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Is Nuclear Physics the Molecular Physics of Coloured Quarks?

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Abstract

We show that nuclear physics provides evidence against the existence of hadronic Van der Waals forces between nucleons.

Introduction

Since Yukawa's (1935) remarkable insight, nuclear forces have been regarded as arising from interchange of pions and possibly other mesons (for a recent review see Measday *et al.* 1978). However, it is becoming increasingly apparent that nucleons are made of quarks (e.g. Harari 1978) and that it is unrealistic to expect nuclear forces to be describable in terms of a local field theory of meson exchange. Instead it is more appropriate to re-examine the nuclear force as a manifestation of the 'molecular physics of quarks' (Glashow 1975).

Forces between two nonpolar molecules are of two types: a long-range Van der Waals force, generated by two-photon exchange, as illustrated in Fig. 1 below; and a short-distance force from the overlapping electron clouds (see e.g. Margeneau and Kestner 1969). Both of these types of force have been applied to the nucleon–nucleon case. Van der Waals forces have been considered by Appelquist and Fischler (1978), Fishbane and Grisaru (1978) and Wiley (1978). Overlap types of forces have been considered by Liberman (1977) and DeTar (1978).

In this paper we shall discuss the Van der Waals type forces. We wish to emphasize that:

(1) in considering the effects of such forces in nuclear matter, the analogous many-body forces of Axelrod–Teller (1943) type should also be considered;

(2) the Coulomb interference phenomenon in p–p scattering can supply empirical limits to the strength of the Van der Waals force between protons.

Before coming to these points it is necessary to review the existing work on nuclear Van der Waals forces.

Nuclear Van der Waals Forces

The quarks in a nucleon are bound together by exchange of massless gluons, in just the same way as electrons in a molecule are bound by exchange of massless photons. Two neutral nonpolar molecules cannot interact, however, by exchange of a single photon, since emission or absorption of a single photon by such a molecule takes it to a polarized state. But two-photon emission, as depicted in Fig. 1, is possible, since the polarized states are virtual. Exchange of two photons leads to the Van der Waals potential

$$V_{\mathrm{mol}} = (-e^2\gamma/4\pi R^6)\langle d^2\rangle, \tag{1}$$

where γ is the molecular polarizability and $e^2\langle d^2\rangle$ is the mean square dipole moment of a molecule. Equation (1), which may be derived semiclassically or from the quantum perturbation theory represented by Fig. 1, is noteworthy because it depends on e^2 and not on e^4 as a glance at Fig. 1 would suggest. This reduction in the order of V with respect to the coupling constant occurs because the excitation energy of the dipole intermediate state in Fig. 1 is also of order e^2.

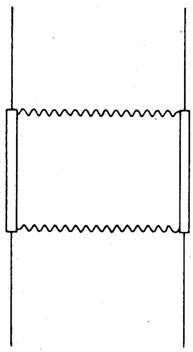

Fig. 1. Illustrating the Van der Waals force. In the electromagnetic case the exchanged particles are photons and the excited atomic states are dipole states. In the chromodynamic case the exchanged particles are gluons and the excited hadronic states are colour octets.

Noting that we have $\gamma \sim a^3$ and $\langle d^2\rangle \sim a^2$, where a is a molecular size parameter, and introducing the fine structure constant $\alpha = e^2/4\pi$, we rewrite the molecular Van der Waals potential (1) as

$$V_{\mathrm{mol}} \approx -\alpha a^5/R^6. \tag{2}$$

Casimir and Polder (1948) pointed out that the classical Van der Waals R^{-6} potential holds only for $R \lesssim \alpha^{-1}a$. At larger separations, the time taken for photons to go from one atom to the other, namely R/c, becomes comparable with the time scale of atomic motions $a/v = a/\alpha c$, and retardation effects must be included. In this case

$$V_{\mathrm{mol}}^{\mathrm{ret}} \approx -\alpha a^6/R^7. \tag{3}$$

The arguments leading to equations (1), (2) and (3) may be, and have been, taken over into nucleon–nucleon physics, by several authors. Appelquist and Fischler (1978) gave a field theoretic static calculation of the Van der Waals force between π mesons. They obtained

$$V_{\mathrm{mes}} \sim -\alpha_{\mathrm{s}}\tfrac{9}{16}x^5/R^6, \tag{4}$$

where x is the static separation of the quarks in the meson, α_{s} is the strong interaction quantum chromodynamic (QCD) 'fine structure constant' and the factor $\frac{9}{16}$ derives from the $SU(3)$ nature of the colour couplings. This has precisely the form of equation (2), demonstrating the QCD–QED analogy, and suggesting that for any hadrons

$$V_{\mathrm{had}} \approx -\alpha_{\mathrm{s}} b^5/R^6, \tag{5}$$

as can be derived by semiclassical considerations. Here b is a hadron size parameter. Wiley (1978) gives

$$V_{\mathrm{had}} \approx -\alpha_s^2 b^4/\Delta E_c R^6,$$

where ΔE_c is the energy difference between the colour singlet and colour octet states. This energy has not been computed, but in many models is taken to be $\Delta E_c \sim \alpha_s b^{-1}$, which recovers equation (5). A potential like (5) was considered by Fishbane and Grisaru (1978), but they introduced a factor of α_s^2, not $\alpha_s^2 \Delta E^{-1}$, and made other errors in their analysis of the effects of this in nuclear physics.

Typically we have $\alpha_s \sim 0\cdot5$ and we may set $b \approx 1$ fm. Then as $\alpha_s^{-1} b \approx 2$ fm is typical of nucleon separations in the nucleus we should almost always use the retarded potentials. No detailed QCD calculations of the Appelquist–Fischler (1978) type have been done for retarded potentials, but semiclassical considerations and the QED analogy suggest a potential

$$V_{\mathrm{had}}^{\mathrm{ret}} \approx -\alpha_s b^6/R^7. \tag{6}$$

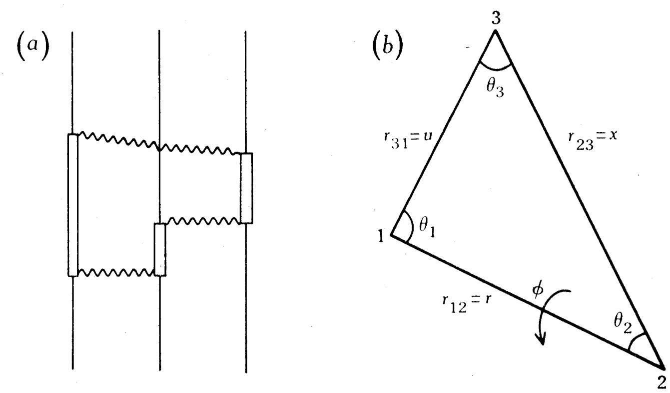

Fig. 2. Diagrams to show (*a*) the Axelrod–Teller forces, and (*b*) the variables defined here for the three-body force.

Many-body Forces

Axelrod and Teller (1943) pointed out that the generalization of Fig. 1 to three-photon exchange and three particles leads to the three-body force of Fig. 2*a*. This force may also be obtained semiclassically from the interaction of dipoles in molecules 2 and 3 induced by the instantaneous moment of 1. In either case

$$V^{\mathrm{AT}} = \tfrac{3}{4} C_1 \gamma (3\cos\theta_1 \cos\theta_2 \cos\theta_3 + 1)/r_{12}^3 r_{23}^3 r_{31}^3, \tag{7}$$

where $V_{\mathrm{mol}} = -C_1/R^6$ and the geometrical parameters are defined in Fig. 2*b*.

Reinterpreting Fig. 2*a* in terms of gluon exchange, we arrive at

$$V_{\mathrm{had}}^{\mathrm{AT}} \approx \alpha_s b^8 (3\cos\theta_1 \cos\theta_2 \cos\theta_3 + 1)/r_{12}^3 r_{23}^3 r_{31}^3. \tag{8}$$

This Axelrod–Teller force between nucleons has a quite different structure to the many-nucleon forces considered heretofore (McKellar and Rajaraman 1979). Similar forces involving more than three particles can be readily generated (Margeneau and Kestner 1969). The important point to notice is that many-body forces do not depend on higher powers of the coupling constant, since the additional energy denominators always cancel the additional powers of α_s. Retarded Axelrod–Teller forces are much more complicated, and have not been considered in detail in molecular physics to our knowledge.

Van der Waals and Axelrod–Teller Forces in Nuclear Matter

In nuclear matter we may write for the contribution of V_{had} to the energy

$$E_2/N = 4\pi\rho_0 \int V_{\text{had}}(r) g(r) r^2 \, dr, \tag{9}$$

where $g(r)$ is the two-nucleon correlation function, ρ_0 is the density of nuclear matter ($\rho_0 = 0{\cdot}18 \text{ fm}^{-3}$) and we have neglected the exchange terms. If we assume the simple cutoff form for $g(r)$ (Blatt and McKellar 1974)

$$\begin{aligned} g(r) &= 1, \qquad r \geqslant r_0, \\ &= 0, \qquad r < r_0, \end{aligned}$$

we obtain

$$\begin{aligned} E_2/N &= -\tfrac{4}{3}\pi \alpha_s b^5 \rho_0 / r_0^3 \qquad \text{(nonretarded)}, && \text{(10a)} \\ &= -\pi \alpha_s b^6 \rho_0 / r_0^4 \qquad \text{(retarded)} && \text{(10b)} \end{aligned}$$

(note that errors in the estimates of Fishbane and Grisaru (1978) have been corrected here). Unfortunately these estimates are sensitive to the size parameter b and the cutoff parameter r_0. A value of $b = 1$ fm seems reasonable for the size parameter, and one may expect r_0 to be somewhere in the range 1–2 fm. Typical values for E_2/N are given in Table 1. Even for the retarded case and $r_0 = 2$ fm the additional binding energy may be difficult to fit into nuclear matter calculations. This conclusion contradicts that of Fishbane and Grisaru. Their more optimistic result is erroneous.

Table 1. Typical contributions to nuclear energy from 'molecular' forces

Cutoff parameter r_0 (fm)	E_2/N (nonretarded)	E_2/N (retarded)	E_3/N (nonretarded)
1·0	−74	−55·5	+8·7
1·5	−22	−10·5	+2·6
2·0	−9	−3·4	+1·1

The three-body force contribution may be calculated in a similar way. From

$$E_3/N = (\rho_0^2/3!\,V) \int d\mathbf{r}_1 \, d\mathbf{r}_2 \, d\mathbf{r}_3 \; V_{\text{had}}^{\text{AT}}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) g(r_{12}) g(r_{23}) g(r_{31}) \tag{11}$$

we find that (details are given in the Appendix)

$$E_3/N = \tfrac{5}{18}\pi^2 \alpha_s b^8 \rho_0^2 / r_0^3 \qquad \text{(nonretarded)}. \tag{12}$$

While E_2 and E_3 depend on the cutoff parameter, E_3/E_2 does not:

$$E_3/E_2 = -\tfrac{5}{24}\pi b^3 \rho_0 = -0{\cdot}11 \qquad \text{for} \qquad b = 1 \text{ fm}.$$

The energy E_3 is also given in Table 1, where it will be seen that, even for $r_0 = 2$ fm, E_3/N is as big as more conventional three-body forces in nuclei (Coon *et al.* 1979).

Nucleon Van der Waals Forces in Proton–Proton Scattering

One can try to estimate the contribution of the Van der Waals force to the p–p scattering amplitude in a Born approximation. Restricting our consideration to the $l = 0$ state at low energies, we have

$$f_0 = (2\mu/4\pi\hbar^2)\int \mathrm{d}^3 r\, V(\boldsymbol{r}), \tag{13}$$

where μ is the reduced mass of the p–p system ($\mu = \frac{1}{2}m$). Since the singularity in $V(r)$ as $r \to 0$ gives an unphysical infinity in f_0, once again we introduce a cutoff parameter r_0 and estimate

$$f_0 \approx \tfrac{1}{3} m\,\alpha_s b^5/r_0^3 \qquad \text{(nonretarded)}, \tag{14a}$$

$$\approx \tfrac{1}{4} m\,\alpha_s b^6/r_0^4 \qquad \text{(retarded)}, \tag{14b}$$

which is again r_0 dependent. The values of f_0 for three values of r_0 are set out below:

	$r_0 = 1\cdot0$	$1\cdot5$	$2\cdot0$ fm
f_0 (retarded)	$0\cdot78$	$0\cdot23$	$0\cdot098$ fm
f_0 (nonretarded)	$0\cdot59$	$0\cdot12$	$0\cdot036$ fm

These amplitudes are very small compared with the nuclear force scattering amplitude $f_N \approx 3\cdot5$ fm at 390 keV. However, at this energy the nuclear scattering is almost exactly cancelled by the Coulomb scattering amplitude; indeed low energy p–p scattering can be used to show that the p–p scattering amplitude due to QED vacuum polarization ($f_{VP} = 0\cdot015$ fm at 390 keV) is within 30% of the calculated value (Brolley 1969). This implies that a new term in the scattering amplitude bigger than 5×10^{-3} fm would have shown up in the analysis. In other words, the Van der Waals scattering amplitudes displayed above would have completely upset the analysis of Brolley (1969). Therefore, if these estimates are correct, there is no QCD analogue of the Van der Waals force in nature.

Conclusions

We have investigated the possibility that the nucleon–nucleon interaction mimics the molecule–molecule interaction to the extent of having power law forces of Van der Waals and Axelrod–Teller types. Our major result is that low energy p–p scattering rules out Van der Waals forces of the expected strength. If we modify the strength parameter, writing

$$V = -\alpha_s C b^5/R^6 \qquad \text{(nonretarded)},$$

$$= -\alpha_s C' b^6/R^7 \qquad \text{(retarded)},$$

we can state our results as

$$Cb^5/r_0^3 \lesssim 6\cdot4\times10^{-3} \qquad \text{or} \qquad C'b^6/r_0^4 \lesssim 8\cdot5\times10^{-3},$$

in terms of the cutoff parameter r_0.

An empirical limit on the strength of Van der Waals forces between hadrons is supported by the fact that such forces contribute unreasonably large energies to the binding energy of nuclear matter. However, the nuclear matter evidence is not as clear cut as that from p–p scattering.

It is perhaps gratifying that the hadron Van der Waals forces are not present in nature, in that it is *a priori* unreasonable to expect low order perturbation theory to be useful in QCD at low energies and large distances. Moreover, the confinement dogma would not permit the gluons to travel large distances, i.e. the coloured states to exist for large times. Thus our negative result is best interpreted as showing that in deriving hadronic Van der Waals forces one is applying perturbative QCD outside its region of validity. If nuclear physics is the molecular physics of coloured quarks, it is much more complex than conventional molecular physics.

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Appendix

We have to calculate

$$\frac{E_3}{N} = \frac{\rho_0^2}{3!\,V}\int \mathrm{d}\boldsymbol{r}_1\,\mathrm{d}\boldsymbol{r}_2\,\mathrm{d}\boldsymbol{r}_3\;V_{\mathrm{had}}^{\mathrm{AT}}(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_3)\,g(r_{12})\,g(r_{23})\,g(r_{31}),$$

where

$$V_{\mathrm{had}}^{\mathrm{AT}} = \alpha_{\mathrm{s}}\, b^8(3\cos\theta_1\cos\theta_2\cos\theta_3+1)/r_{12}^3\,r_{23}^3\,r_{31}^3,$$

and the variables are defined in Fig. 2*b*. Simplifying the notation by putting

$$r_{12} = r, \qquad r_{23} = x, \qquad r_{31} = u,$$

we change the variables of integration to x, u and r, and get

$$\frac{E_3}{N} = \frac{\rho_0^2}{3!\,V}\int \mathrm{d}\boldsymbol{r}_1\int \mathrm{d}\Omega\int \mathrm{d}\phi\int \mathrm{d}r\int \mathrm{d}x\int \mathrm{d}u\; rxu\,V_{\mathrm{had}}^{\mathrm{AT}}\,g(r)\,g(x)\,g(u).$$

With the substitution

$$\cos\theta_1 = (u^2+r^2-x^2)/2ur$$

and the corresponding expressions for $\cos\theta_2$ and $\cos\theta_3$, we find

$$T = rxu\, V_{\text{had}}^{\text{AT}} = \frac{3\alpha_s b^8}{8}\left(\frac{1}{r^2x^4}+\frac{1}{r^2u^4}+\frac{1}{x^2r^4}+\frac{1}{x^2u^4}+\frac{1}{u^2r^4}+\frac{1}{u^2x^4}\right.$$
$$\left.+\frac{2}{3r^2x^2u^2}-\frac{x^2}{u^4r^4}-\frac{u^2}{x^4r^4}-\frac{r^2}{x^4u^4}\right).$$

Since the integrand is a function of r, x and u only, we can do the angle integrations and the integration with respect to $\boldsymbol{r}_1$, giving $8\pi^2V$. The integrations with respect to r, x and u are subject to the restriction that they form the sides of a triangle. Therefore we have

$$\frac{E_3}{N} = \frac{8\pi^2\rho_0^2}{3!}\left(\int_0^\infty \mathrm{d}r\int_0^r \mathrm{d}x\int_{r-x}^{r+x}\mathrm{d}u\; T g(r)g(x)g(u)\right.$$
$$\left.+\int_0^\infty \mathrm{d}r\int_r^\infty \mathrm{d}x\int_{x-r}^{x+r}\mathrm{d}u\; T g(r)g(x)g(u)\right).$$

For $g(s) = \theta(s-a)$, where

$$\theta(t) = 1,\qquad t\geqslant 0,$$
$$= 0,\qquad t<0,$$

the intervals of integration are further restricted, giving

$$\frac{E_3}{N} = \frac{8\pi^2\rho_0^2}{3!}\left(\int_a^{2a}\mathrm{d}r\int_a^r\mathrm{d}x\int_a^{r+x}\mathrm{d}u\;T+\int_a^{2a}\mathrm{d}r\int_r^{r+a}\mathrm{d}x\int_a^{x+r}\mathrm{d}u\;T\right.$$
$$+\int_a^{2a}\mathrm{d}r\int_{r+a}^\infty\mathrm{d}x\int_{x-r}^{x+r}\mathrm{d}u\;T+\int_{2a}^\infty\mathrm{d}r\int_a^{r-a}\mathrm{d}x\int_{r-x}^{r+x}\mathrm{d}u\;T$$
$$+\int_{2a}^\infty\mathrm{d}r\int_{r-a}^{r+a}\mathrm{d}x\int_a^{r+x}\mathrm{d}u\;T+\int_{2a}^\infty\mathrm{d}r\int_r^{r+a}\mathrm{d}x\int_a^{x+r}\mathrm{d}u\;T$$
$$\left.+\int_{2a}^\infty\mathrm{d}r\int_{r+a}^\infty\mathrm{d}x\int_{x-r}^{x+r}\mathrm{d}u\;T\right)$$

$$= \frac{8\pi^2\rho_0^2}{3!}\left(\int_a^{2a}\mathrm{d}r\int_a^{r+a}\mathrm{d}x\int_a^{r+x}\mathrm{d}u\;T+\int_a^\infty\mathrm{d}r\int_{r+a}^\infty\mathrm{d}x\int_{x-r}^{x+r}\mathrm{d}u\;T\right.$$
$$+\int_{2a}^\infty\mathrm{d}r\int_a^{r-a}\mathrm{d}x\int_{r-x}^{r+x}\mathrm{d}u\;T+\int_{2a}^\infty\mathrm{d}r\int_{r-a}^{r+a}\mathrm{d}x\int_a^{r+x}\mathrm{d}u\;T$$
$$\left.+\int_{2a}^\infty\mathrm{d}r\int_r^{r+a}\mathrm{d}x\int_a^{x+r}\mathrm{d}u\;T\right).$$

These integrations are straightforward and give the result

$$E_3/N = \left(8\pi^2\rho_0^2/3!\right)\left(\tfrac{5}{24}\alpha_s b^8/a^3\right),$$

or

$$E_3/N = \tfrac{5}{18}\pi^2 \alpha_s b^8 \rho_0^2/a^3,$$

as given in the text.

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