

The Quark Presence in Nuclei*

G. E. Brown

Department of Physics, State University of New York at Stony Brook,
Stony Brook, NY 11794, U.S.A.

Abstract

The strong short-range repulsion seen in the nucleon–nucleon interaction is, in the classical solution discussed here, produced by the loss of mean-field pion energy when two three-quark systems are pushed on top of each other. The intermediate- and long-range interactions come from boson exchange.

1. Introduction

In 1950, when I went to Birmingham, Stuart Butler was just completing his thesis on the theory of deuteron stripping. The shell model, as developed by Maria Goeppert-Mayer and J. H. D. Jensen, was still young and nobody suspected that the motion of nucleons within the nucleus was as simple as it turned out to be. Butler's work showed that, in going from one nucleus to the next, particles were dropped into orbits with definite angular momentum l . How the deuteron got to the edge of the nucleus and how the neutron got out—the question of nuclear distortion—was a somewhat complicated matter, and it took some years to develop distorted wave codes; also, computers were highly inadequate in 1950. But Stuart Butler's work was trail breaking in pinning down what was actually going on in the nucleus.

Three decades later we are at roughly the same position with the nucleon. High-energy deep inelastic scattering has shown that the three quarks inside the nucleon behave as free particles when struck with a high momentum transfer. The problem is different from the one discussed above, in that these quarks cannot be singly added to or removed from the nucleon. This has disadvantages in that one cannot take the quarks out and 'look at them'. On the other hand, it has advantages in that the three-body problem—the spectrum of the nucleon—is much simpler than the three-body problem in nuclear physics because there is no continuum; the quarks cannot go off to infinity because they are confined. Thus, the spectrum of low-lying excited states is a very simple one, with just the quantum numbers one would expect (Brown and Rho 1983).

The size of the nucleon can be pinned down from its excitation spectrum (Brown and Rho 1983). Perhaps a more direct and unambiguous determination of nucleon size is that made from pion photoproduction in the quark model. Copley *et al.* (1969)

* Dedicated to the memory of Professor S. T. Butler who died on 15 May 1982.

found that the absence of backward photoproduction of pions through the $D_{13}(1520)$ excited state of the nucleon could be explained by an interference of E1 and M2 contributions. Since these multipoles have different dependences upon radius, such a cancellation determines the radius parameter of the nucleon. For the harmonic oscillator model used by these authors, the oscillator parameter turned out to be

$$a = 0.48 \text{ fm}, \quad (1)$$

and this implies, after removing spurious centre of mass effects, an r.m.s. radius for the nucleon of the same value, since in the oscillator $\langle r^2 \rangle^{\frac{1}{2}} = a$.

Thus, the radius of the nucleon is ~ 0.5 fm; naively, one would expect to begin to see quark effects when the internucleon distance is less than 1 fm. This is not so much less than the average distance between nucleons in the nucleus of

$$r_0 \equiv 1.2 \text{ fm}. \quad (2)$$

The questions are: Why have we not seen effects from the quark substructure of nucleons in nuclear physics? Have we seen such effects, but called them by other names?

I shall venture an answer here in terms of pion cloud effects. First of all, these clouds give mechanisms for boson-exchange models, in that the boson exchange between two nucleons is simply the interference of one cloud with the other, as they begin overlapping. Secondly, classical mean-field solutions of the three- or six-quark systems surrounded by pion clouds suggest that the two three-quark systems do not fuse into a six-quark system until brought very close together, i.e. $r_{12} \lesssim 0.4$ fm, where r_{12} is the distance between the two three-quark clusters. At larger distances, each three-quark cluster tends to retain its shape and identity, in order to benefit from large attractive energies produced by coupling of the pion cloud to the three quarks. The coupling of the pion cloud to the six-quark system is much less favourable energetically, so this delays, until very short distances, the fusing of the two clusters.

Since the surface region of each nucleon acts as a source for pions, as long as each nucleon more or less retains its identity, the boson-exchange interaction between nucleons will persist. At very short distances, when the two clusters do indeed fuse, the interaction must be cut-off or 'regularized' in a way we shall discuss.

2. Development of Model

I wish to outline here a model of the nucleon-nucleon interaction, especially the short-range part, which has been developed together with Mannque Rho and Vincent Vento.

In the interior of the quark confinement region, chiral symmetry is realized in the Wigner-Weyl mode; at a certain radius, chiral symmetry is broken, and exterior to this, it is realized in the Goldstone mode. Across this interface, the operator equation

$$\partial J_\mu^5 / \partial x_\mu = 0 \quad (3)$$

must hold; here J_μ^5 is the axial vector current. Equation (3) is strictly true in the underlying Yang-Mills theory, and would remain true if the quark masses were zero. In fact, one knows that the up and down quark masses are not zero, but a few MeV

in magnitude. These small masses are supposedly relics of interactions at very high energies, and are irrelevant from the point of view of strong-interaction physics.

Since the quarks are confined, they cannot carry the axial current in the exterior region. In fact, the pion can do so, and the pionic coupling to the surface region of the nucleon, more precisely the interface between different modes of chiral symmetry discussed above, can be determined from the condition that the axial current is to be continuous across this interface (Chodos and Thorn 1975; Callan *et al.* 1978; Brown and Rho 1979; Brown *et al.* 1979). This re-establishes the asymptotic pion field traditionally associated with Yukawa theory; the pion coupling is the usual pseudovector one, with coupling constant

$$f^2/4\pi = 0.08. \quad (4)$$

Within simplified models, such as the MIT bag model (Chodos *et al.* 1974; DeGrand *et al.* 1975), one can then write an energy functional for the system involving three quarks in the interior, coupled to an exterior (nonlinear) pion cloud in such a way that equation (3) holds. The resulting equations for the inside and outside regions and for the surface are as follows:

$$\text{Inside: } \gamma \cdot \partial \psi = 0, \quad \text{Outside: } \mathcal{D}_\mu^2 \phi = 0. \quad (5a, b)$$

On the surface:

$$\bar{\psi} \{1 + (i \boldsymbol{\tau} \cdot \boldsymbol{\phi}/f_\pi) \gamma_5\} \psi = 0, \quad f_\pi \mathbf{n} \cdot \mathcal{D} \phi = i \bar{\psi} \gamma_5 \mathbf{n} \cdot \boldsymbol{\gamma} \frac{1}{2} \boldsymbol{\tau} \psi, \quad (5c, d)$$

$$B = -\frac{1}{2} (1 + \phi^2/f_\pi^2)^{-\frac{1}{2}} \mathbf{n} \cdot \partial \{ \bar{\psi} (1 + i \boldsymbol{\tau} \cdot \boldsymbol{\phi} \gamma_5 / f_\pi) \psi \} - \mathcal{L}_\phi. \quad (5e)$$

Here ψ is the quark field (massless u and d quarks), ϕ the nonlinear pion field, $\boldsymbol{\tau}$ the isospin Pauli matrix, f_π the pion decay constant, B the bag constant, \mathcal{D}_μ the nonlinear derivative $(1 + \phi^2/f_\pi^2)^{-1} \partial_\mu$ and \mathcal{L}_ϕ the nonlinear chiral Lagrangian. Equation (5c) is the linear boundary condition that imposes confinement, (5d) the axial-current continuity condition and (5e) gives the condition for pressure balance on the surface. In the limit $f_\pi^{-1} \rightarrow 0$ with the radius of the bag fixed at 'large' R , the pions decouple from the bag and the equations go over to those of the MIT bag.

Because of the nonlinearities, these equations are not easy to solve, even classically. It has, however, been possible to obtain spherical solutions by introducing a new quantum number K :

$$K = J + I = L + S + I, \quad (6)$$

with I the isospin of the quark. For the three-quark system, the lowest energy state is obtained by filling up the even-parity $K = 0$ ($L = 0, J = \frac{1}{2}$) shell by three quarks of different colour; this solution has been discussed by Vento *et al.* (1980).

The solution is referred to as a 'hedgehog' solution because the spin and isospin vectors $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$ point in the radial direction \hat{r} . It can be shown to be the only spherically symmetric solution of the classical equations. The physical nucleon must be deformed, since the pion couples chiefly at the poles and exerts more pressure there than at the equator, so there is the question of the relation of the hedgehog solution to the actual nucleon. Roughly speaking, I believe the analogy in nuclear physics to be the relation between spherical and deformed nuclei. We know that almost all nuclei are deformed; yet we get a good first orientation by looking at spherical nuclei. Energies gained by

deformation are not large, usually only a few MeV out of several hundred. This is because once one has minimized the energy of a system, one can make quite large changes in the structure without changing this energy very much. These are, of course, qualitative comments, and must be made more firm before they are believable. Also, the role of topology is likely to be more important in the solution for the nucleon.

I now wish to discuss, within the framework of the hedgehog solutions, what happens when two bags merge. We follow the unpublished work of V. Vento, M. Rho and J. F. Logeais; a brief report of this work was given by Vento *et al.* (1981).

Although the pion coupling constant (4) may not appear to be large, the self-energy from the mean-field pion coupling in the case of the three-quark hedgehog solutions contains a numerical factor of 27 from the expectation value of

$$\sum_{i,j} \sigma_i \cdot \sigma_j \tau_i \cdot \tau_j; \quad (7)$$

furthermore, the self-energy goes as $(f_\pi^2 R^3)^{-1}$, where f_π is the pion weak decay constant with the experimental value of 93 MeV. Thus, for $R \ll f_\pi^{-1}$, the energy from pionic coupling becomes large; indeed, the lowest order perturbation result goes as

$$\delta E_\pi = -12m_n c^2 / (Rm_n c/\hbar)^3 \quad (8)$$

(where m_n is the nucleon mass), although this is cut down somewhat by nonlinear terms. The chief point is that the pion self-energy is just as large as quark kinetic energies for a small bag radius R . This means that the pion coupling will 'warp' the quark orbits. Indeed, only three quarks can go into the lowest $K = 0$ orbit, in contrast to the perturbative situation where twelve quarks can go into the $1s_{\frac{1}{2}}$ state. This may seem strange, but remember that in the perturbative case, the twelve quarks form a closed shell, so that the pion coupling, which goes through the axial-vector operator $A = \sum_i \sigma_i \tau_i$, is zero, and the twelve-quark system will thus have a zero mean pion field. In the hedgehog solution, the pion coupling goes linearly to zero, as one goes from three to twelve quarks. Thus, the six-quark bag has two-thirds of the coupling of the three-quark one; i.e. in terms of the expectation values of the axial-vector operator

$$(g_A)_{6q} / (g_A)_{3q} = \frac{2}{3}, \quad (9)$$

and the energy of the six-quark bag will be lowered only $(\frac{2}{3})^2$ as much as that of the three-quark bag from the mean-field pion coupling. Thus, when two three-quark bags are pushed on top of each other, the loss of pion self-energy is

$$\Delta E_\pi = (2 - \frac{4}{9}) 12m_n c^2 / (Rm_n c/\hbar)^3, \quad (10)$$

where the radii of the three- and six-quark bags have been taken to be equal; this will be of the order of GeV for a small bag radius R . This is the mechanism adduced by Vento *et al.* (1981) for the short-range repulsion in the nucleon-nucleon interaction.

In obtaining equation (8), the weak interaction parameter f_π occurring in the pion coupling has been re-expressed in terms of the strong interaction f by use of the Goldberger-Treiman relation

$$f_\pi^{-1} = 2f/g_A m_\pi. \quad (11)$$

There are some uncertainties in the magnitude of δE_π , not only because it is not clear that the hedgehog solution is directly relevant, but also because the value of g_A in

the hedgehog solution is too large. We therefore will now make the same picture of short-range repulsion in terms of a phenomenology, where the parameters can be set from the nucleon mass spectrum. In order to do this, we must first sketch a treatment of zero-point energy in the bag motion.

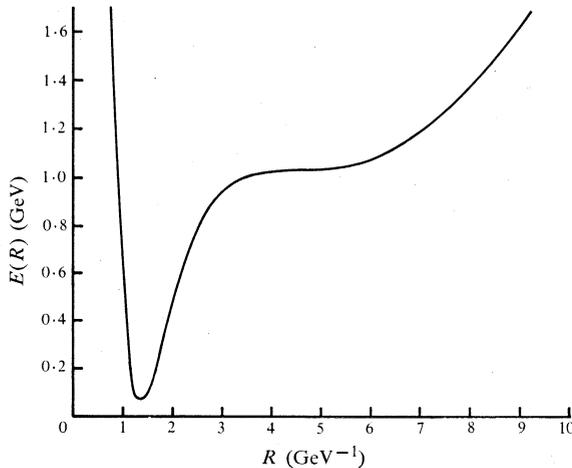


Fig. 1. Bag energy as a function of the bag radius R .

3. Zero-point Bag Motion

Following Brown *et al.* (1983), we shall take the energy functional of the chiral bag to be

$$E(R) = \frac{3\Omega_0}{R} + \frac{4}{3}\pi R^3 B - \frac{Z_0}{R} - \frac{1.71}{m_n^2} \frac{g_{\pi NN}^2}{4\pi} \frac{1}{R^3 + R_0^3}. \quad (12)$$

Here the first term on the right is the quark kinetic energy with $\Omega_0 = 2.04$ (Chodos *et al.* 1974; DeGrand *et al.* 1975). We shall take the bag constant to be $B^\ddagger = 0.137$ GeV. The term Z_0/R corrects, in a rough way, for spurious centre of mass energy, with $Z_0 = 0.75$. The last term comes from pion coupling. Setting $R_0 \approx R_\pi$ gives a cut-off which we determine phenomenologically in order to obtain the correct energy for the ground state of the nucleon. With $g_{\pi NN}^2/4\pi = 14.6$ and $R_0 = 0.33$ fm, we obtain the curve shown in Fig. 1.

The pionic coupling, the last term in equation (12), is that of (8) with the numerical factor of 27 (see equation 7) replaced by 57, the value appropriate for the nucleon (Jaffe 1979) when both nucleon and Δ intermediate states are included in the mean-field coupling. If, in the spirit of Vento *et al.* (1981), only the nucleon intermediate states are included, the factor would be 25, not far from that of the hedgehog, and R_0 would have to be decreased by a factor of $\sim 2^{\frac{1}{3}}$ to get equivalent fits to the nucleon mass.

The curve in Fig. 1 was viewed by Brown *et al.* (1983) as a collective potential $U(R) = E(R)$, and the wavefunction $\psi(R)$, the probability amplitude for the nucleon having bag radius R , is calculated from

$$-\frac{1}{2M_{\text{eff}}} \frac{d^2\psi(R)}{dR^2} + U(R)\psi(R) = \epsilon\psi(R). \quad (13)$$

The parameter R_0 is set in order to position the lowest eigenvalue ε_0 at 1.04 GeV, 100 MeV above the ground state of the nucleon, because gluon-exchange interactions between the quarks will lower this ε_0 to the empirical value. The M_{eff} is taken to be the mass that would follow from a collective scaling motion of breathing type:

$$M_{\text{eff}} = 0.48 m_n. \quad (14)$$

The resulting $\psi(R)$ is given by Brown *et al.* (1983); the r.m.s. bag radius is 0.7 fm, which gives an r.m.s. quark radius of 0.5 fm, in agreement with equation (1).

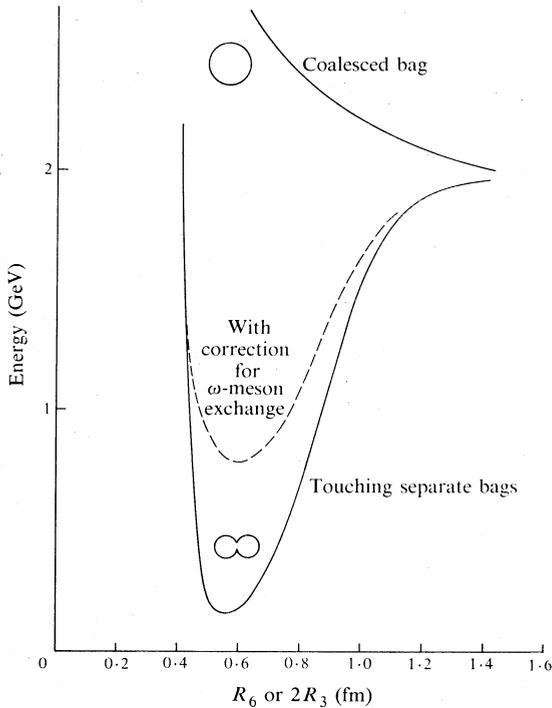


Fig. 2. Energy functionals of a six-quark bag and two three-quark bags plotted against R_6 in the former case, and $2R_3$ in the latter. A correction upwards in energy for the two three-quark bags from ω -meson exchange is shown (dashed curve).

4. Interaction Energy

Let us now discuss the energy function of the six-quark bag. For the pionic coupling, we take four-ninths of that for the three-quark bag, following the discussion of Section 2. This is a guess. The deuteron has $I = 0$, so the mean-field pion coupling would be zero, as it would be for two nucleons merging in an $I = 1, S = 0$ state. Thus, it might be better to take the pion coupling to be zero for the six-quark bag, but this will not appreciably change our argument. We have no idea of what quantum fluctuations will do; it is fashionable in QCD to obtain mean-field solutions, and then later allow for quantum fluctuations about them. It is conceivable that quantum fluctuations might restore some or all of the difference between pionic energies of the three- and six-quark states; however, we proceed on the belief that they do not restore much of this difference.

One immediately sees that the energy of the six-quark bag will be determined at short distances almost completely by the kinetic energy [of course, $3\Omega_0$ in equation (12) goes to $6\Omega_0$ for the six-quark bag]. On the other hand, the large attractive

pionic energies will strongly influence the energy of two touching three-quark bags. From the energy functionals, we can calculate the energies of two touching three-quark bags and of the six-quark bag. Fig. 2 clearly shows that until a very small bag radius, it is energetically preferable for the two touching three-quark bags to retain their identity, while each decreases in radius, rather than merge into a six-quark bag. Of course, there will be some tunnelling of quarks between the bags, but we neglect this. The final merging will occur only for $2R_3 \sim R_6 \sim 0.3\text{--}0.4$ fm.

In the case of the two three-quark bags, the surface of each can be a source of mesons and we include a correction for ω -meson exchange, where we have used an ω -meson coupling constant given by

$$g_\omega^2/4\pi = 11. \tag{15}$$

We include, then, a repulsive energy

$$E_\omega = 11 \exp(-m_\omega r_{12})/r_{12}, \tag{16}$$

where the internucleonic distance r_{12} is chosen to be equal to $2R_3$.

At the minimum in the energy functional of Fig. 1 each of the three-quark clusters has its energy lowered by $2.18 m_n c^2$ by coupling to the pions. The pionic energy of the six-quark bag, which we take to represent the situation of completely overlapping bags, has a pion energy of only $(\frac{2}{3})^2 = \frac{4}{9}$ of this:

$$(\delta E_\pi)_{6q} = -0.97 m_n c^2. \tag{17}$$

Thus the total energy lost, from the classical minimum in the curve in Fig. 1, is $3.4 m_n c^2$. Since each three-quark bag will 'breathe' about its classical minimum when the bags are well separated, we take this value of $3.4 m_n c^2$ to represent the repulsion encountered when two nucleons are shoved right on top of each other so that $r_{12} = 0$.

Boson-exchange models treat the modifications from the internal structure of the nucleons at short distances by regularizing the boson-exchange potential. Thus, a regularized ω -exchange interaction is represented by

$$(V_\omega)_{\text{reg}} = \frac{g_\omega^2}{4\pi} \left(\frac{\exp(-m_\omega r)}{r} - \frac{\exp(-\Lambda r)}{r} \right). \tag{18}$$

Taking the limit $r \rightarrow 0$, we find that

$$(g_\omega^2/4\pi)(\Lambda - m_\omega) = 3.4 m_n c^2. \tag{19}$$

This would give a regularization mass Λ in the range $1.1\text{--}1.2$ GeV, which is the general size needed in boson-exchange models.

5. Discussion

Building on the classical solutions of hedgehog type found by Vento *et al.* (1980, 1981) and on the phenomenology by Brown *et al.* (1983), we have shown that within the approximation of mean-field solutions, considerable repulsive energy is encountered in pushing two nucleons on top of each other. The situation is not unlike that of the short-range repulsions encountered in van der Waals forces. The pion couples particularly strongly to the three-quark system, reducing its energy (~ 2 GeV in

our phenomenology) and 'warping' the quark orbitals. Thus, three quarks form a closed shell in the hedgehog solution, and we believe this is likely to persist more generally, rather than twelve quarks forming a closed shell as in the situation where the pion coupling is neglected. Once one has a picture in which three quarks form a closed shell, then it is obviously energetically unfavourable to push six quarks on top of each other, since the additional three quarks must be promoted into higher orbitals.

Although our estimated energies are large, the estimate of $1.1\text{--}1.2$ GeV we find for the regularization mass Λ is reasonable.

It is interesting to consider the relation of the solution by Vento *et al.* (1980) and of the model of the short-range repulsion by Vento *et al.* (1981) to perturbative calculations such as those by Liberman (1977) and DeTar (1978, 1979a, 1979b). In these calculations a repulsion results at short distances because it is unfavourable, from the standpoint of colour magnetic energy, to push six quarks on top of each other. This repulsion is not separate from, but additional to, that discussed in Section 2. (The discussion in Section 3 aimed to put the results of Section 2 into some phenomenology, and to introduce zero-point motion.)

As indicated in the discussion by Vento *et al.* (1980), the dependence of the energy of the solution on the coupling constant goes as f_π^2 or, equivalently, as N_c , the number of quark colours. Since the pion coupling to the bag goes as f_π^{-1} (or $N_c^{-\frac{1}{2}}$) this dependence was that expected of a soliton. Recently, it has been shown (Rho *et al.* 1983) that the solution is indeed a soliton, and has a topological (winding) number, the baryon number. This baryon number is, of course, unity for the solution by Vento *et al.* (1980).

Perturbative expansions such as those by Liberman (1977) and DeTar (1978, 1979a, 1979b) miss completely the terms of order N_c , which are responsible for the topology, and those of order unity, the gluon-exchange terms going as N_c^{-1} , the same as the pionic terms. It is no surprise to us that our model estimates a short-range repulsion of ~ 3 GeV, while for the perturbative calculations it is ~ 300 MeV. The latter should be down from the former by two orders in the expansion parameter N_c^{-1} . Indeed, the two repulsions differ by a factor of $\sim N_c^2$.

Zero-point energy, which is of order unity in the expansion being discussed, was introduced in Section 3. Since we are dealing with an effect which is of order N_c higher than those of pion and gluon exchange, it should be treated. An undesirable feature in our collective potential for zero-point motion is its sensitivity to the pionic size parameter R_0 . With R_0 chosen so as to give the correct nucleon mass, the Roper resonance came out at the right energy, and this was a check. Furthermore, our value of $R_0 = 0.33$ fm is close to the pion r.m.s. radius $R_\pi = 0.41$ fm determined from fitting the pion weak decay and $\pi_0 \rightarrow \gamma + \gamma$ reaction by Brodsky *et al.* (1983) and also to the r.m.s. value of ~ 0.35 fm from current bag model phenomenology (Carlson *et al.* 1983).

There is a transition from the short-range repulsion to the boson-exchange interaction in the region $\sim 0.4\text{--}0.7$ fm. As noted earlier, since the nucleon-nucleon wavefunction is so small in this region due to the strongly repulsive interaction at shorter distances, it is unlikely that low-energy nuclear physics will be sensitive to this region. Electromagnetic exchange currents, for example, are not at all sensitive because the nucleon-nucleon interaction is chiefly governed by the exchange of neutral objects, and the wavefunction is small, anyway.

When Stuart Butler entered nuclear physics, the field was in a mess. The shell model was just being developed, and then his work on deuteron stripping and pickup showed how nuclear reactions could give us detailed spectroscopic information. The development of boson-exchange models came chiefly during his lifetime; mean-field treatments, once the strong short-range repulsions had been regularized, gave us an underlying basis for the shell model. We conjecture that the same type of mean-field approximations, applied at the field theory level, can give us an understanding of the strong short-range repulsions in the nucleon–nucleon interaction.

Note added in proof. The work of Rho *et al.* (1983) makes it clear that the value of g_A for the hedgehog solution found in Vento *et al.* (1980) should be decreased by a factor ~ 2 due to baryon number fractionalization between interior and exterior regions. Detailed mean-field calculations by Vento and Rho (to be published) give repulsions about half the size of those estimated here.

Acknowledgments

I am grateful to my collaborators Mannque Rho and Vincent Vento who developed most of the picture described here. This work was supported by USDOE under Contract No. DE-AC02-76-ER-13001.

References

- Brodsky, S. J., Huang, T., and Lepage, G. P. (1983). SLAC-PUB-2868.
Brown, G. E., Durso, J. W., and Johnson, M. B. (1983). *Nucl. Phys. A* **397**, 447.
Brown, G. E., and Rho, M. (1979). *Phys. Lett. B* **82**, 177.
Brown, G. E., and Rho, M. (1983). *Phys. Today* **36** (No. 2), 24.
Brown, G. E., Rho, M., and Vento, V. (1979). *Phys. Lett. B* **84**, 383.
Callan, C. G., Dashen, R. F., and Gross, D. J. (1978). *Phys. Lett. B* **78**, 307.
Carlson, C., Hansson, H., and Peterson, C. (1983). *Phys. Rev. D* **27**, 1556.
Chodos, A., Jaffe, R. L., Johnson, K., and Thorn, C. B. (1974). *Phys. Rev. D* **10**, 2599.
Chodos, A., and Thorn, C. B. (1975). *Phys. Rev. D* **12**, 2733.
Copley, L. A., Karl, G., and Obryk, E. (1969). *Nucl. Phys. B* **13**, 303.
DeGrand, T., Jaffe, R. L., Johnson, K., and Kiskis, J. (1975). *Phys. Rev. D* **12**, 2060.
DeTar, C. (1978). *Phys. Rev. D* **17**, 323.
DeTar, C. (1979a). *Phys. Rev. D* **19**, 1028.
DeTar, C. (1979b). *Phys. Rev. D* **19**, 1451.
Jaffe, R. L. (1979). Proc. Erice Summer School 'Ettore Majorana', 31 July to 12 August 1979.
Lieberman, D. A. (1977). *Phys. Rev. D* **16**, 1542.
Rho, M., Goldhaber, A. S., and Brown, G. E. (1983). *Phys. Rev. Lett.* **51**, 747.
Vento, V., Rho, M., and Brown, G. E. (1981). *Phys. Lett. B* **103**, 285.
Vento, V., Rho, M., Nyman, E. M., Jun, J. H., and Brown, G. E. (1980). *Nucl. Phys. A* **345**, 413.

