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Sea Level Muon Spectrum and Muon Charge Ratio derived from CERN Results for Nucleon–Nucleus Inelastic Interactions

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Abstract

The sea level cosmic ray spectrum and muon charge ratio have been estimated by using the energy moments of the cross section for proton–air inelastic collisions. These energy moments have been determined by interpolation from CERN results for proton–nucleus inelastic interactions in pion production. The derived results are compared with previous work.

Introduction

Until now most of the theoretical calculations for cosmic ray pion or muon spectra have been based on nucleon–nucleon collision data. However, it is probably more realistic to use data on proton–nucleus collisions for such calculations, and in the present work we have taken data from nucleon–nucleus collisions at parameter values determined by CERN experiments on pion production (Stevenson 1976) to derive a sea level cosmic ray spectrum. The charge ratio of muons has also been estimated by conventional calculations, following Frazer *et al.* (1972) and Morrison and Elbert (1973). The derived results are compared with those estimated by previous authors.

Theoretical Aspects

The history of theoretical models of multiple-meson production in p–p, p–n or p–nucleus collisions has been summarized by Ramakrishnan (1962), while recent theories are reviewed by Ranft and Ranft (1970). In particular, the thermodynamical model (fireballs) is described by Grote *et al.* (1970). Ranft (1971) has developed a formula for pion production which agrees with the predictions of the thermodynamical model and has the correct scale invariance character, i.e. Feynman (1969) scaling behaviour. This formula, in a centre of mass frame, is

$$\frac{\mathrm{d}^2N^*}{\mathrm{d}p_{\mathrm{L}}^*\,\mathrm{d}p_{\mathrm{T}}}=\frac{A_1\exp(-A_2\,p_{\mathrm{L}}^{*2}/E_{\mathrm{cm}}^2)\,p_{\mathrm{T}}\{\exp(-A_3\,p_{\mathrm{T}}^2)+A_4\exp(-A_5\,p_{\mathrm{T}})\}}{(p_{\mathrm{L}}^{*2}+p_{\mathrm{T}}^2+m_\pi^2)^{\frac{1}{2}}},\tag{1}$$

where N^* is the number of secondary pions emitted in proton–nucleus collisions, p_{L}^* and p_{T} are the longitudinal and transverse momenta of the pions in the centre of mass frame, E_{cm} is the total centre of mass energy of the collision (GeV) and m_π is the pion mass. The quantities A_1–A_5 are mass-dependent parameters for the various inelastic interactions that result in pion production. The values of these parameters used in the present calculations were taken from Stevenson (1976), and they are listed in Table 1.

For a nuclear reaction with cross section σ, the structure function $f(x, p_T)$ (with $x \equiv p_L^*/p_{max} = 2p_L^*/E_{cm}$) is given by

$$f(x, p_T) = E\left(\frac{d^3\sigma}{d^3p}\right) = \frac{E}{\pi}\frac{d^2\sigma}{dp_L^* \, dp_T^2} = \frac{E\sigma}{2\pi p_T}\frac{d^2N^*}{dp_L^* \, dp_T}$$

$$= (\sigma A_1/2\pi)\exp(-\tfrac{1}{4}A_2 x^2)\{\exp(-A_3 p_T^2) + A_4 \exp(-A_5 p_T)\}, \quad (2)$$

while the fractional energy moments $Z_{p\pi\pm}$ for pion production by proton–nucleus inelastic collisions with cross section σ_{in} have the form

$$Z_{p\pi\pm} = (\pi/\sigma_{in}) \int_0^\infty \int_0^1 x^{\gamma-2} f(x, p_T)\, dx\, dp_T^2$$

$$= A_1 \int_0^\infty \int_0^1 \exp(-\tfrac{1}{4}A_2 x^2)\, p_T \{\exp(-A_3 p_T^2) + A_4 \exp(-A_5 p_T)\} x^{\gamma-2}\, dx\, dp_T$$

$$\approx A_1 \Gamma(\tfrac{1}{2}(\gamma-1))\,(\tfrac{1}{2}A_3^{-1} + A_4/A_5^2)/2(\tfrac{1}{4}A_2)^{\frac{1}{2}(\gamma-1)}. \quad (3)$$

Table 1. Interaction parameters and energy moments for pion production

Pion type	Target nucleus	Mass number A	Interaction parameters A_1	A_2	A_3	A_4	A_5	Energy moments $Z_{p\pi\pm}$
π^+	H	1	4·94	33·83	6·11	0·69	4·12	0·0600
	Be	9·01	1·81	33·39	3·01	5·12	7·34	0·0474
	Al	26·98	1·54	35·54	3·70	3·03	4·94	0·0380
	Cu	63·54	2·36	37·21	5·83	0·76	3·22	0·0345
	Pb	207·12	1·79	38·60	6·04	0·96	3·23	0·0280
π^-	H	1	2·81	44·08	5·17	0·81	4·34	0·0313
	Be	9·01	1·52	42·74	5·33	0·82	3·53	0·0199
	Al	26·98	1·54	44·62	5·67	0·83	3·17	0·0208
	Cu	63·54	1·60	46·52	6·47	0·93	3·05	0·0217
	Pb	207·12	1·55	47·16	6·02	0·56	2·66	0·0190

Each $Z_{p\pi}$ gives the required ratio of the flux of resulting singly charged pions to the flux of incident particles at the same energy after each incident particle has undergone a single interaction. With these formulae, the pion production spectrum $\Pi(E)\,dE$ at the top of the atmosphere can be estimated from the relation

$$\Pi(E)\,dE = (Z_{p\pi+} + Z_{p\pi-})\,N(E)\,dE, \quad (4)$$

where $N(E)\,dE$ is the differential primary proton spectrum.

The sea level differential muon spectrum $\mu(E)\,dE$ can be estimated from the pion spectrum (4) by using the solution of the convential pion atmospheric diffusion equation given by Bhattacharyya *et al.* (1976). This solution has the form

$$\mu(E)\,dE = \frac{\Pi(E)\,dE\; A\, r^{\gamma-1}\, Br\, h(E)\, y(E)}{\lambda_n(Br + E)}, \quad (5)$$

where the parameters $r = 0\cdot 76$ and $B = 118$ GeV, the function $h(E)$ depends on muon–electron decay and energy losses of muons in the atmosphere, and $y(E)$ is a correction factor that depends on the K/π meson ratio at production, with equal nucleon absorption length Λ and pion interaction length λ_π in nucleon–air collisions, that is, $\Lambda = \lambda_\pi = 120\ \mathrm{g\,cm^{-2}}$, and a nucleon interaction length $\lambda_n = 71\ \mathrm{g\,cm^{-2}}$ (Allkofer 1975).

The charge ratio of sea level muons can be estimated from the relation (Frazer *et al.* 1972)

$$\mu^+/\mu^- = (Z^+_{p\pi}/Z^-_{p\pi} + S_{\pi-}/S_{\pi+})/(1 + S_{\pi-}\, Z^+_{p\pi}/S_{\pi+}\, Z^-_{p\pi}), \tag{6}$$

where

$$Z^+_{p\pi} = Z_{p\pi+} + Z_{p\pi-}, \qquad Z^-_{p\pi} = Z_{p\pi+} - Z_{p\pi-},$$

with the moments $Z_{p\pi\pm}$ as given by equation (3), and

$$S_{\pi\pm} = \frac{\ln(\Lambda_\pi/\Lambda_N)}{\Lambda_\pi/\Lambda_N - 1} \pm \frac{\Delta_0}{N_0}\frac{\ln(\Lambda_\pi/\Lambda'_N)}{\Lambda_\pi/\Lambda'_N - 1},$$

with values of $\Lambda_N = 120\ \mathrm{g\,cm^{-2}}$, $\Lambda'_N = 93\cdot 2\ \mathrm{g\,cm^{-2}}$, $\Lambda_\pi = 226\cdot 7\ \mathrm{g\,cm^{-2}}$ and

$$\Delta_0/N_0 = (p_0 - n_0)/(p_0 + n_0) \approx 0\cdot 504,$$

p_0 and n_0 being the amplitudes of the incident proton and neutron spectrum at the mean depth of meson production, that is, $n_0/p_0 \approx 0\cdot 33$.

A simplified model developed by Morrison and Elbert (1973) can also be used to estimate the charge ratio of muons. On the assumption that muons are produced only by pions from the first interaction of primaries in the atmosphere, this model yields the formula

$$\mu^+/\mu^- = (N_p Z_{p\pi+} + N_n Z_{p\pi-})/(N_p Z_{p\pi-} + N_n Z_{p\pi+}) = (1+\delta\varepsilon)/(\delta+\varepsilon), \tag{7}$$

where the ratio $\delta = N_n/N_p$ of incident neutrons to incident protons is assumed to be $0\cdot 33$ and ε is the ratio $Z_{p\pi-}/Z_{p\pi+}$.

Results and Discussion

In a recent survey of the radiation environment in proton accelerators and storage rings, Stevenson (1976) has discussed the Ranft (1971) formula (1) and has given the relevant parameters for positive and negative pion production from the interaction of protons with nuclei of different mass numbers, namely hydrogen, beryllium, aluminium, copper and lead. The values of the energy moments $Z_{p\pi+}$ and $Z_{p\pi-}$ calculated from the relation (3) are included in Table 1 above. As can be seen, the results show a general tendency for the energy moments to decrease slowly with the mass number of the target nuclei. Assuming a mass number for air of around $14\cdot 75$, we obtain by interpolation the following energy moments for proton–air inelastic interactions

$$Z_{p\pi+} = 0\cdot 043, \qquad Z_{p\pi-} = 0\cdot 022. \tag{8}$$

A power law fit to the primary nucleon spectrum of Ramaty *et al.* (1973) has the form

$$N(E)\,\mathrm{d}E = N_0 E^{-\gamma}\,\mathrm{d}E, \tag{9}$$

where the spectral amplitude $N_0 = 1\cdot 84$ (with a nucleon–proton ratio of $1\cdot 33$ at the production depth of the incident flux) and the spectral index $\gamma = 2\cdot 64$; the proton energy E and intensity I are expressed in units of GeV and $(\mathrm{cm}^2\,\mathrm{s\,sr\,GeV})^{-1}$ respectively. From the relation (9), the pion production spectrum (4) at the top of the atmosphere has been estimated to be given by

$$\Pi(E)\,\mathrm{d}E = 0\cdot 12\,E^{-2\cdot 64}\,\mathrm{d}E. \tag{10}$$

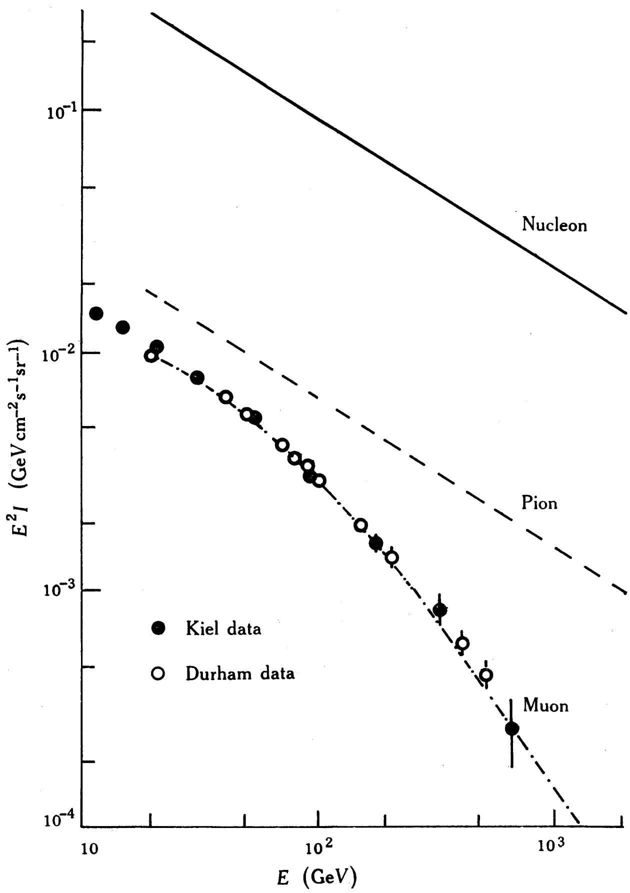

Fig. 1. Comparison of the estimated sea level muon spectrum from the present work (dot–dash curve), which includes a 10% contribution from K meson decay, with the magnetic spectrograph data obtained by the Kiel and Durham groups (Allkofer *et al.* 1971; Ayre *et al.* 1975). Also shown is the fit to the primary nucleon spectrum of Ramaty *et al.* (1973) and the pion spectrum derived here.

The vertical sea level muon spectrum has been deduced from the relation (10) using the solution (5) of the pion atmospheric diffusion equation. The calculated muon intensity has been increased by approximately 10% to account for the contribution of kaons via $\mathrm{K} \to \mu\nu$ decays; this correction was justified in a previous investigation (Bhattacharyya 1976). The calculated results are plotted in Fig. 1 along with the magnetic spectrograph data of the Kiel group (Allkofer *et al.* 1971) and the Durham group (Ayre *et al.* 1975). It is evident from the figure that the calculated muon spectrum from the primary nucleon spectrum of the Goddard Space Flight Center group (Ramaty *et al.* 1973) is in accord with the spectrograph data.

Finally, we have made a calculation of the charge ratio of sea level muons, using the fractional energy moments (8) for proton–air collisions and the models of Frazer *et al.* (1972) and Morrison and Elbert (1973). The results are compared in Table 2 with those obtained by other authors. Although the sea level muon spectrum derived from our work is in accord with the measured data of the Kiel and Durham groups, the observed muon charge ratio (Kasha *et al.* 1975; Baxendale *et al.* 1975; Asatiani *et al.* 1975; Carstensen *et al.* 1975) is still lower than predicted. This is despite the fact that we have made an improved calculation by using data for proton–nucleon interactions, which are more relevant than the proton–proton data previously considered. A similar discrepancy between the observed and predicted muon charge ratio has been observed by previous authors, but they have drawn different conclusions (see e.g. Hoffman 1975; Bhadwar *et al.* 1977). Hoffman (1975) suggested that a resolution of the discrepancy required either a large neutron–proton ratio in the primary flux or the introduction of a new factor for high energy interactions, or both.

Table 2. Comparison of calculated charge ratios for sea level muons

Reference for calculation	Interaction considered	Integral spectral index	Muon charge ratio μ^+/μ^-
Frazer *et al.* (1972)	proton–proton	1·70	1·56
Yekutieli (1972)	proton–proton	1·70	1·55
Garaffo *et al.* (1973)	proton–proton	1·70	1·38
Liland and Pilkuhn (1973)	proton–proton	1·70	1·56
Adair (1974)	proton–proton	1·70	1·53
Erlykin *et al.* (1974)	proton–proton	1·62	1·36
Hoffman (1975)	proton–proton	1·75	1·54
Present work	proton–air	1·64	1·58[A] 1·39[B]

[A] From the model of Frazer *et al.* (1972), as given by equation (6).
[B] From the model of Morrison and Elbert (1973), as given by equation (7).

In the present work, we have used a rather large value for the neutron–proton ratio, as favoured by some authors (e.g. Adair *et al.* 1977) but the discrepancy still remains. It is possible that there is a larger flux of heavy nuclei in primary cosmic rays at very high energies than is observed at lower energies, and this may explain the discrepancy. In any case, it seems clear that a more accurate estimate of the muon charge ratio will only be obtained when the composition of the primary flux and details of the hadron–hadron interactions are better established. It may turn out that meson production from nucleus–nucleus collisions at high energy makes a significant contribution to the muon charge ratio.

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