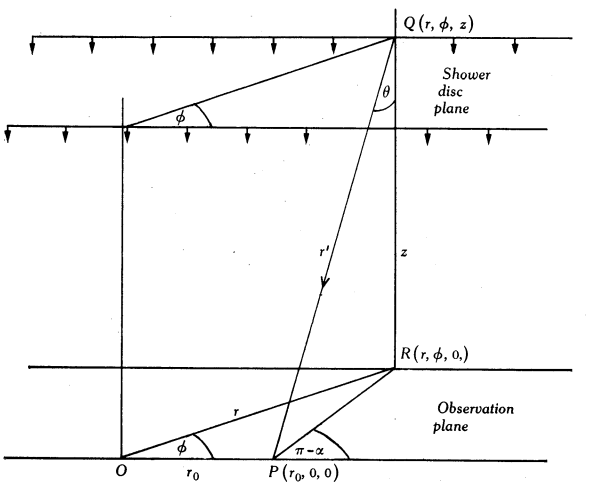

Fig. 2. Cylindrical coordinate representation of a shower disc at time $t = -z/c$, which passes the observation plane at $t = 0$. The diagram illustrates the geometry used to calculate the electric field at point P due to electrons produced by the passage of the shower disc. Note that

$$\cos\alpha = -r\sin\phi/r'\sin\theta = (r_0 - r\cos\phi)/(r_0^2 + r^2 - 2rr_0\cos\phi)^{\frac{1}{2}}.$$

Here $\boldsymbol{v}$ and $\dot{\boldsymbol{v}}$ are the drift velocity and acceleration of the electron swarm respectively. The field (3) has a time dependence that is determined by those of $\boldsymbol{v}$ and $\dot{\boldsymbol{v}}$ in Fig. 1, but starting from a time t given by $t = (r' - z)/c$. The total field is obtained by an integral over the atmosphere, care being taken to add the components of the field as well as to keep account of the time dependence. The first term in equation (3) is independent of the velocity and acceleration, and it will be exactly cancelled by the field due to the positive ions. The positive ions have a velocity that is nearly 10^4 times smaller than that of the electrons, and hence their contribution to the field will be negligible. Thus the field at P will include only the velocity- and acceleration-dependent terms of equation (3). As the electron swarm suffers loss due to attachment, the expression for E, including only the $\boldsymbol{v}$- and $\dot{\boldsymbol{v}}$-dependent terms will be multiplied by $\exp(-t'/\tau)$, where τ is the time constant for loss (10 ns) while t' is the time measured from the instant the swarm was created. I performed the required integration numerically on a DEC-10 computer at the Tata Institute of Fundamental Research, Bombay.

Features of Field

From the symmetry of the shower and the form of equation (3), the following general features can be deduced. The field at an arbitrary point has only vertical and radial components. On the axis, the field is completely vertical, but it becomes

increasingly radially polarized as we move away from the axis. Because the dependences of E_z and E_r on time differ, the net polarization will be a function of time. If the shower development, as indicated by its lateral and longitudinal structure, does not change with size (i.e. the number of particles) then the field will also be proportional to size. But changes in shower development will be followed by changes in the magnitude and time structure of the field, so that the field eventually ceases to be proportional to size. Figs 3*a* and 3*b* respectively show the time dependence of E_r and E_z at different radial distances from the axis of the shower.

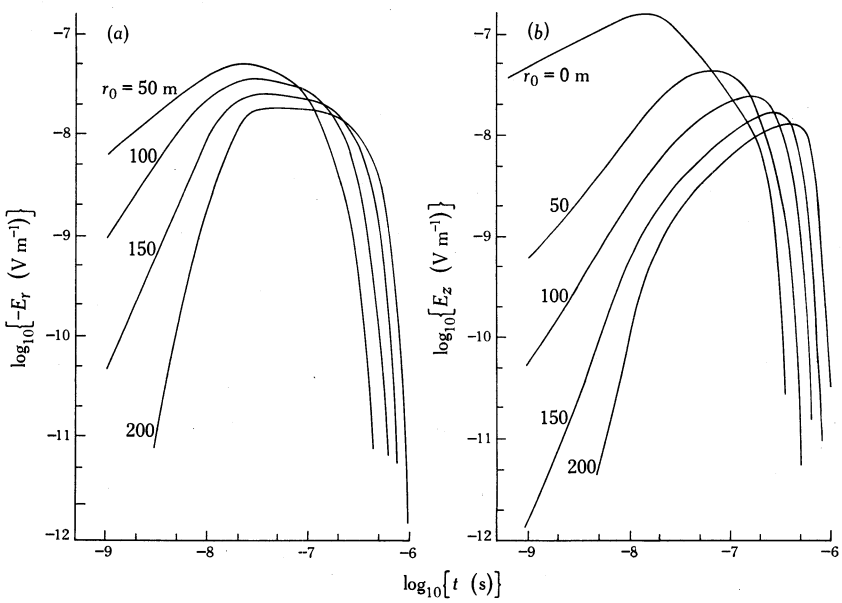

Fig. 3. Plots as a function of time t of (*a*) the radial electric field E_r and (*b*) the vertical electric field E_z at the indicated distances r_0 from the axis of a shower containing 10^6 electrons.

To make a comparison with experimental observations one needs to know the response of detectors with finite bandwidths to the fields shown in Figs 3*a* and 3*b*. The bandwidth-limited pulse $E_{\mathrm{bw}}(t)$, is given by

$$E_{\mathrm{bw}}(t) = \mathrm{Re}\left((2\pi)^{-1}\int_{\omega_1}^{\omega_2}\exp(\mathrm{i}\omega t)\,\mathrm{d}\omega\int_0^{\infty}E(t')\exp(-\mathrm{i}\omega t')\,\mathrm{d}t'\right), \tag{6}$$

where ω_1 and ω_2 are the limiting angular frequencies of the pass band, with $\omega = 2\pi f$. The pulse E_{bw} has been calculated for the bandwidths and central frequencies used in the experiments. To make the comparison complete, I calculated for each case the r.m.s. value of the bandwith-limited pulse, integrated over 10 times the smallest period in the pass band, by assuming the response to be flat across the pass band.

Table 1 shows a comparison of the observed and calculated fields, the latter having been linearly scaled from calculations made for fields associated with a shower of 10^6 particles. Variation in size due to different altitudes of observation has been ignored, all observations having been made near sea level. The observations of Allan *et al.* (1970) and Clay *et al.* (1973) are for vertical fields, while all others are for horizontal fields. The lateral distribution of the field is rather flat, the field at 200 m

being reduced by a factor of only two compared with that near the axis of the shower. Owing to the different time structure (frequency content) of the field at different distances, the frequency spectrum of the field is a function of distance.

The dependence of the field on the properties of the electron swarm has also been examined. Since $\boldsymbol{E}$ is proportional to $\boldsymbol{v}$ and $\dot{\boldsymbol{v}}$, any increase in $\boldsymbol{v}$ or $\dot{\boldsymbol{v}}$ without a change in their time structures will only produce a proportional increase in both E and E_{bw}. However, changes in τ affect the time dependence of the field and hence they affect E_{bw}. The field at $r_0 = 50$ m, calculated for a time constant $\tau = 50$ ns, is given in column 6 of Table 1. It can be seen that E_{bw} increases almost linearly with τ, but this linearity need not hold for large increases in τ. R. W. Crompton (personal communication) felt that τ could not be very much larger than 100 ns, and hence the expected fields given in columns 5 and 7 of Table 1 cannot be increased by more than a factor of 10.

Table 1. Comparison between observed and calculated field strengths

The tabulated quantities are expressed in the following units: observation frequency f_0 in MHz, shower size S in 10^6 particles, observed field E_{obs} in $\mathrm{pV\,m^{-1}\,Hz^{-1}}$ and calculated field $E_{r_0,\tau}$ in $\mathrm{fV\,m^{-1}\,Hz^{-1}}$ (note that these units are 1000 times smaller than those for E_{obs}). The parameters r_0 and τ are expressed in metres and nanoseconds respectively

Observer	$f_0 \pm \Delta f$	S	E_{obs}	$E_{50,10}$	$E_{50,50}$	$E_{100,10}$
Allan *et al.* (1970)	1·8 ±0·9	100	300·0	60	290	20
Hough *et al.* (1971)	3·6 ±0·2	0·5	2·5±0·6	0·64	2·57	0·28
Stubbs (1971)	2·0 ±0·035	0·04	1·0	0·085	0·375	0·045
Felgate and Stubbs (1972)	5·96±0·04	0·2	0·65[A]	0·17	0·489	0·067
Atrashkevich *et al.* (1973)	1·9 ±0·1	32	6·7[B]	63	180	36
Clay *et al.* (1973)	0·1 ±0·04	0·5	500·0	1·0	5·39	0·66
Allan *et al.* (1975)	1·8 ±0·1	100	0·6	200	970	100
Clay *et al.* (1975)	2·0 ±0·35	0·2	0·5	0·22	1·93	0·12
Clay *et al.* (1975)	3·6 ±0·2	1	0·7	1·3	5·13	0·55

[A] This is the mean of the following two values: 0·8±0·2 (EW) and 0·5±0·2 (NS).
[B] This value is for $r_0 = 1$ km.

Conclusions

From Table 1 it is evident that the observations divide into two classes: one in which the observed upper limits are consistent with the calculated fields and the other in which the expected fields fall short of those observed by a factor in the range 10^{-2}–10^{-6}. It should be noted here that those observers who reported large fields in earlier experiments (Allan *et al.* 1970; Clay *et al.* 1973) have not observed them since (Allan *et al.* 1975; Clay *et al.* 1975). Either the field variability is very large or the earlier experiments contained unsuspected sources of error. However, if the earlier observations are assumed to be correct then such fields can be produced by the geoelectric mechanism only if the electric field is several orders of magnitude larger than the normally prevalent $100\ \mathrm{V\,m^{-1}}$. The anomalous events observed by Allan *et al.* (1970) may have had a geoelectric origin, as they all occurred during or after thunderstorms. For the 15% of cases, in which the geomagnetic origin of the field is not established (Prescott *et al.* 1971), the mechanism could be geoelectric. The diurnal variations of the EAS-produced fields have been observed to be similar to those of the geoelectric field (Clay *et al.* 1973). However, correlation between the

two has not been clearly established, and continuous monitoring of the latter during an experiment is necessary.

According to Allan (1971), a near-vertical shower of 100 PeV primaries produces a field of 1 $\mathrm{pV\,m^{-1}\,Hz^{-1}}$ at 200 m due to the geomagnetic mechanism (either dipole moment or charge excess) in the tens-of-megahertz region. The present calculation yields a field of 100 $\mathrm{fV\,m^{-1}Hz^{-1}}$ for a similar shower. At lower frequencies ($\sim$100 kHz), both mechanisms produce fields that are nearly equal. Hence the geoelectric mechanism considered here does not yield fields larger than those produced by the geomagnetic mechanism. It would seem that the usefulness of purely radio studies of EAS for eliciting information on either the shower structure or the primary radiation has not yet been established.

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