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Parametric Instabilities in a Magnetized and Collisional Plasma

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Abstract

The dispersion relation for a magnetized, collisional and hot plasma in the presence of a pump wave is developed for the case where the pump frequency ω_0 is large compared with the cyclotron frequency ω_c and the plasma frequency ω_p. Formulae for the growth rate, the damping rate for the free electron plasma wave and the threshold power are derived and discussed numerically under different conditions. It is found that in a hot plasma (for magnetic fields with $\omega_c/\omega_p = 1$ and 10) the threshold power P_T is less than or greater than that in a cold plasma for the $(\mathrm{Re}\,\omega_2)_+$ or $(\mathrm{Re}\,\omega_2)_-$ modes respectively. In a weak magnetic field ($\omega_c/\omega_p = 0 \cdot 1$), P_T does not vary with the direction θ of the magnetic field for the $(\mathrm{Re}\,\omega_2)_+$ mode. However, P_T for the $(\mathrm{Re}\,\omega_2)_-$ mode is a minimum at $\theta = 30°$ and 10° for $\omega_c/\omega_p = 1$ and 10 respectively, and it becomes very large (10^5–10^7 times its value in a cold unmagnetized plasma) for $\omega_c/\omega_p = 0 \cdot 1$. The results for the growth rate are found to be just the reverse of those for the threshold power.

Introduction

The study of the parametric decay of an intense electromagnetic (pump) wave into a scattered electromagnetic wave and an electron plasma wave is of much current interest because of the relevance to the problems of heating a magnetically confined plasma and of providing an explanation for anomalous absorption and scattering of radio waves in the ionosphere and guidance for ionospheric modification experiments. Recently various aspects of an electron plasma wave (stimulated Raman scattering) have been investigated by Lee (1974), Forslund *et al.* (1975), Bodner and Eddleman (1972), Drake *et al.* (1974), Koch and Albritton (1975), Silin and Starodub (1975), Fuchs (1976), Sodha *et al.* (1976), Willett and Maraghechi (1978) and many others. In particular, Willett and Maraghechi analysed the problem by neglecting the pressure tensor and collision terms in the electron momentum equation.

In the present work we extend the analysis of Willett and Maraghechi (1978) by including the pressure tensor and collision terms. On solving Maxwell's equations with the electron continuity and momentum equations in a straightforward manner, we obtain the dispersion relation and formulae for the growth rate α, the damping rate α_2 for the free electron plasma wave and the threshold power P_T. The resulting variations of α_2, α and P_T as functions of the plasma thermal velocity v_t, cyclotron frequency ω_c and direction θ of the magnetic field with respect to the electromagnetic wave are shown graphically.

Basic Assumptions and Dispersion Relation

We consider a large amplitude plane polarized electromagnetic pump wave (ω_0, k_0) propagating in an infinite collisional and hot plasma that is embedded in a uniform static magnetic field $\boldsymbol{B}_0$ $(=B_0 \hat{\boldsymbol{b}}_0)$. To simplify matters, we neglect the ion motion at the outset since it has negligible influence on the high frequency waves under consideration. The resulting parametric decay may be described as a three-wave process involving the pump wave (ω_0, k_0), a backscattered electromagnetic wave (ω_1, k_1) and an electron plasma wave (ω_2, k_2). These waves satisfy the phase matching conditions

$$\omega_1 = \omega_2 - \omega_0\,, \qquad k_1 = k_2 - k_0\,, \tag{1}$$

where $\operatorname{Re}\omega_1$ is negative and $\operatorname{Re}\omega_2$, ω_0, k_1, k_2 and k_0 are positive. The cyclotron frequency ω_c, plasma frequency ω_p and electron plasma frequency $\operatorname{Re}\omega_2$ are taken to be small compared with the pump frequency ω_0. To avoid relativistic effects, the amplitude of the electron velocity v_0 is also assumed to be very small in comparison with the speed of light c.

For the parametric decay, the electron number density n, electron gas velocity $\boldsymbol{v}$ and electric and magnetic fields $\boldsymbol{E}$ and $\boldsymbol{B}$ are taken as

$$n = n^0 + n_2'\,, \qquad \boldsymbol{v} = \boldsymbol{v}^0 + \boldsymbol{v}_1' + \boldsymbol{v}_2'\,, \tag{2a}$$

$$\boldsymbol{E} = \boldsymbol{E}^0 + \boldsymbol{E}_1' + \boldsymbol{E}_2'\,, \qquad \boldsymbol{B} = \boldsymbol{B}^0 + \boldsymbol{B}_1' + \boldsymbol{B}_2'\,, \tag{2b}$$

where

$$A_{1,2}' = \mathscr{A}_{1,2}' \exp\{\mathrm{i}(\boldsymbol{k}_{1,2}\cdot\boldsymbol{r} - \omega_{1,2}\,t)\} + \text{c.c.}\,, \tag{3a}$$

$$\boldsymbol{v}^0 = \mathrm{i}\,v_0\,\hat{\boldsymbol{e}}_0 \exp\{\mathrm{i}(\boldsymbol{k}_0\cdot\boldsymbol{r} - \omega_0\,t)\} + \text{c.c.}\,, \tag{3b}$$

$$\boldsymbol{E}^0 = E_0\,\hat{\boldsymbol{e}}_0 \exp\{\mathrm{i}(\boldsymbol{k}_0\cdot\boldsymbol{r} - \omega_0\,t)\} + \text{c.c.}\,, \tag{3c}$$

$$\boldsymbol{B}^0 = B_0\,\hat{\boldsymbol{b}}_0 + [(m/e)ck_0\,v_0\,\hat{\boldsymbol{k}}_0 \times \hat{\boldsymbol{e}}_0 \exp\{\mathrm{i}(\boldsymbol{k}_0\cdot\boldsymbol{r} - \omega_0\,t)\}] + \text{c.c.}\,. \tag{3d}$$

Here A denotes n, v, E or B, the superscript 0 and the prime denote unperturbed and perturbed states respectively, and the subscripts 1 and 2 indicate the perturbed quantities associated with the backscattered wave and the electron plasma wave.

Solving the linearized equations of continuity and momentum and the Maxwell equations, using the relations (1), (2) and (3), we obtain after some straightforward algebra:

$$v_0 = eE_0/m\omega_0\,, \tag{4}$$

$$\omega_0^2 = \omega_p^2 + c^2 k_0^2\,, \qquad (\operatorname{Re}\omega_1)^2 = \omega_p^2 + c^2 k_1^2\,, \tag{5}$$

$$\begin{aligned} 1 - \frac{\omega_c^2\beta^2}{\omega_2^2}\Big(1 - \frac{2\mathrm{i}\nu}{\omega_2}\Big) + \Big(\frac{c^2 k_2^2 \omega_p^2 \beta}{\omega_2^2(\omega_2^2 - k_2^2 c^2)} - \frac{v_t^2 k_2^2 \beta}{\omega_2^2}\Big)\Big(1 - \frac{\mathrm{i}\nu}{\omega_2}\Big)\Big\{1 - \frac{\omega_c^2\beta^2\cos^2\theta}{\omega_2^2}\Big(1 - \frac{2\mathrm{i}\nu}{\omega_2}\Big)\Big\} \\ = -\frac{\omega_p^2 v_0^2 k_2^2 \beta}{\omega_2^2 Q_1}\Big(1 - \frac{\mathrm{i}\nu}{\omega_2}\Big)\Big\{1 - \frac{\omega_c^2\beta^2\cos^2\theta}{\omega_2^2}\Big(1 - \frac{2\mathrm{i}\nu}{\omega_2}\Big)\Big\}, \end{aligned} \tag{6}$$

where

$$\beta = (k_2^2 c^2 - \omega_2^2)/(k_2^2 c^2 - \omega_2^2 + \omega_p^2), \qquad Q_1 = k_1^2 c^2 - \omega_1^2 + \omega_p^2,$$

$$v_t^2 = \kappa T/m, \qquad \omega_c = eB_0/mc, \qquad \omega_p^2 = 4\pi n^0 e^2/m, \qquad \hat{\boldsymbol{k}}_0 . \hat{\boldsymbol{b}}_0 = \cos\theta,$$

and ν, κ and T are the electron collision frequency, Boltzmann constant and absolute temperature respectively. In deriving equation (6) we have neglected the nonresonant anti-Stokes wave $(\omega_2 + \omega_0, k_2 + k_0)$.

Under the restriction that $\mathrm{Re}\,\omega_2$, ω_p and ω_c are all very much less than $2\omega_0$, and for small ν, equation (6) reduces to

$$1 - \frac{\omega_c^2}{\omega_2^2}\left(1 - \frac{2\mathrm{i}\nu}{\omega_2}\right) - \left(\frac{\omega_p^2}{\omega_2^2} + \frac{v_t^2 k_2^2}{\omega_2^2}\right)\left(1 - \frac{\mathrm{i}\nu}{\omega_2} - \frac{\omega_c^2 \cos^2\theta}{\omega_2^2} + \frac{3\mathrm{i}\nu\omega_c^2\cos^2\theta}{\omega_2^3}\right)$$

$$= -\frac{\omega_p^2 v_0^2 k_2^2}{\omega_2^2 Q_1}\left(1 - \frac{\mathrm{i}\nu}{\omega_2} - \frac{\omega_c^2\cos^2\theta}{\omega_2^2} + \frac{3\mathrm{i}\nu\omega_c^2\cos^2\theta}{\omega_2^3}\right). \tag{7}$$

This is the dispersion relation for the magnetized, collisional and hot plasma in the presence of a large amplitude pump wave. It describes the parametric excitation of a low frequency electron plasma wave (ω_2, k_2) and a backscattered electromagnetic wave (ω_1, k_1).

Calculation of α_2, α and P_T

To calculate the damping rate α_2 for a free electron plasma wave $(\alpha_2 = -\alpha)$ and the growth rate α and threshold power P_T for stimulated Raman backscattering, we assume

$$\omega_1 = \mathrm{Re}\,\omega_1 + \mathrm{i}\alpha, \qquad \omega_2 = \mathrm{Re}\,\omega_2 + \mathrm{i}\alpha. \tag{8}$$

Under the restriction of weak coupling $(v_0 \to 0)$, equation (7) leads to the following equations for $\mathrm{Re}\,\omega_2$ and α_2:

$$(1 - v_t^2 c^{-2})(\mathrm{Re}\,\omega_2)^6 - (\omega_c^2 + \omega_p^2 - v_t^2 c^{-2}\omega_c^2\cos^2\theta + 5\nu\alpha v_t^2 c^{-2})(\mathrm{Re}\,\omega_2)^4$$

$$+ \{\omega_p^2\omega_c^2\cos^2\theta - 3\nu\alpha(2\omega_c^2 + \omega_p^2 - 3v_t^2 c^{-2}\omega_c^2\cos^2\theta)\}(\mathrm{Re}\,\omega_2)^2 + 3\nu\alpha\omega_p^2\omega_c^2\cos^2\theta = 0, \tag{9a}$$

$$\alpha_2 = \tfrac{1}{2}\nu\left(\frac{v_t^2 c^{-2}(\mathrm{Re}\,\omega_2)^4 + (2\omega_c^2 + \omega_p^2 - 3v_t^2 c^{-2}\omega_c^2\cos^2\theta)(\mathrm{Re}\,\omega_2)^2 - 3\omega_p^2\omega_c^2\cos^2\theta}{3(1 - v_t^2 c^{-2})(\mathrm{Re}\,\omega_2)^4 - 2(\omega_c^2 + \omega_p^2 - v_t^2 c^{-2}\omega_c^2\cos^2\theta)(\mathrm{Re}\,\omega_2)^2 + \omega_p^2\omega_c^2\cos^2\theta}\right). \tag{9b}$$

Inserting equations (8) into the dispersion relation (7) and applying the approximation $\alpha \ll \mathrm{Re}\,\omega_2 \ll \mathrm{Re}\,\omega_1$, we obtain

$$\alpha^2 + \nu\alpha(P + Q) = R, \tag{10a}$$

where

$$P = \frac{3 - 2v_t^2 c^{-2} - (\omega_c^2 + 2\omega_p^2)(\mathrm{Re}\,\omega_2)^{-2}}{2\{1 - v_t^2 c^{-2} - \omega_p^2\omega_c^2\cos^2\theta(\mathrm{Re}\,\omega_2)^{-4}\}}, \tag{10b}$$

$$Q = \frac{21\,v_0^2}{4c^2(\mathrm{Re}\,\omega_2)(\omega_0 - \mathrm{Re}\,\omega_2)}\,\frac{(1 - v_t^2 c^{-2})(\mathrm{Re}\,\omega_2)^2 - \omega_c^2(1 - v_t^2 c^{-2}\cos^2\theta)}{1 - v_t^2 c^{-2} - \omega_p^2\,\omega_c^2\cos^2\theta(\mathrm{Re}\,\omega_2)^{-4}}, \tag{10c}$$

$$R = \tfrac{1}{21}(\mathrm{Re}\,\omega_2)^2 Q. \tag{10d}$$

The growth rate for instability above the threshold is given by the positive root of equation (10a) while the growth rate just above the threshold is

$$\alpha \approx (R/\nu P)(1 - Q/P). \tag{11}$$

Substituting $\alpha = 0$ in equation (11), we obtain the minimum threshold power P_T ($\propto v_0^2$), in terms of its value in a cold unmagnetized plasma, as

$$P_\mathrm{T} = \frac{\omega_p(\mathrm{Re}\,\omega_2)(\omega_0 - \mathrm{Re}\,\omega_2)}{\omega_0 - \omega_p}\,\frac{3 - 2v_t^2 c^{-2} - (\omega_c^2 + 2\omega_p^2)(\mathrm{Re}\,\omega_2)^{-2}}{(1 - v_t^2 c^{-2})(\mathrm{Re}\,\omega_2)^2 - \omega_c^2(1 - v_t^2 c^{-2}\cos^2\theta)}. \tag{12}$$

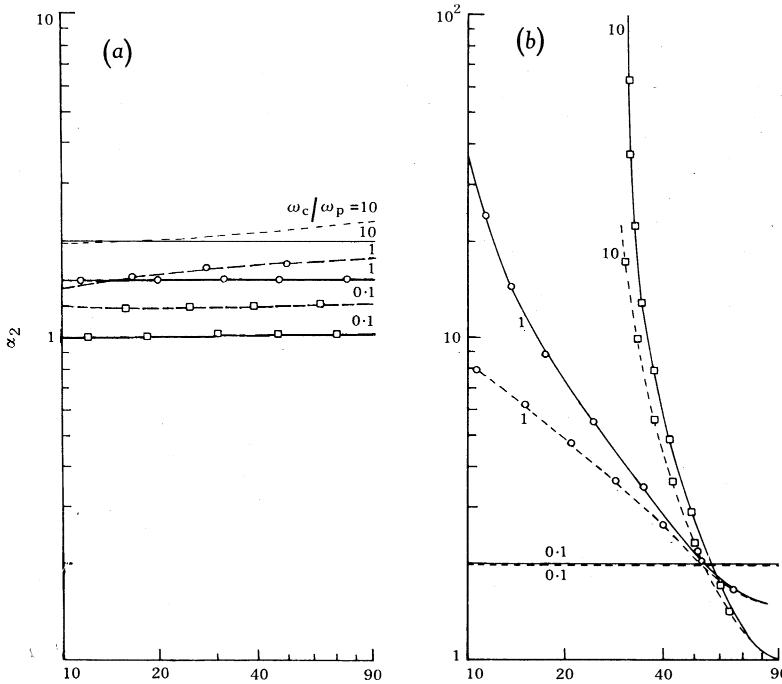

Fig. 1. Variation of the damping rate α_2 of the free electron plasma wave as a function of the direction θ of the magnetic field for (*a*) the $(\mathrm{Re}\,\omega_2)_+$ mode and (*b*) the $(\mathrm{Re}\,\omega_2)_-$ mode. The curves are for the parameter values $\nu/\omega_p = 10^{-3}$, $\omega_0/\omega_p = 10^2$, $v_0/c = 10^{-2}$ and the indicated values of ω_c/ω_p, with $(v_t/c)^2 = 0$ and $0 \cdot 2$ for the full and dashed curves respectively in each case.

Results and Discussion

At the beat frequencies $\omega_0 - |\omega_1|$ and $\omega_0 - \omega_2$ the electron plasma wave and the backscattered wave are resonantly excited and the energy is added to these modes (at the expense of the pump wave) at the resonance if their natural damping rates are small. The nonresonant modes are not important in the backscattering process.

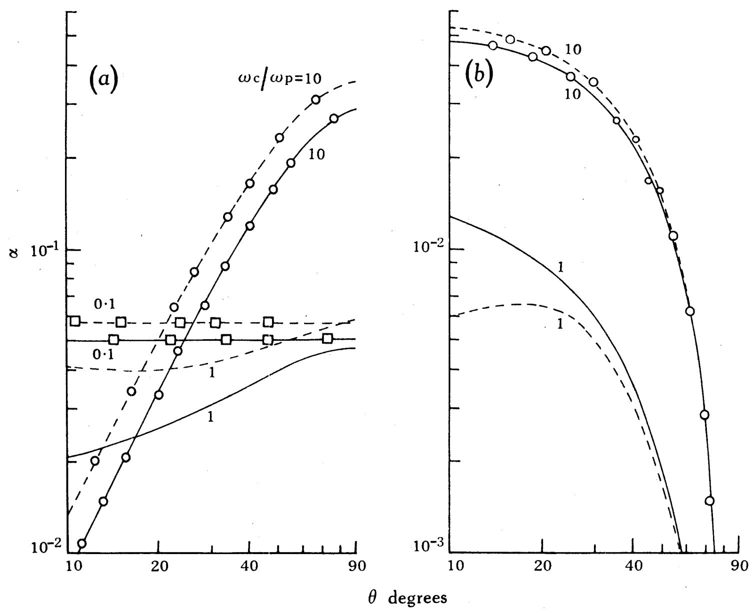

Fig. 2. Variation of the growth rate α as a function of θ for (*a*) the $(\mathrm{Re}\,\omega_2)_+$ mode and (*b*) the $(\mathrm{Re}\,\omega_2)_-$ mode. The curves are for $\nu/\omega_p = 10^{-3}$, $\omega_0/\omega_p = 10^2$, $v_0/c = 10^{-2}$ and the indicated values of ω_c/ω_p, with $(v_t/c)^2 = 0$ and $0\cdot2$ for the full and dashed curves respectively in each case.

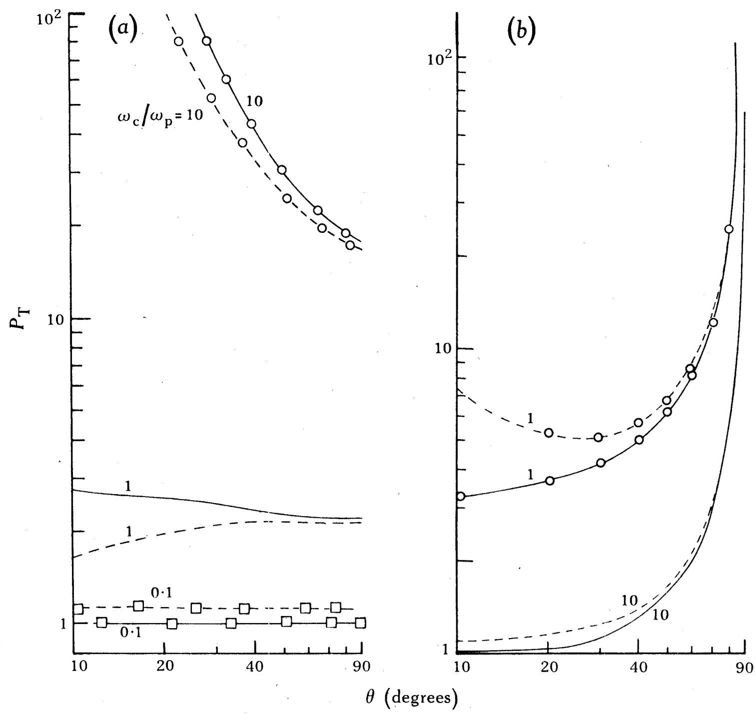

Fig. 3. Variation of the threshold power P_T as a function of θ for (*a*) the $(\mathrm{Re}\,\omega_2)_+$ mode and (*b*) the $(\mathrm{Re}\,\omega_2)_-$ mode. The curves are for $\omega_0/\omega_p = 10^2$ and the indicated values of ω_c/ω_p, with $(v_t/c)^2 = 0$ and $0\cdot2$ for the full and dashed curves respectively in each case.

The mode frequencies, which are independent of the wave number k_2 and the collision frequency ν, are determined by the two roots of equation (9a) (at $\nu = 0$), namely

$$(\mathrm{Re}\,\omega_2)^2 = [(\omega_c^2 + \omega_p^2 - v_t^2 c^{-2}\omega_c^2 \cos^2\theta) \pm \{(\omega_c^2 + \omega_p^2 - v_t^2 c^{-2}\omega_c^2 \cos^2\theta)^2 - 4(1 - v_t^2 c^{-2})\omega_p^2 \omega_c^2 \cos^2\theta\}^{\frac{1}{2}}]/2(1 - v_t^2 c^{-2}). \qquad (13)$$

The calculated results for the two modes $(\mathrm{Re}\,\omega_2)_+$ and $(\mathrm{Re}\,\omega_2)_-$ are plotted in Figs 1, 2 and 3, which show the variation with θ of α_2, α and P_T (in terms of their values for Langmuir mode excitation in an unmagnetized cold plasma) at parameter values of $\omega_c/\omega_p = 0\cdot 1$, 1 and 10 and $v_t^2/c^2 = 0$ and $0\cdot 2$, with $v_0/c = 10^{-2}$, $\nu/\omega_p = 10^{-3}$ and $\omega_0/\omega_p = 10^2$.

From Fig. 1 it can be seen that, for the $(\mathrm{Re}\,\omega_2)_+$ mode (Fig. 1*a*), the damping rate α_2 is very weakly dependent on θ in a cold plasma and increases slightly with θ as the plasma temperature is raised; whereas, for the $(\mathrm{Re}\,\omega_2)_-$ mode (Fig. 1*b*), α_2 is independent of θ for $\omega_c/\omega_p = 0\cdot 1$ but an increase in plasma temperature and magnetic field (ω_c/ω_p) results in a marked decrease of α_2 with θ.

Fig. 2*a* shows that the growth rate α for the $(\mathrm{Re}\,\omega_2)_+$ mode is independent of θ in a weak magnetic field $(\omega_c/\omega_p = 0\cdot 1)$ but increases with θ as the magnetic field is strengthened. At small angles the value of α for $\omega_c/\omega_p = 10$ is less than for $\omega_c/\omega_p = 0\cdot 1$ and 1 but, after about $\theta = 26°$, α is greatest for the strongest magnetic field. An increase in plasma temperature results in an increase in α for all magnetic field strengths at all angles. In contrast, for the $(\mathrm{Re}\,\omega_2)_-$ mode (Fig. 2*b*), the growth rate is negligible for $\omega_c/\omega_p = 0\cdot 1$ and a maximum at 10° for $\omega_c/\omega_p = 1$ and 10. An increase in the plasma temperature results in an increase in α for the strongest magnetic field but in a large decrease in α for $\omega_c/\omega_p = 1$ at small angles, which becomes less marked with increasing θ up to about 70°, when the growth rate becomes independent of θ.

From Fig. 3*a* it can be seen that, for the $(\mathrm{Re}\,\omega_2)_+$ mode, the threshold power P_T is high for a strong magnetic field, and an increase in the plasma temperature reduces P_T for $\omega_c \geqslant \omega_p$ but has the reverse effect for $\omega_c < \omega_p$. In the case of the $(\mathrm{Re}\,\omega_2)_-$ mode (Fig. 3*b*), for $\omega_c/\omega_p = 0\cdot 1$, P_T is found to be 10^5–10^7 times its value in an unmagnetized cold plasma (not shown in the figure) while, for $\omega_c/\omega_p = 1$ and 10 (in a hot plasma), P_T is a minimum at angles of 30° and 10° respectively. At small angles an increase in the plasma temperature results in an increase in P_T up to about $\theta = 80°$. Beyond this angle the plasma temperature has no effect.

The formulae derived here for the dispersion relation and the threshold and growth rate of the $(\mathrm{Re}\,\omega_2)_\pm$ modes for backscattering, when evaluated at $\nu = 0$ and $v_t = 0$, agree with the results obtained by Willett and Maraghechi (1978) and, at $\theta = 90°$, also reduce to those obtained by Lee (1974).

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