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Elastic Scattering of Heavy Ions and the Compressibility of Nuclear Matter

K. A. Gridnev,[A] *E. F. Hefter,*[B] *K. Mikulaš,*[A] *V. M. Semjonov*[A] *and V. B. Subbotin*[A]

[A] NIIF, Leningrad State University, Leningrad B-164, USSR.
[B] Institut für Theoretische Physik, Universität Hannover,
Applestrasse 2, D-3000 Hannover 1, Federal Republic of Germany.

Abstract

Analogies to hydrodynamics led recently to the derivation of a nonlinear Schrödinger equation for the elastic scattering of heavy ions (Delion *et al.* 1978). In the present paper, the numerical solution of this equation is shown to yield realistic values for the nuclear compressibility modulus $K = 9C$, with very small uncertainties.

1. Introduction

Recently, analogies to hydrodynamics have been shown to yield a nonlinear Schrödinger equation (NOSE) describing the elastic scattering between heavy ions (Delion *et al.* 1978). In preliminary applications the nonlinearity has been successfully approximated by phenomenological repulsive potentials (Delion *et al.* 1978; Gridnev *et al.* 1978). In another study (Delion *et al.* 1980) the compression of nuclear matter in the region of contact between the colliding ions has been discussed with the aid of simplified trapezoidal form factors for the nuclear charge densities involved.

In each of these studies, the compressibility modulus of (finite) nuclear matter $K = 9C$ was the only adjustable parameter (apart from the rather *a priori* choices for the phenomenological form factors of the matter/potential distributions and for the repulsive potentials used so far). The values for K arrived at range from 170 to 230 MeV, i.e. they are fully in line with results in the literature based on more conventional approaches (cf. e.g. Blaizot *et al.* 1976; Bohigas *et al.* 1979). The point to note is that, within the model used (see Sections 2 and 3), there is very little if any freedom in the determination of this fundamental quantity.

The aim of this paper is to present the first results obtained upon solving the NOSE numerically. In doing so, attention is drawn to the unexpected feature that, even with an incomplete knowledge of the required experimental data (i.e. the angular distributions for the differential cross sections $d\sigma/d\Omega$ of elastically scattered heavy ions), it is possible to determine $K = 9C$ with surprisingly small uncertainties.

In Sections 2 and 3 we present the basic philosophy and the relevant formulae, respectively, associated with the derivation of the NOSE. Sections 4 and 5 give a description of the numerical procedure and a discussion of the results for a single test case. A short summary follows in Section 6.

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2. General Philosophy

The physical picture giving rise to the NOSE is a rather simple one. Previous studies of quasi-molecular states and of anomalous large angle scattering of α particles by light and intermediate (α-cluster) nuclei showed (see e.g. Darwisch *et al.* 1977, and references therein) that the rise in the elastic differential cross section $d\sigma/d\Omega$ for backward angles, together with the measured resonances E_x of the di-nuclear systems and their decay widths Γ_x, may be described by (the same parameters of) a crude 'effective surface potential' V_{ESP}, arrived at by supplementing the optical model interaction V_{OM} by a repulsive (hard or soft) core potential V_r:

$$V_{ESP} \approx V_{OM} + V_r . \tag{1}$$

(This potential has sometimes been chosen to be l-dependent.) Here V_{OM} facilitates the correct evaluation of $d\sigma/d\Omega$ for forward angles, while V_r gives the additional potential contribution which allows for an appropriate description of the measured E_x and Γ_x, together with the backward rise in $d\sigma/d\Omega$ observed for a scattering angle $\theta_{cm} > 90°$. Quasi-molecular features similar to those seen in these α-scattering experiments have also been observed in elastic collisions between heavy ions (cf. e.g. Cindro 1978; Barrette and Kahana 1980).

By knowing that semi-classical and in particular hydrodynamical approaches work nicely when applied to heavy-ion physics, it seems appropriate to model the elastic scattering of two heavy ions in terms of colliding liquid drops with diffuse surfaces. By assuming that the 'membranes' or surfaces of the two drops remain impenetrable, one thus obtains automatically a compression of the densities in the surface regions of the two touching drops. The physical motivation for this boundary condition is then that this compression, giving rise to a repulsive spring-like force proportional to V_r, vanishes for separations

$$r > r_0 \approx R_1 + R_2 + \tfrac{1}{2}(a_1 + a_2) \tag{2}$$

between the two ions or drops, i.e. $V_r \to 0$ for $r > r_0$. Here R_i and a_i denote the radius and diffuseness of the ith ion ($i = 1, 2$). Thus the dynamical interaction between the two nuclei is characterized by their unperturbed or uncompressed asymptotic densities $\rho_{0i}(R_i, a_i)$ (for $r > r_0$), corresponding to the measured (charge) densities, and by the perturbed or compressed densities $\rho_i(r; R_i, a_i)$ (for $r < r_0$).

3. Nonlinear Schrödinger Equation

The central feature emerging from the considerations of Section 2, together with the use of the Euler equation and some algebra in connection with the neglect of higher order derivatives, is the NOSE (Delion *et al.* 1978):

$$-(\hbar^2/2m)\nabla^2\Psi + V\Psi - C(1-\rho/\rho_0)\Psi = E\Psi; \quad \rho = |\Psi|^2, \; \rho_0 = \rho_{01} + \rho_{02}; \tag{3a, b, c}$$

where we use the conventional notation with $C = \frac{1}{9}K$ for the nuclear compressibility. Here ρ and ρ_0 denote the perturbed and unperturbed densities respectively. To disentangle the different contributions to Ψ, we write it as the product of the total unperturbed internal (bound-state) wavefunction Φ and the 'modulating' (continuous) wavefunction χ, with

$$\rho = |\chi|^2 + |\Phi|^2 \approx \Delta\rho(r) + \rho_0, \qquad \rho_0 \approx \rho_{01} + \rho_{02} = |\Phi|^2 . \tag{4a, b}$$

The modulating wavefunction contains the entire dynamics of the system. Equations (4) after some algebra [and the neglect of a term proportional to $(\nabla\Phi)(\nabla\chi)/\Phi$] allow us to reduce equation (3a) to the simpler form

$$-(\hbar^2/2m)\nabla^2\chi + V_\chi \chi + C|\chi|^2\chi = E_\chi \chi, \tag{5a}$$

where the quantities V_χ and E_χ are introduced to absorb the additional terms which arise. If we bear in mind that fluxes such as that transferred from the elastic to the inelastic channels (due to inelastic collisions) have so far been ignored, it is quite obvious that realistic results can only be expected if we replace the linear parts of the interaction in equation (3a), i.e. the V_χ of equation (5a), by an effective (phenomenological and complex) potential. Hence, we obtain for equation (5a)

$$-(\hbar^2/2m)\nabla^2\chi + V_{\mathrm{OM}} \chi + C|\chi|^2\chi = E\chi, \tag{5b}$$

where the optical model interaction V_{OM} is to contain as usual the Coulomb V_{Cl} and nuclear $V + \mathrm{i}\,W + V_{\mathrm{so}}$ interactions. For $C = 0$ the quantity χ is simply the optical model wavefunction.

4. Numerical Procedure

The numerical solution of the NOSE follows the traditional procedure of decomposing χ in terms of the Legendre polynomials $\mathrm{P}_l(\cos\theta)$:

$$\begin{aligned} C|\chi|^2 &= C\sum_{m,n} \frac{u_m(r)\,u_n^*(r)}{r^2}\mathrm{P}_m(\cos\theta)\,\mathrm{P}_n(\cos\theta) \\ &= C\sum_L \Big(\sum_{m,n} \frac{u_m u_n^*}{r^2}(C^{L0}_{m0,n0})^2\Big)\mathrm{P}_L(\cos\theta), \end{aligned} \tag{6}$$

leading to a system of coupled (radial nonlinear) equations (that allow us at least in principle to also take into account intermediate virtual quasi-molecular states). At present however we neglect the contributions containing the cross terms to arrive at, for the lth partial wave,

$$C|\chi_l|^2 = C(2l+1)\sum_{L,m} \frac{|u_m|^2}{r^2}(C^{L0}_{l0,l0}\,C^{L0}_{m0,m0})^2/(2L+1) \tag{7}$$

for the nonlinearity of equations (5). (Further details and technicalities are to be discussed in a forthcoming paper.) The $C^{L0}_{l0,l0}$ and u_m in equation (7) denote the Clebsch–Gordan coefficients and radial wavefunctions respectively. Equation (5b) together with the interaction (7) is now solved by the following iterative procedure.

First we use $C = 0$ to solve the usual optical model equation (with the wavefunctions normalized for large r to their Coulomb solutions) and obtain $\chi^{(0)}$. This result is then inserted into the nonlinear term of the equation

$$-(\hbar^2/2m)\nabla^2\chi^{(n)} + V_{\mathrm{OM}}\chi^{(n)} + C|\chi^{(n-1)}|^2\chi^{(n)} = E\chi^{(n)}; \quad n = 1, 2, \ldots, \tag{8}$$

to evaluate $\chi^{(1)}$, i.e. for $n = 1$. The procedure is repeated up to those $\chi^{(n-1)}$ and $\chi^{(n)}$ for which the differences in the phase shifts and hence also the differences in the cross sections are negligible. The convergence turns out to be rapid, so that there are no problems as far as the numerical aspects are concerned.

5. Discussion of Numerical Results

In the discussion of our results we examine the elastic scattering of ^{9}Be on ^{16}O (at $E_{\mathrm{Be}} = 27$ MeV). Here we take exactly the same optical model parameters as used before in a phenomenological discussion of V_r, so that the only possible difference in the results would arise from the more exact treatment of the nonlinearity. Similarly, as in all previous (phenomenological) calculations of this type, the optical model parameters (with no adjustments) give a suitable description of $d\sigma/d\Omega$ for forward angles. The nonlinear term provides the repulsion needed to account for the anomalous rise in $d\sigma/d\Omega$ observed for large angles (and hopefully also for the resonances, a point we are not as yet able to pursue further). The only adjustable parameter we are left with is the compressibility modulus $K = 9C$.

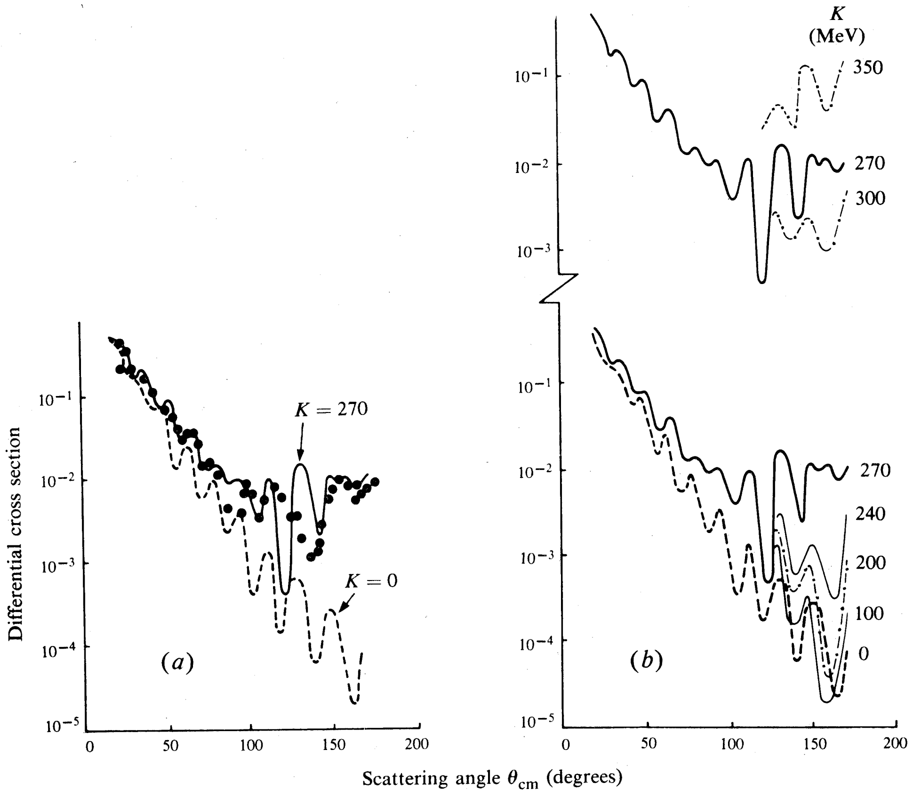

Fig. 1. Experimental and theoretical differential cross sections for elastic scattering of ^{9}Be ions ($E = 27$ MeV) incident on ^{16}O. In (*a*) the experimental points are as measured at the Kurchatov Institute, the calculated curves corresponding to $K = 0$ and 270 MeV. The lower part of (*b*) shows the steady increase of $d\sigma/d\Omega$ for $\theta_{\mathrm{cm}} \gtrsim 150°$ as K increases from 0 to 270 MeV. The upper part illustrates the abrupt changes in $d\sigma/d\Omega$ for $K > 270$ MeV. The optical model parameters used are $V = 26$ MeV, $W = 18$ MeV, $R_V = R_W = R_C = 1\cdot2$ fm and $a_V = a_W = 0\cdot7$ fm (Gridnev *et al.* 1978).

Following the procedure of Section 4 we evaluate $\chi^{(0)} = \chi_{\mathrm{OM}}$, corresponding to the angular distribution in Fig. 1 (dashed curve) for $K = 0$. If we increase K from 0 to 270 MeV, the magnitude of $d\sigma/d\Omega$ for $\theta_{\mathrm{cm}} \gtrsim 150°$ rises steadily. The accompanying changes in the shape of $d\sigma/d\Omega$ are smaller but by no means negligible. As may be verified by the curves given in the lower part of Fig. 1*b*, the changes in $d\sigma/d\Omega$ go in the desired direction and improve the agreement between theory and experiment for large angles (see Fig. 1*a*). However, as shown in the upper part of Fig. 1*b*, a further increase in K from 270 to 300 MeV is seen to lead to a sudden *reduction* in $d\sigma(\theta_{cm} \gtrsim 150°)/d\Omega$! But the angular distribution based on the still larger value of $K = 350$ MeV displays again a rise in $d\sigma/d\Omega$ compared with the cases for $K = 270$ and 300 MeV. In such a way the monotonic rise of $d\sigma(\gtrsim 150°)/d\Omega$ as a function of K changes to an oscillatory or iterative regime for K greater than the critical value $K_{cr} = 270$ MeV.

With a linear equation we would naturally expect a monotonic dependence of $d\sigma/d\Omega$ on K, but the nonlinearity (coupling degrees of freedom related to the internal and relative motions with each other) leads to a drastic qualitative change in $d\sigma(K)/d\Omega$ for different regions of the variable $K = 9C$. From our experience with nonlinear equations, however, these 'jumps' are not unusual; for example, a change of sign in the nonlinearity of the one-dimensional analogue to equation (5b) (or a gradual transition of C from a positive to a negative value) modifies its stationary wave solutions from a sech form to a tanh form. Hence, the peculiar behaviour of $d\sigma(K)/d\Omega$ obviously has to be interpreted in terms of the nonlinearity of the system; a rather general comment which, unfortunately, reflects the fact that we have not yet managed to develop a satisfactory understanding in terms of the underlying physics. (Suggestions would be appreciated.)

As far as the extraction of the compressibility modulus is concerned, the rather unexpected implications of this extraordinary behaviour are that a distinct meaning has to be assigned to K_{cr}. Even in the absence of any experimental data for backward angles and without any knowledge of the physics involved, one feels tempted to speculate that K_{cr} should be approximately equal to the best-fit value K_{bf}. Indeed, in our test case we have $K_{cr} = K_{bf}$ and the improvement in the angular distribution is not just restricted to backward angles but holds also for the forward hemisphere. (Further preliminary results for other cases are at least consistent with $K_{cr} \approx K_{bf}$.) However, as long as the peculiar behaviour of $d\sigma(K)/d\Omega$ and the role of K_{cr} are not yet fully understood, it is obviously K_{bf} which is the significant quantity.

In our test case the present result of $K(\mathrm{Be+O}) = 270$ MeV automatically removes the only real discrepancy noted in previous calculations, with phenomenological substitutes for the nonlinearity, which led to the value of $K = 90$ MeV (cf. Gridnev *et al.* 1978). Apparently the self-consistent treatment here eliminates some unphysical features involved in the *ad hoc* choice of the phenomenological repulsion previously used.

The crux of the method is that $K = K_{bf}$ is determined with rather small uncertainties: at present the confidence limits for the error bars are $+10\%$ and -15% (to be discussed in more detail in a forthcoming paper containing a larger body of data). The primary source of error in K_{bf} is believed to be due to the use of phenomenological optical model parameters that have no fixed relationship with the measured (charge) densities of the nuclei involved, unlike the one established within the microscopic folding model. More accurate information on these potential parameters is expected to reduce the error bars by about 50%. Yet, the uncertainties arising are already a lot smaller than those inherent in traditional approaches based on random phase approximation or Hartree–Fock calculations (with a rather ambiguous input in terms of phenomenological effective nucleon–nucleon interactions) or in approaches relying on rather problematic background separations and interpretations of experimentally measured excitation functions (understood as evidence for monopole resonances), as given for example by Blaizot *et al.* (1976), Bohigas *et al.* (1979) and Marty *et al.* (1979, and references therein). On the other hand, the study of the monopole resonance by forward angle inelastic α scattering is by now a well-established method for obtaining the nuclear compressibility, and one which yields reasonably well-defined error bars for the resulting compressibility modulus (cf. the more recent contributions by Youngblood *et al.* 1981, and references therein).

6. Summary

We have reviewed the physical picture leading to the derivation of a nonlinear Schrödinger equation as applied to the elastic interaction of heavy ions. By using phenomenological optical model parameters taken from the literature for the linear part of the potential, the compressibility modulus $K = 9C$ remains the only adjustable parameter.

It is observed that by increasing K steadily from zero, the magnitude of $\mathrm{d}\sigma(\theta_{\mathrm{cm}} \gtrsim 150°)/\mathrm{d}\Omega$ rises monotonically until a critical value is reached (which in our test case has the value of 270 MeV); for $K > K_{\mathrm{cr}}$ further increases in K induce oscillatory fluctuations of $\mathrm{d}\sigma(\theta_{\mathrm{cm}} \gtrsim 150°)/\mathrm{d}\Omega$ around its value for K_{cr}. Since $K \approx K_{\mathrm{cr}}$ characterizes theoretical curves for $\mathrm{d}\sigma/\mathrm{d}\Omega$ that display a nice (best-fit) correspondence to the experimental data over the whole angular range, we speculate that a definite meaning should be attributed to K_{cr}. Yet, in the absence of more convincing arguments for this idea, the value of K_{bf} is the significant quantity to be considered.

The peculiar features of $\mathrm{d}\sigma/\mathrm{d}\Omega$ as a function of K together with the experimental data impose severe restrictions on the possible values of the compressibility modulus arising from such an analysis. The accuracy of the resulting K (with estimated uncertainties of $+10\%$ and -15%) could presumably be improved by using potentials that have a more direct relation to the measured (charge) distributions of the nuclei involved. But a final assessment of the value of this approach, its generality and its limitations will naturally have to await the results of more extensive calculations.

Because of the simplicity of this approach (the unconventional nonlinearity may readily be implemented in existing optical model codes), the results presented here are expected to be of practical value for other groups interested in experimental and theoretical studies of anomalous large angle scattering and the associated quasi-molecular structures.

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References

Barrette, J., and Kahana, S. (1980). *Comments Nucl. Part. Phys.* **9**, 67.
Blaizot, J. P., Gogny, D., and Grammaticos, B. (1976). *Nucl. Phys.* A **265**, 315.
Bohigas, O., Lane, A. M., and Martorell, J. (1979). *Phys. Rep.* **51**, 268.

Cindro, N. (Ed.) (1978). 'Nuclear Molecular Phenomena' (North-Holland: Amsterdam).

Darwisch, N. Z., Gridnev, K. A., Hefter, E. F., and Semjonov, V. M. (1977). *Nuovo Cimento* A **42,** 303.

Delion, D. S., Gridnev, K. A., Hefter, E. F., and Semjonov, V. M. (1978). *J. Phys.* G **4,** 125.

Delion, D. S., Gridnev, K. A., Hefter, E. F., and Semjonov, V. M. (1980). *Z. Phys.* A **297**, 115.

Gridnev, K. A., *et al.* (1978). *Izv. Akad. Nauk SSSR* **42,** 2361.

Marty, N., Willis, A., Morlet, M., Frascaria, R., Comparat, V., and Kitching, P. (1979). Proc. Fourth Balaton Conf. on Nuclear Physics, Keszthely, Hungary, p. 92 (Central Research Inst. for Physics: Budapest).

Youngblood, D. H., Bogucki, P., Bronson, J. D., Garg, U., Lui, Y.-W., and Rozsa, C. M. (1981). *Phys. Rev.* C **23**, 1997.

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