# A Semi-empirical Law for Sputtering in Plasma Devices

## J. L. Cook, E. K. Rose and J. W. Connolly

Australian Atomic Energy Commission Research Establishment, Private Mail Bag, Sutherland, N.S.W. 2232.

#### Abstract

A general law is given for sputtering coefficients as a function of the energy of hydrogen and helium isotope ions incident on metallic and graphite walls and limiters. The average over a Maxwellian spectrum is performed analytically.

#### 1. Introduction

Conceptual design studies of fusion reactor performance, such as the INTOR project, are being undertaken by many nations. A recent review paper by Hershman and Sigmar (1981) gives full details of the effect of impurities in plasmas; another by Ashby and Hughes (1981) combines the evaluation of radiation loss for impurities (Post *et al.* 1977) with transport codes to conclude that in INTOR the temperature radial profile will collapse inwards from the wall. Post *et al.* claimed that the plasma temperature near the wall would be about 200 eV, a temperature at which impurities, particularly from heavy metals, have quite large radiation losses. It is not clear which wall reflective properties were considered to contribute to such losses, but these authors pointed out that the main source of impurities which has considerable power rate coefficients is sputtering from the walls and limiters.

At the AAEC Research Establishment there is insufficient effort to carry out extensive compilations of sputtering coefficients for the hydrogen and helium isotopes, but such a compilation was carried out by Thomas *et al.* (1979). At about the same time, McCracken and Stott (1979) reviewed the semi-empirical theory of sputtering, but their use of the spectral law, developed by Bohdansky, and scaling coefficients did not lead to a particularly good fit of the data and no maximum was predicted in the spectral curve, which occurs in almost all of the data around 1 keV. We have used all available data from Thomas *et al.* to fit the spectrum and to discover the appropriate scaling rules.

### 2. Semi-empirical Laws

We have chosen the evaluations by Thomas *et al.* to fit the elements Be, C, Ti, Fe, Co, Ni, Cu, Nb, Mo, Ag, Ta, W, Au and U to the rational function

$$S(E) = A(m_1, m_2) \{ (E - E_T) / (E + B)^2 \},$$
(1)

where  $E \ge E_T$ , S(E) is the sputtering coefficient for the incident ion beam energy E, and

$$E_{\rm T} = E_{\rm S} / \{\gamma(1-\gamma)\}, \qquad \gamma = 4m_1 m_2 / (m_1 + m_2)^2; \qquad (2a, b)$$

 $m_1$  and  $m_2$  are the atomic masses of the elements in the wall and incident beam respectively and  $A(m_1, m_2)$  is the scaling coefficient. The threshold energy  $E_{\rm T}$  can be obtained directly from the sublimation energy of the wall material, defined as  $E_{\rm S}$  (eV), which, in turn, can be obtained from the relation

$$E_{\rm S} = 0.0435 L_{\rm H},\tag{3}$$

where  $L_{\rm H}$  is the latent heat of sublimation (Smithells 1976). If the latent heat is not tabulated, one can use the rough approximations

$$E_{\rm s} \approx 2 \cdot 8 \times 10^{-3} M_{\rm p}$$
 or  $1 \cdot 48 \times 10^{-3} B_{\rm p}$ , (4)

where  $M_p$  is the melting point and  $B_p$  is the boiling point in K. We have used the second correlation for graphite and silicon only.

It was not possible to find sufficient data to check whether  $E_{\rm T}$  in equation (2a) is the experimental threshold, so we have assumed the theoretical value and computed the constants A and B from the fits to the equation

$$\{(E - E_{\rm T})/S(E)\}^{\pm} = a_0 + a_1 E,\tag{5}$$

in which  $A = 1/a_1^2$  and  $B = a_0/a_1$ . The elements used were Be, C, Ti, Fe, Co, Ni, Cu, Nb, Mo, W, Au and U. The maximum of the spectrum occurs when

$$E_{\rm max} = B + 2E_{\rm T} \tag{6}$$

and the width is so broad that the fits are very insensitive to B. We observed the correlation

$$\bar{E}_{\max} = b_0 + b_1 Z_1^{\frac{1}{2}} \tag{7}$$

for Ti, Fe, Ni, Mo, W and Au, where  $Z_1$  is the atomic number of the element in the wall and, with an error weighting of 5%, obtained the values

$$b_0 = -2.5429$$
 keV and  $b_1 = 0.7879$  keV.

We could just as well have found a rough correlation of  $\overline{E}_{max}$  with  $m_1$  but, for interpolation purposes, equation (7) suffices.

We note that the scaling factor  $A(m_1, m_2)$  in equation (1) can be well approximated by the law

$$A(m_1, m_2) = A(m_1, 1) \exp\{\bar{c}(m_2 - 1)\},$$
(8)

where  $\bar{c}$  is an average value of  $m_1$  and  $m_2$ . For this average, we used the data from Thomas *et al.* (1979) for Ti, Fe, Ni, Mo and Au to find

$$\bar{c} = 1 \cdot 0. \tag{9}$$

Element	<i>m</i> <sub>2</sub>	Es (eV)	E <sub>T</sub> (keV)	γ	$A(m_1,m_2)$	$\frac{B(Z_1)}{(\text{keV})}$
<sup>4</sup> Be	1	3.44	$1.490 \times 10^{-2}$	$3.619 \times 10^{-1}$	$4 \cdot 335 \times 10^{-1}$	6.150
	2	3.44	$1.430 \times 10^{-2}$	$5.972 \times 10^{-1}$	$7.050 \times 10^{-2}$	$2 \cdot 025 \times 10^{-1}$
	4	3.44	$2.726 \times 10^{-2}$	$8 \cdot 518 \times 10^{-1}$	$5.048 \times 10^{-1}$	$1.750 \times 10^{-1}$
۴C	1	7.42	$3.635 \times 10^{-2}$	$2.857 \times 10^{-1}$	$1.672 \times 10^{-2}$	$4 \cdot 360 \times 10^{-1}$
	2	7.42	$2.968 \times 10^{-2}$	$4.919 \times 10^{-1}$	$3 \cdot 373 \times 10^{-2}$	$3 \cdot 209 \times 10^{-1}$
	4	7.42	$3.956 \times 10^{-2}$	$7 \cdot 499 \times 10^{-1}$	$6.868 \times 10^{-1}$	1.585
<sup>22</sup> Ti	1	4.88	$6.576 \times 10^{-2}$	$8 \cdot 073 \times 10^{-2}$	$1.703 \times 10^{-2}$	$9.681 \times 10^{-1}$
	2	4.88	$3.728 \times 10^{-2}$	$1.549 \times 10^{-1}$	$4 \cdot 275 \times 10^{-2}$	1.009
	3	4·88	$2.813 \times 10^{-2}$	$2 \cdot 234 \times 10^{-1}$	$1 \cdot 209 \times 10^{-1}$	1 · 268
	4	4.88	$2 \cdot 396 \times 10^{-2}$	$2.847 \times 10^{-1}$	$3 \cdot 395 \times 10^{-1}$	1.024
2617-		4·14	$6.507 \times 10^{-2}$	$6.829 \times 10^{-2}$	$5 \cdot 223 \times 10^{-2}$	1.278
<sup>26</sup> Fe	1	4·14 4·14	$3.617 \times 10^{-2}$	$1 \cdot 319 \times 10^{-1}$	$1 \cdot 224 \times 10^{-1}$	1.056
	2 3	4·14 4·14	$2.676 \times 10^{-2}$	$1.913 \times 10^{-1}$	$2.497 \times 10^{-1}$	1.128
	3 4	4.14	$2 \cdot 070 \times 10^{-2}$ $2 \cdot 237 \times 10^{-2}$	$2.452 \times 10^{-1}$	1.071	1.751
			$2 237 \times 10^{-2}$ $7 \cdot 158 \times 10^{-2}$	$6.612 \times 10^{-2}$	$1 \cdot 207 \times 10^{-1}$	3.056
<sup>27</sup> Co	1	4·42	$3.965 \times 10^{-2}$	$1 \cdot 278 \times 10^{-1}$	$2.807 \times 10^{-1}$	2.192
	2	4.42	$2.436 \times 10^{-2}$	$1^{\circ}273 \times 10^{\circ}$ $2 \cdot 382 \times 10^{-1}$	5.955	4.562
<b>2</b> 0	4	4.42	$2^{4}30 \times 10^{-2}$ 7.197 × 10 <sup>-2</sup>	$6.638 \times 10^{-2}$	$9.769 \times 10^{-2}$	1.513
<sup>28</sup> Ni	1	4.46	$3.988 \times 10^{-2}$	$1.283 \times 10^{-1}$	$2 \cdot 430 \times 10^{-1}$	1.365
	2	4.46	$3.988 \times 10^{-2}$ $2.942 \times 10^{-2}$	$1 \cdot 283 \times 10^{-1}$ $1 \cdot 863 \times 10^{-1}$	$7.519 \times 10^{-1}$	1.797
	3	4.46	$2.942 \times 10^{-2}$ $2.452 \times 10^{-2}$	$1^{-303 \times 10}$ $2 \cdot 390 \times 10^{-1}$	1.972	2.004
	4	4.46		$6 \cdot 147 \times 10^{-2}$	$1.849 \times 10^{-1}$	$6.632 \times 10^{-1}$
<sup>29</sup> Cu	1	3.55	$6 \cdot 153 \times 10^{-2}$	$1.191 \times 10^{-1}$	$5 \cdot 394 \times 10^{-1}$	$1.632 \times 10^{-1}$
	2	3.55	$3 \cdot 383 \times 10^{-2}$	$1 \cdot 191 \times 10^{-1}$ $2 \cdot 230 \times 10^{-1}$	$5 \cdot 394 \times 10$ 5 \cdot 167	3.298
	4	3.55	$2 \cdot 049 \times 10^{-2}$		$3.266 \times 10^{-2}$	3.110
<sup>41</sup> Nb	1	7.50	$1.844 \times 10^{-1}$	$4 \cdot 246 \times 10^{-2}$	$8 \cdot 207 \times 10^{-2}$	3.252
	2	7.50	$9.846 \times 10^{-2}$	$8 \cdot 307 \times 10^{-2}$	3.152	8.552
	4	$7 \cdot 50$	$5 \cdot 626 \times 10^{-2}$	$1.584 \times 10^{-1}$		3.877
42Mo	1	6.90	$1.749 \times 10^{-1}$	$4 \cdot 115 \times 10^{-2}$	$3 \cdot 180 \times 10^{-2}$	2.157
-	2	6.90	$9.316 \times 10^{-2}$	$8.056 \times 10^{-2}$	$6.546 \times 10^{-2}$	2.137
	3	6.90	$6.607 \times 10^{-2}$	$1 \cdot 185 \times 10^{-1}$	$1 \cdot 432 \times 10^{-1}$	2.355
	3	6.90	$6.607 \times 10^{-2}$	$1 \cdot 185 \times 10^{-1}$	$3 \cdot 288 \times 10^{-1}$ $5 \cdot 575 \times 10^{-1}$	2.273
	4	6.90	$5 \cdot 302 \times 10^{-2}$	$1.538 \times 10^{-1}$		2·273 2·534
74W	1	8.81	$4 \cdot 152 \times 10^{-1}$	$2 \cdot 169 \times 10^{-2}$	$4 \cdot 012 \times 10^{-3}$	
	2	8.81	$2 \cdot 147 \times 10^{-1}$	$4 \cdot 288 \times 10^{-2}$	$1 \cdot 793 \times 10^{-2}$	3·219 5·134
	4	8.81	$1 \cdot 152 \times 10^{-1}$	$8 \cdot 341 \times 10^{-2}$	$7.737 \times 10^{-1}$	
<sup>79</sup> Au	1	3.94	$1.985 \times 10^{-1}$	$2 \cdot 026 \times 10^{-2}$	$3 \cdot 162 \times 10^{-1}$	5.042
	2	3.94	$1.024 \times 10^{-1}$	$4 \cdot 008 \times 10^{-2}$	$7 \cdot 477 \times 10^{-1}$	4·189
	3	3.94	$7.034 \times 10^{-2}$	$5.956 \times 10^{-2}$	$1 \cdot 252$	3.884
	3	3.94	$7.034 \times 10^{-2}$	$5.956 \times 10^{-2}$	2.570	4·342
	. 4	3.94	$5 \cdot 474 \times 10^{-2}$	$7 \cdot 808 \times 10^{-2}$	2.761	3.197
<sup>92</sup> U	1	4.76	$2.883 \times 10^{-1}$	$1.679 \times 10^{-2}$	$3 \cdot 627 \times 10^{-2}$	18.16
	4	4.76	$7 \cdot 826 \times 10^{-2}$	$6.506 \times 10^{-2}$	1.519	22.39

Table 1. Experimentally fitted data

Thomas *et al.* reported old experimental values for S(E) by Kenknight and Wehner (1964) for eighteen elements at the energies of 2.33 and 3.5 keV for incident H<sup>+</sup> ions. They warned that the quoted error of 3% was unrealistic and differed from more recent measurements by factors of two. However, their graphs show quite definite regions,

(i)  $22 \le Z_1 \le 29$ , (ii)  $41 \le Z_1 \le 47$ , (iii)  $74 < Z_1 \le 79$ , (10)

$$A(m_1, 1) = \exp(c_0 + c_1 m_1) \tag{11}$$

holds. For a least-squares fit over region (i) for five atoms, we obtained

$$c_{01} = -11 \cdot 393, \qquad c_{11} = 0 \cdot 15362.$$
 (12)

Assuming the same slope in region (ii), we found, from the accurate Mo data, that

$$c_{02} = -18 \cdot 187, \qquad c_{12} = c_{11};$$
 (13)

		Table 2.Predicted values of $A$ and $B$								
Element	<i>m</i> <sub>2</sub>	Es (eV)	E <sub>T</sub> (keV)	γ	$A(m_1,m_2)$	$\frac{B(Z_1)}{(\text{keV})}$				
<sup>13</sup> Al	1	4.09	$3 \cdot 421 \times 10^{-2}$	$1.388 \times 10^{-1}$	$4 \cdot 503 \times 10^{-2}$	$2 \cdot 294 \times 10^{-1}$				
	2	4.09	$2 \cdot 134 \times 10^{-2}$	$2 \cdot 585 \times 10^{-1}$	$1 \cdot 257 \times 10^{-1}$	$2 \cdot 551 \times 10^{-1}$				
	3	4.09	$1.770 \times 10^{-2}$	$3 \cdot 624 \times 10^{-1}$	$3.461 \times 10^{-1}$	$2 \cdot 624 \times 10^{-1}$				
	4	4.09	$1.653 \times 10^{-2}$	$4 \cdot 500 \times 10^{-1}$	$9.225 \times 10^{-1}$	$2 \cdot 648 \times 10^{-1}$				
<sup>14</sup> Si	1	4.68	$4 \cdot 039 \times 10^{-2}$	$1.338 \times 10^{-1}$	$8 \cdot 435 \times 10^{-4}$	$3 \cdot 243 \times 10^{-1}$				
	2	4.68	$2 \cdot 498 \times 10^{-2}$	$2 \cdot 497 \times 10^{-1}$	$2 \cdot 325 \times 10^{-3}$	$3 \cdot 243 \times 10^{-1}$ $3 \cdot 551 \times 10^{-1}$				
	3	4.68	$2 \cdot 055 \times 10^{-2}$	$3.510 \times 10^{-1}$	$6.380 \times 10^{-3}$	$3.640 \times 10^{-1}$				
	4	4.68	$1.902 \times 10^{-2}$	$4 \cdot 367 \times 10^{-1}$	$1 \cdot 699 \times 10^{-2}$	$3.670 \times 10^{-1}$				
<sup>21</sup> Sc	1	3.91	$4.986 \times 10^{-2}$	$8.578 \times 10^{-2}$	$1 \cdot 126 \times 10^{-2}$	$9.679 \times 10^{-1}$				
	2	3.91	$2 \cdot 850 \times 10^{-2}$	$1.642 \times 10^{-1}$	$1 120 \times 10^{-2}$ $3 \cdot 103 \times 10^{-2}$	$9.6/9 \times 10^{-1}$ 1.011				
	3	3.91	$2 \cdot 167 \times 10^{-2}$	$2 \cdot 362 \times 10^{-1}$	$8.515 \times 10^{-2}$	1.011				
	4	3.91	$1.861 \times 10^{-2}$	$3.003 \times 10^{-1}$	$2 \cdot 267 \times 10^{-1}$	1.024				
<sup>23</sup> V	1	4.88	$6.941 \times 10^{-2}$	$7.610 \times 10^{-2}$						
	2	4.88	$3.906 \times 10^{-2}$	$1.463 \times 10^{-1}$	$5 \cdot 044 \times 10^{-2}$ $1 \cdot 418 \times 10^{-1}$	1.097				
	3	4.88	$2 \cdot 926 \times 10^{-2}$	$2 \cdot 116 \times 10^{-1}$	$1.418 \times 10^{-1}$ $3.917 \times 10^{-1}$	1.158				
	4	4.88	$2 \cdot 475 \times 10^{-2}$	$2 \cdot 702 \times 10^{-1}$	1.046	1.177				
<sup>24</sup> Cr	1	5.36	$7.763 \times 10^{-2}$	$7.461 \times 10^{-2}$		1.186				
	2	5.36	$4.358 \times 10^{-2}$	$1.436 \times 10^{-1}$	$3 \cdot 319 \times 10^{-2}$	1.162				
	3	5.36	$3.257 \times 10^{-2}$	$1.436 \times 10^{-1}$ 2.077 × 10 <sup>-1</sup>	$9.151 \times 10^{-2}$	1.230				
	4	5·36	$2.749 \times 10^{-2}$	$2.655 \times 10^{-1}$	$2 \cdot 511 \times 10^{-1}$	1.252				
<sup>30</sup> Zn <sup>40</sup> Zr	1	1.37	$2 \cdot 743 \times 10^{-2}$ $2 \cdot 437 \times 10^{-2}$		$6.684 \times 10^{-1}$	1.262				
	2	$1 \cdot 37$ 1 · 37	$1.336 \times 10^{-2}$	$5.980 \times 10^{-2}$	$2 \cdot 593 \times 10^{-1}$	1.724				
	3	1.37 1.37	$1.336 \times 10^{-2}$ 9.756 × 10 <sup>-3</sup>	$1 \cdot 160 \times 10^{-1}$	$7.150 \times 10^{-1}$	1.746				
	4	1.37 1.37	$3.736 \times 10^{-3}$ $8.051 \times 10^{-3}$	$1.690 \times 10^{-1}$	1.962	1.753				
				$2 \cdot 174 \times 10^{-1}$	5.222	1.756				
	1	6·36	$1.538 \times 10^{-1}$	$4 \cdot 323 \times 10^{-2}$	$4 \cdot 144 \times 10^{-2}$	2.133				
	2	6·36	$8 \cdot 217 \times 10^{-2}$	$8 \cdot 454 \times 10^{-2}$	$1 \cdot 177 \times 10^{-1}$	$2 \cdot 276$				
	3	6.36	$5 \cdot 847 \times 10^{-2}$	$1 \cdot 242 \times 10^{-1}$	$3 \cdot 262 \times 10^{-1}$	2.323				
	4	6.36	$4.707 \times 10^{-2}$	$1 \cdot 611 \times 10^{-1}$	$8.724 \times 10^{-1}$	2.346				
<sup>45</sup> Rh	1	5.78	$1.565 \times 10^{-1}$	$3 \cdot 842 \times 10^{-2}$	$9 \cdot 248 \times 10^{-2}$	2.429				
	2	5.78	$8 \cdot 300 \times 10^{-2}$	$7.531 \times 10^{-2}$	$2 \cdot 550 \times 10^{-1}$	2.576				
	3	5.78	$5 \cdot 861 \times 10^{-2}$	$1 \cdot 109 \times 10^{-1}$	$6.996 \times 10^{-1}$	2.625				
	4	5.78	$4 \cdot 685 \times 10^{-2}$	$1 \cdot 442 \times 10^{-1}$	1.862	2.649				
<sup>47</sup> Ag	1	1.78	$5.037 \times 10^{-2}$	$3.668 \times 10^{-2}$	$3 \cdot 122 \times 10^{-1}$	2.758				
	2	1.78	$2 \cdot 665 \times 10^{-2}$	$7 \cdot 197 \times 10^{-2}$	$8.679 \times 10^{-1}$	2.805				
	4	1 · 78	$1 \cdot 496 \times 10^{-2}$	$1.380 \times 10^{-1}$	6.365	2.829				
<sup>73</sup> Ta	1	8.13	$3.773 \times 10^{-1}$	$2 \cdot 203 \times 10^{-2}$	$1.778 \times 10^{-2}$	3.434				
	2	8.13	$1.952 \times 10^{-1}$	$4 \cdot 355 \times 10^{-2}$	$5 \cdot 102 \times 10^{-2}$	3.798				
	4	8.13	$1.049 \times 10^{-1}$	$8.469 \times 10^{-2}$	$3 \cdot 800 \times 10^{-1}$	3.979				

Table 2. Predicted values of A and B

similarly, using the Au information,

$$c_{03} = -31 \cdot 409, \qquad c_{13} = c_{11}.$$
 (14)

## 3. Details of Data

As was the case with the ADL-1 atomic data library (Clancy *et al.* 1981), preparation of the sputtering coefficient data library was based on the use of all experimental information, resorting to the constants in the fitted law (1) only for interpolation and extrapolation when experimental data were unavailable. The best fits for  $E_{\rm s}$ ,  $E_{\rm T}$ ,  $\gamma$ , A and B are displayed in Table 1. The predicted values of A and B are shown in Table 2 for Al, Si, Sc, V, Cr, Zn, Zr, Rh, Ag and Ta. The elements Al, Zr and Ta can be normalized to the measured value of  $A(m_1, 1)$ .

The compilation by Thomas *et al.* gives tables and plots for the sputtering of incident  $H^+$ ,  $D^+$ ,  $T^+$ ,  $He_3^+$  and  $He_4^+$  ion beams from the elements Be, C, Ti, Fe, Co, Ni, Cu, Nb, Mo, Ag, Ta, W, Au and U, although the information is sometimes very scanty indeed. The fitted spectra of some of these elements are given in Fig. 1 on a log-log scale. It is noted that the deviation from the experimental values is the largest at very low energies; we believe that this is because S(E) is very small in this region and that the predicted values of  $E_T$  could be in error. The data here are very sensitive to the value of  $E_T$ .

McCracken and Stott (1979) also gave a general curve for  $\overline{S}(T)$ , the average value of S(E) over a Maxwellian ion energy distribution, which is appropriate for plasmas. Because of their use of the Bohdansky spectral law, the necessary integral cannot be evaluated analytically. In contrast, the form of equation (1) can be averaged analytically.

#### 4. Maxwellian Averaged Sputtering Coefficients

The integral which has to be calculated is

$$\bar{S}(T) = 2\pi^{-\frac{1}{2}} T^{-3/2} \int_{E_{T}}^{\infty} dE \, S(E) E^{\frac{1}{2}} \exp(-E/T), \qquad (15)$$

where T is the temperature of the plasma edge in keV. This can be written

$$\bar{S}(T) = 2\pi^{-\frac{1}{2}}T^{-3/2} \left( \int_{0}^{\infty} dE S(E) E^{\frac{1}{2}} \exp(-E/T) - \int_{0}^{E_{T}} dE S(E) E^{\frac{1}{2}} \exp(-E/T) \right)$$
  
=  $I_{1} - I_{2}$ . (16)

Substituting equation (1) into (16), we find that (Gradshteyn and Ryzhik 1965)

$$\int_{0}^{\infty} dE S(E) E^{\frac{1}{2}} \exp(-E/T) = A(I_{1}^{1} - E_{T}I_{2}^{1}), \qquad (17)$$

where

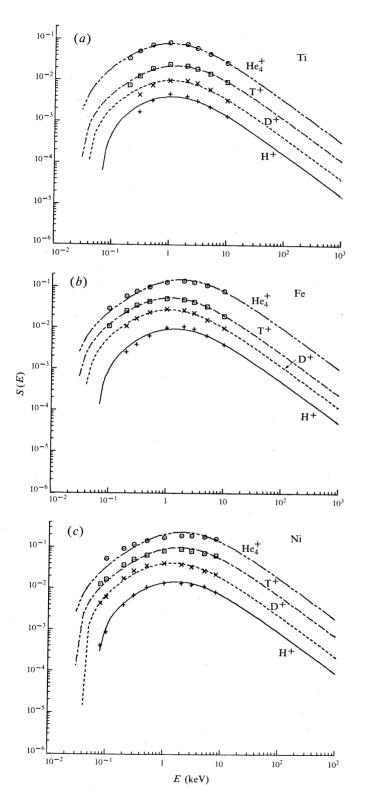
$$V_1^1 = 4\Gamma_2^{\frac{5}{2}} \mu^{-\frac{1}{2}} \exp(\frac{1}{4}z^2) D_{-4}(z), \qquad (18)$$

$$I_{2}^{1} = 2^{3/2} \Gamma_{\frac{3}{2}} B^{-\frac{1}{2}} \exp(\frac{1}{4}z^{2}) D_{-3}(z), \qquad (19)$$

and A is defined by equation (8). In these equations

$$\mu = 1/T, \qquad z = (2B/T)^{\frac{1}{2}},$$
 (20)

.....





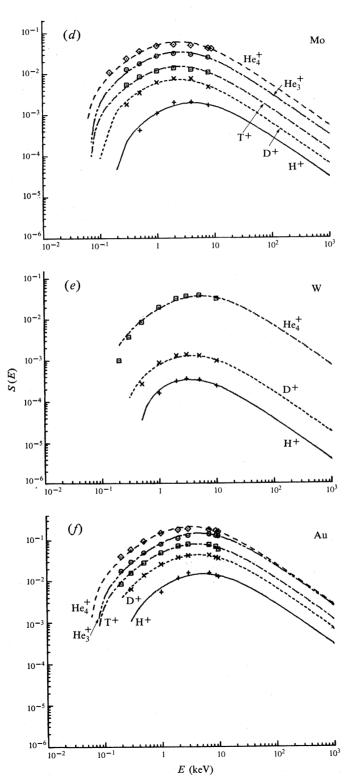


Fig. 1. Calculated curves and experimental points showing sputtering coefficients as a function of energy for the sputtering of incident  $H^+$ ,  $D^+$ ,  $T^+$ ,  $He_3^+$  and  $He_4^+$ ion beams from various elements. and  $D_{\nu}(z)$  is the parabolic cylinder function. This function is tabulated by Abramowitz and Stegun (1965) for  $0 \le z \le 5$ , and can be computed from the recurrence relation of Erdélyi *et al.* (1953)

$$D_{\nu-1}(z) = \nu^{-1} \{ z D_{\nu}(z) - D_{\nu+1}(z) \}.$$
(21)

For the first two cases

$$D_{-1}(z) = (\frac{1}{2}\pi)^{\frac{1}{2}} \exp(\frac{1}{4}z^2) \operatorname{erfc}(\sqrt{\frac{1}{2}}z), \qquad (22)$$

$$D_{-2}(z) = z D_{-1}(z) - \exp(\frac{1}{4}z^2), \qquad (23)$$

where

$$\operatorname{erfc}(x) = 2\pi^{-\frac{1}{2}} \int_{x}^{\infty} \exp(-t^{2}) dt.$$

For very low temperatures, the asymptotic series

$$\exp(\frac{1}{4}z^2) D_{\nu}(z) = z^{\nu} \left( 1 - \frac{\nu(\nu-1)}{2z^2} + \frac{\nu(\nu-1)(\nu-2)(\nu-3)}{2!(2z^2)^2} \right)$$
(24)

can be used, and higher terms can be found in equation (21).

The second integral in equation (16) can be approximated very well by expanding both the exponential term and the denominator into Taylor series and integrating term-by-term. From this we get

$$I_{2} \approx (A/B^{2}) \sum_{n=0}^{\infty} \{(-1)^{n}/n!\} E_{T}^{5/2} (E_{T}/T)^{n} [\{1 + (2E_{T}/B)\}(n + \frac{5}{2})^{-1} - (n + \frac{3}{2})^{-1} - (2E_{T}/B)(n + \frac{7}{2})^{-1}], \quad (25)$$

with a maximum error of about 1% close to threshold.

### 5. Discussion

It will be noticed that in Fig. 1 the first few experimental points sometimes fall below the calculated threshold, in the low temperature region; various effects can account for this. The most probable explanation is that the thermal spread of the local bonding in the region of the incident ion impact smears out the values of the sublimation energy, so that sub-threshold sputtering becomes possible. Unfortunately, none of the data reported by Thomas *et al.* (1979) contain point-by-point errors, hence we could not perform a systematic error analysis on the data fits to equation (1).

McCracken and Stott (1979) found an appreciable upward shift in their Maxwellian average maximum temperature. This is because the Bohdansky law predicts a continuous rise in the spectrum, and the contribution from the high temperature region is much too large. We found a much smaller upward shift.

A full library of Maxwellian averages for all of the elements analysed by Thomas *et al.* is available at the AAEC. It is proposed that this information be included in the ADL-1 atomic data library for computations of plasma behaviour in conceptual fusion reactor design studies. A zero-dimensional code, SCORCH, has been written to enable benchmark calculations to be carried out as a check on the data from the plasma devices now in use.

#### References

Abramowitz, M., and Stegun, I. A. (1965). 'Handbook of Mathematical Functions', p. 702 (Dover: New York).

Ashby, D. E. T. F., and Hughes, M. H. (1981). Nucl. Fusion 21, 911.

Clancy, B. E., Cook, J. L., and Rose, E. K. (1981). ADL-1: An atomic data library for the use in computing the behaviour of plasma devices including fusion reactors. AAEC Rep. No. E515.

Erdélyi, A., Magnus, W., Oberhettinger, F., and Tricomi, F. G. (1953). 'Higher Transcendental Functions', Vol. II, p. 122 (McGraw-Hill: New York).

Gradshteyn, I. S., and Ryzhik, I. M. (1965). 'Tables of Integrals, Series and Products', p. 319 (Academic: New York).

Hershman, S. P., and Sigmar, D. J. (1981). Nucl. Fusion 21, 1079.

Kenknight, C. E., and Wehner, G. K. (1964). J. Appl. Phys. 35, 322.

McCracken, G. M., and Stott, P. E. (1979). Nucl. Fusion 19, 889.

Post, D. E., Jensen, R. V., Tarter, C. B., Grasberger, W. H., and Lokke, W. A. (1977). At. Data Nucl. Data Tables 20, 5.

Smithells, C. J. (1976). 'Metals Reference Book' (Butterworths: London).

Thomas, E. W., Hawthorne, S. W., Meyer, F. W., and Farmer, B. J. (1979). Atomic data for controlled fusion research. ORNL Rep. No. 5207/Rl.

Manuscript received 30 March, accepted 18 August 1982

