

Study of Approximations for Electron–Atom Direct Reactions*

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Abstract

The electron–hydrogen system is a true three-body system which provides an excellent test for theories of reactions in many-body systems that approximately involve only three-body degrees of freedom. The coupled-channels optical approximation reproduces experimental data in most cases within experimental error. The approximation may be extended to a larger space of coupled channels by various approximations which are tested with the example of 54.42 eV electron scattering on the 1s, 2s and 2p space for hydrogen, extended by the addition of 3s and 3p channels. Channels outside this five-state space are treated by including the corresponding polarization potentials.

1. Introduction

Stuart Butler pioneered the use of the Born approximation for describing angular distributions for non-elastic reactions at relatively high energies in nuclear physics (Butler 1950, 1951; Austern *et al.* 1953). This work helped considerably in the identification of, for example, direct reactions, i.e. those that can be described by a potential scattering theory using relatively few degrees of freedom. In this case two elementary particles (nucleons) were assumed to interact in the presence of an inert core formed by the remainder of the nucleus. The rearrangement and inelastic scattering problems were regarded as three-body problems. Butler (1951) also foresaw improvements in the form of the distorted-wave Born approximation, which takes into account the particle–core potential that is neglected in unbound two-body subsystems in the simple Born approximation.

Nuclear reactions are complicated by several factors in addition to the solution of the three-body problem. The extent to which non-three-body degrees of freedom are important in the reaction is initially unknown. Also, the potentials in the various two-body subsystems are not known very well. For example, off-shell matrix elements of the nucleon–nucleon potential are still undetermined, and it is not clear to what extent many-body forces play a part. It is important to find a reliable approach to the three-body problem so that one of these unknown factors can be eliminated.

The scattering of electrons by small atoms provides a quantum system with a massive core and fewer degrees of freedom. In fact for a hydrogen-atom target we have a true three-body problem. In addition, there is strong evidence that the Coulomb

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potential adequately describes the individual two-body interactions. The electron-hydrogen problem is an ideal testing ground for a three-body reaction theory.

The Born approximation for atomic inelastic scattering was pioneered by Massey and Mohr (1931). Early inelastic scattering experiments were performed on inert gas targets by Mohr and Nicoll (1932*a*, 1932*b*). With the advent of improved detector technology we now have accurate data for electron scattering to the 1s, 2s and 2p states of hydrogen and for coincidence of electrons with the photons resulting from the decay of the 2p state (Teubner *et al.* 1974; Williams 1975; van Wingerden *et al.* 1977; Frost and Weigold 1980; Williams 1981). Of course the atomic scattering problem has great intrinsic interest in addition to its role as an analogue computer for the three-body problem.

2. Electron-Hydrogen Scattering Theory

The theory that has been applied to electron-hydrogen scattering with most success up to 1981 is the close-coupling approximation (Burke and Smith 1962; Kingston *et al.* 1976). Here a set of reaction channels projected from the whole channel space by an operator P is treated by explicitly solving the set of coupled integrodifferential equations which constitute the coordinate-space representation of the Schrödinger equation for the problem, truncated to include only P space. In the spirit of this approximation the remainder of channel space, projected by an operator Q , may be included in the coupled set by adding a set of pseudochannels represented by square-integrable functions for the target system orthogonalized to the states in P space. The most comprehensive calculation of this type by Bransden *et al.* (1982) included several pseudochannels in the form of second-order potentials which were added to the coupling potentials for the P -space close-coupling problem. In all these calculations P space has consisted of the 1s, 2s and 2p states for hydrogen.

The computational techniques for the coordinate-space close-coupling method seem far removed from the simplicity of the Born approximation, or even the distorted-wave Born approximation which has been applied to the electron-hydrogen problem by Madison *et al.* (1983). The Born approximation for each transition within P space, however, is the driving term for the corresponding integral equation in the coupled set that constitutes the momentum-space representation of the Schrödinger equation for the problem. Furthermore, the potential factors in the kernels of the integral equations are simply off-shell Born amplitudes. Given a numerical method of solving a set of coupled integral equations, it is necessary only to compute Born amplitudes to solve the whole P -space problem! McCarthy and Stelbovics (1983*a*, 1983*b*) have made the first successful application of this method to the electron-hydrogen problem.

The explicit inclusion of Q space has been developed by McCarthy and Stelbovics (1980). In the P -space coupled equations one uses an optical potential formed by adding to the potential V felt by the incident electron a complex polarization term computed from amplitudes for excitation into channels outside P space. Here we investigate the effect of different approximations for R space, comprising several channels adjacent in energy to P space. The remaining channels, including the continuum, comprise Q space, which is thus redefined. The corresponding projection operators obey

$$P+Q+R = 1. \quad (1)$$

That part of the Q -space optical potential $V^{(Q)}$ describing continuum excitations is computed explicitly from the Born-Oppenheimer approximation to the ionization amplitudes, which is known to be a good approximation for total ionization cross sections.

The R space may be included in the calculation in several ways, the most straightforward but computationally most difficult being a complete calculation of the coupled-channels problem for $P+R$ space with the potential $V^{(Q)}$. The complete calculation of the polarization potential for R space to second order in $V^{(Q)}$ is equivalent to solving the $P+R$ space problem omitting matrix elements of the operator $RV^{(Q)}R$. We call this the second-order optical approximation. The practical computation of the polarization potential for R space involves the peaking approximation for the exchange amplitudes and the equivalent-local approximation (McCarthy *et al.* 1981). Thus the R -space polarization-potential approximation is an approximation to the second-order optical approximation.

The three approximations have been tested on the entrance (1s) channel in the 1s, 2s and 2p model problem for hydrogen at 100 eV (McCarthy and Stelbovics 1982). Here P space consisted of the 1s channel and R space of the 2s and 2p channels. The second-order optical approximation was accurate to about 5% for differential and 2% for total elastic cross sections. The polarization-potential approximation was about as good.

In the present paper we compare the polarization-potential approximation, the second-order optical approximation and the full solution of the $P+R$ problem for 54.42 eV electron scattering on hydrogen. The P space consists of the 1s, 2s and 2p channels, while R space consists of the 3s and 3p channels. We also include diagonal polarization potentials for Q space. The only significant discrete contributions to the optical potential $V^{(Q)}$ are 1s-4p and 2s-4p. In addition we include the continuum polarization potentials for 1s, 2s and 2p.

In this way we not only test the polarization-potential approximation for R space for inelastic data as well as entrance-channel data, but examine the question of whether truncation of $P+R$ space by omission of the matrix elements of $RV^{(Q)}R$ gives a sufficiently accurate calculation of all P -space data. This is very important for the justification of this approximation as a way of performing calculations with large numbers of channels in R space. Note that this second-order optical approximation includes matrix elements of $PV^{(Q)}R$ to all orders. It is not equivalent to the second Born approximation for P space which includes $PV^{(Q)}R$ only to second order.

It is possible to test a further approximation which we call the selected-transition approximation. Here we choose to omit matrix elements of certain transitions $PV^{(Q)}R$, chosen because we believe them to be small. For example, dipole transitions are much stronger than monopole transitions. In the present work the transitions to be omitted are considered small on the basis of calculations of the corresponding polarization potentials.

The object of all the approximations is to reproduce the P -space transitions arising from a full coupled-channels calculation in $P+R$ space with the smallest possible number of matrix elements of the type $PV^{(Q)}R$.

3. Formal Theory

The Hamiltonian for the electron-hydrogen problem is

$$H = K_1 + K_2 + v_1 + v_2 + v_3, \quad (2)$$

where K and v denote kinetic energy and potential operators, the subscripts 1 and 2 refer to electron-proton subsystems, of which 2 is bound, and v_3 is the electron-electron potential. The orbital ϕ_i and separation energy ε_i of the target are defined by

$$(\varepsilon_i - K_2 - v_2)\phi_i = 0. \quad (3)$$

Since we are ignoring spin-orbit coupling, scattering occurs in singlet and triplet states, $S = 0$ and 1, where S is the spin of the two-electron subsystem. The Schrödinger equation for the electron-hydrogen problem is

$$\{E^{(+)} - K_1 - K_2 - v_1 - v_2 - v_3 - (-)^S(H - E)P_r\}\Psi_S^{(\pm)} = 0, \quad (4)$$

where P_r is the space-exchange operator for the coordinates of the electrons.

A channel in P space is characterized by the corresponding target state. We choose the subscripts i, j, l for P space. The reformulation of the problem in terms of an optical potential for the space $Q+R$ has been discussed by McCarthy and Stelbovics (1980). The integral equation for reactions involving channels in P space is

$$\langle \mathbf{k}_r \phi_i | T | \phi_j \mathbf{k}_j \rangle = \langle \mathbf{k}_r \phi_i | V^{(Q+R)} | \phi_j \mathbf{k}_j \rangle + \sum_l \int d^3 k_r' \langle \mathbf{k}_r \phi_i | V^{(Q+R)} | \phi_l \mathbf{k}_r' \rangle \frac{1}{E^{(+)} - \varepsilon_l - \frac{1}{2}k_r'^2} \langle \mathbf{k}_r' \phi_l | T | \phi_j \mathbf{k}_j \rangle, \quad (5)$$

where the T matrix elements for on-shell momenta \mathbf{k}_j give the scattering amplitudes. The on-shell condition is

$$k_j^2 = 2(E + \varepsilon_j). \quad (6)$$

Equation (5) is abbreviated as follows:

$$T_{ij} = V_{ij}^{(Q+R)} + \sum_l V_{il}^{(Q+R)} G_l T_{lj}; \quad i, j, l \in P. \quad (7)$$

We are interested only in reactions within P space but we want to treat channels in R space more explicitly.

The full set of coupled equations for $P+R$ space is split into a set for exciting P -space channels and a set for exciting R -space channels, where we use subscripts m and n for R space:

$$T_{ij} = V_{ij}^{(Q)} + \sum_l V_{il}^{(Q)} G_l T_{lj} + \sum_m V_{im}^{(Q)} G_m T_{mj}, \quad (8a)$$

$$T_{nj} = V_{nj}^{(Q)} + \sum_l V_{nl}^{(Q)} G_l T_{lj} + \sum_m V_{nm}^{(Q)} G_m T_{mj}, \quad (8b)$$

with $i, j, l \in P$ and $m, n \in R$. Projecting from $P+R$ to P one obtains the formal definition of $V^{(Q+R)}$ in terms of $V^{(Q)}$. The second-order potential approximation to $V_{ij}^{(Q+R)}$ for excitation of channels in R space is

$$V_{ij}^{(Q+R)} = V_{ij}^{(Q)} + \sum_m V_{im}^{(Q)} G_m V_{mj}^{(Q)}. \quad (9)$$

Using this potential for the P -space coupled equations (7) we have the second-order optical approximation T'_{ij} to T_{ij} :

$$T'_{ij} = \left(V_{ij}^{(Q)} + \sum_m V_{im}^{(Q)} G_m V_{mj}^{(Q)} \right) + \sum_l \left(V_{il}^{(Q)} + \sum_m V_{im}^{(Q)} G_m V_{ml}^{(Q)} \right) G_l T_{lj}. \quad (10)$$

This equation may be rearranged to give

$$T'_{ij} = V_{ij}^{(Q)} + \sum_l V_{il}^{(Q)} G_l T'_{lj} + \sum_m V_{im}^{(Q)} G_m \left(V_{mj}^{(Q)} + \sum_l V_{ml}^{(Q)} G_l T'_{lj} \right). \quad (11)$$

We make a similar rearrangement for equation (8a), describing the exact T matrix elements for transitions within P space, by iterating once for T_{mj} , using (8b):

$$T_{ij} = V_{ij}^{(Q)} + \sum_l V_{il}^{(Q)} G_l T_{lj} + \sum_m V_{im}^{(Q)} G_m \left(V_{mj}^{(Q)} + \sum_l V_{ml}^{(Q)} G_l T_{lj} + \sum_n V_{mn}^{(Q)} G_n T_{nj} \right). \quad (12)$$

The kernel matrix for (12) is identical to that of (11) if we set

$$V_{mn}^{(Q)} = 0; \quad m, n \in R. \quad (13)$$

The use of approximation (13) in equations (8) for $P+R$ space is therefore identical to the second-order optical approximation. Note that the remainder (8b) of the coupled equations for $P+R$ space involves the T matrix elements T_{nj} with driving terms $V_{nj}^{(Q)}$. The second-order optical approximation keeps $RV^{(Q)}P$ matrix elements such as $V_{ml}^{(Q)}$ in the kernel matrix and in the driving terms. They are therefore taken into account to all orders in the calculation of T_{ij} .

4. Comparison of Approximations

In Tables 1-4 we compare data for transitions within the 1s, 2s and 2p P space for 54.42 eV electrons incident on hydrogen. The accuracy of the calculations decreases from left to right. In all cases the Q -space transitions taken into account in $V^{(Q)}$ are 1s-4p, 2s-4p, 1s-continuum, 2s-continuum and 2p-continuum.

The column CCO5 is the full five-state $P+R$ space coupled-channels optical approximation in which R space consists of 3s and 3p channels. The column SOO is the second-order optical approximation in which the five-state coupled-channels optical approximation is calculated with the omission of matrix elements of $RV^{(Q)}R$. Specifically, these transitions are 3s-3s, 3p-3p and 3s-3p. The STA denotes the selected-transition approximation, in which omitted matrix elements in addition to those for SOO are 1s-3s and 2s-3s, while CCO3 is the three-state coupled-channels optical calculation in which the optical potential $V^{(Q+R)}$ includes the same set of PR transitions that are included in STA.

Comparison of SOO with CCO5 tests the importance of RR transitions in the CCO calculation, while comparison of STA with CCO5 tests the omission of some PR transitions. Comparison of CCO3 with STA tests the validity of the peaking approximation for exchange and the equivalent-local approximation, which are used in the calculation of the optical potentials for PR transitions.

The numerical analysis (McCarthy and Stelbovics 1983b) is designed to produce an accuracy of order 1%. Two significant figures (three when the first digit is 1) are therefore given in the tables. Differences are sometimes exaggerated by rounding.

We first notice that the approximations make almost no difference to the entrance-channel data within the 1% computational accuracy limits (Table 1). For the 2s differential cross sections (Table 2) SOO is usually not more than 5% different from CCO5, although the difference can be as high as 10%. The omission of the two

monopole channels in STA seems to compensate for some errors, although this is probably fortuitous. At angles other than 0° the R -space polarization potentials in CCO3 represent the PR transitions very well, usually within 5%. Two sets of experimental data due to Williams (1981) are included in Table 2 to give an idea of the overall quality of the calculations in comparison with experiment.

Table 1. Differential elastic cross sections, total elastic σ_E and total reaction σ_R cross sections for electron-hydrogen scattering at 54.42 eV

CCO5, five-state coupled-channels optical approximation; SOO, second-order optical approximation; STA, selected-transition approximation; CCO3, three-state coupled-channels optical approximation with PR transitions corresponding to the STA. Units are a_0^2 . Powers of 10 are indicated by a superscript

θ (deg.)	CCO5	SOO	STA	CCO3
0	$8 \cdot 2^0$	$8 \cdot 3^0$	$8 \cdot 2^0$	$8 \cdot 3^0$
10	$3 \cdot 9^0$	$3 \cdot 9^0$	$3 \cdot 9^0$	$3 \cdot 9^0$
20	$1 \cdot 74^0$	$1 \cdot 76^0$	$1 \cdot 75^0$	$1 \cdot 73^0$
30	$9 \cdot 0^{-1}$	$9 \cdot 1^{-1}$	$9 \cdot 0^{-1}$	$9 \cdot 1^{-1}$
40	$5 \cdot 0^{-1}$	$5 \cdot 0^{-1}$	$5 \cdot 0^{-1}$	$5 \cdot 1^{-1}$
60	$1 \cdot 76^{-1}$	$1 \cdot 76^{-1}$	$1 \cdot 76^{-1}$	$1 \cdot 77^{-1}$
80	$7 \cdot 5^{-2}$	$7 \cdot 4^{-2}$	$7 \cdot 5^{-2}$	$7 \cdot 5^{-2}$
100	$4 \cdot 2^{-2}$	$4 \cdot 3^{-2}$	$4 \cdot 3^{-2}$	$4 \cdot 3^{-2}$
120	$2 \cdot 7^{-2}$	$2 \cdot 8^{-2}$	$2 \cdot 8^{-2}$	$2 \cdot 7^{-2}$
140	$2 \cdot 0^{-2}$	$2 \cdot 0^{-2}$	$2 \cdot 1^{-2}$	$2 \cdot 0^{-2}$
160	$1 \cdot 69^{-2}$	$1 \cdot 72^{-2}$	$1 \cdot 73^{-2}$	$1 \cdot 71^{-2}$
180	$1 \cdot 61^{-2}$	$1 \cdot 64^{-2}$	$1 \cdot 65^{-2}$	$1 \cdot 61^{-2}$
σ_E	3.21	3.24	3.22	3.22
σ_R	6.70	6.62	6.57	6.69

Table 2. Differential cross sections for inelastic scattering to 2s state

For details see Table 1. EXP1, EXP2 are experimental data of Williams (1981). Experimental error in the final digits is in parentheses

θ (deg.)	CCO5	SOO	STA	CCO3	EXP1	EXP2
0	$2 \cdot 6^0$	$2 \cdot 5^0$	$2 \cdot 6^0$	$3 \cdot 3^0$	—	—
10	$5 \cdot 2^{-1}$	$4 \cdot 7^{-1}$	$5 \cdot 2^{-1}$	$5 \cdot 4^{-1}$	$3 \cdot 80^{-1}(128)$	$5 \cdot 99^{-1}(94)$
20	$9 \cdot 9^{-2}$	$8 \cdot 9^{-2}$	$1 \cdot 01^{-1}$	$9 \cdot 5^{-2}$	$9 \cdot 00^{-2}(18)$	$1 \cdot 16^{-1}(17)$
30	$3 \cdot 0^{-2}$	$2 \cdot 8^{-2}$	$3 \cdot 2^{-2}$	$3 \cdot 1^{-2}$	$3 \cdot 50^{-2}(39)$	$3 \cdot 75^{-2}(40)$
40	$1 \cdot 19^{-2}$	$1 \cdot 17^{-2}$	$1 \cdot 33^{-2}$	$1 \cdot 31^{-2}$	$1 \cdot 65^{-2}(128)$	$1 \cdot 84^{-2}(47)$
60	$7 \cdot 3^{-3}$	$7 \cdot 2^{-3}$	$7 \cdot 9^{-3}$	$8 \cdot 1^{-3}$	$1 \cdot 01^{-2}(48)$	$1 \cdot 06^{-2}(31)$
70	$6 \cdot 3^{-3}$	$6 \cdot 1^{-3}$	$6 \cdot 7^{-3}$	$6 \cdot 7^{-3}$	$7 \cdot 9^{-3}(34)$	$7 \cdot 04^{-3}(218)$
80	$5 \cdot 2^{-3}$	$5 \cdot 1^{-3}$	$5 \cdot 5^{-3}$	$5 \cdot 4^{-3}$	$6 \cdot 15^{-3}(247)$	$5 \cdot 27^{-3}(163)$
90	$4 \cdot 0^{-3}$	$3 \cdot 8^{-3}$	$4 \cdot 2^{-3}$	$4 \cdot 4^{-3}$	$5 \cdot 43^{-3}(117)$	$5 \cdot 70^{-3}(128)$
100	$3 \cdot 3^{-3}$	$3 \cdot 1^{-3}$	$3 \cdot 5^{-3}$	$3 \cdot 6^{-3}$	$3 \cdot 85^{-3}(105)$	$3 \cdot 85^{-3}(129)$
120	$2 \cdot 4^{-3}$	$2 \cdot 3^{-3}$	$2 \cdot 5^{-3}$	$2 \cdot 6^{-3}$	$2 \cdot 82^{-3}(98)$	$3 \cdot 21^{-3}(114)$
140	$2 \cdot 0^{-3}$	$1 \cdot 89^{-3}$	$2 \cdot 1^{-3}$	$2 \cdot 1^{-3}$	$2 \cdot 18^{-3}(87)$	$2 \cdot 24^{-3}(87)$
160	$1 \cdot 70^{-3}$	$1 \cdot 60^{-3}$	$1 \cdot 78^{-3}$	$1 \cdot 89^{-3}$	—	—
180	$1 \cdot 84^{-3}$	$1 \cdot 71^{-3}$	$1 \cdot 89^{-3}$	$1 \cdot 83^{-3}$	—	—
σ_{2s}	0.255	0.237	0.262	0.277	—	—

Table 3. Differential cross sections for inelastic scattering to 2p state

For details see Tables 1 and 2

θ (deg.)	CCO5	SOO	STA	CCO3	EXP
0	3.9 ¹	3.8 ¹	3.8 ¹	3.8 ¹	—
10	7.2 ⁰	7.0 ⁰	6.9 ⁰	7.3 ⁰	7.54 ⁰ (71)
20	9.9 ⁻¹	9.2 ⁻¹	9.1 ⁻¹	1.03 ⁰	1.04 ⁰ (11)
30	1.63 ⁻¹	1.45 ⁻¹	1.43 ⁻¹	1.68 ⁻¹	1.57 ⁻¹ (21)
40	4.4 ⁻²	3.8 ⁻²	3.7 ⁻²	4.2 ⁻²	4.36 ⁻² (69)
60	1.14 ⁻²	9.6 ⁻³	9.6 ⁻³	1.06 ⁻²	1.19 ⁻² (21)
70	7.2 ⁻³	6.0 ⁻³	5.9 ⁻³	6.6 ⁻³	6.12 ⁻³ (131)
80	4.4 ⁻³	3.8 ⁻³	3.7 ⁻³	4.2 ⁻³	4.05 ⁻³ (87)
90	3.1 ⁻³	2.6 ⁻³	2.6 ⁻³	2.9 ⁻³	3.56 ⁻³ (56)
100	2.3 ⁻³	1.92 ⁻³	1.86 ⁻³	2.2 ⁻³	2.16 ⁻³ (46)
120	1.33 ⁻³	1.19 ⁻³	1.14 ⁻³	1.32 ⁻³	1.59 ⁻³ (36)
140	9.2 ⁻⁴	8.7 ⁻⁴	8.2 ⁻⁴	9.6 ⁻⁴	1.03 ⁻³ (28)
160	7.4 ⁻⁴	7.1 ⁻⁴	6.7 ⁻⁴	8.1 ⁻⁴	—
180	7.3 ⁻⁴	7.1 ⁻⁴	6.7 ⁻⁴	7.6 ⁻⁴	—
σ_{2p}	2.62	2.52	2.51	2.63	

Table 4. Angular correlation parameters λ and R

For details see Tables 1 and 2

θ (deg.)	CCO5	SOO	STA	CCO3	EXP
(a) Parameter λ					
10	3.0 ⁻¹	2.8 ⁻¹	2.8 ⁻¹	2.7 ⁻¹	3.41 ⁻¹ (45)
15	2.1 ⁻¹	2.0 ⁻¹	2.0 ⁻¹	1.91 ⁻¹	1.88 ⁻¹ (32)
20	2.1 ⁻¹	1.95 ⁻¹	1.97 ⁻¹	1.94 ⁻¹	2.25 ⁻¹ (25)
30	4.0 ⁻¹	3.7 ⁻¹	3.8 ⁻¹	3.8 ⁻¹	5.53 ⁻¹ (39)
40	7.6 ⁻¹	7.5 ⁻¹	7.6 ⁻¹	7.4 ⁻¹	8.17 ⁻¹ (56)
60	9.4 ⁻¹	9.3 ⁻¹	9.4 ⁻¹	9.3 ⁻¹	9.22 ⁻¹ (62)
70	8.9 ⁻¹	8.7 ⁻¹	8.9 ⁻¹	8.8 ⁻¹	8.16 ⁻¹ (39)
90	8.2 ⁻¹	7.9 ⁻¹	8.1 ⁻¹	8.2 ⁻¹	4.83 ⁻¹ (52)
100	8.1 ⁻¹	7.9 ⁻¹	8.1 ⁻¹	8.3 ⁻¹	3.54 ⁻¹ (33)
120	8.9 ⁻¹	8.7 ⁻¹	8.8 ⁻¹	9.0 ⁻¹	5.25 ⁻¹ (42)
140	9.5 ⁻¹	9.4 ⁻¹	9.4 ⁻¹	9.6 ⁻¹	9.01 ⁻¹ (43)
(b) Parameter R					
10	3.0 ⁻¹	3.0 ⁻¹	3.0 ⁻¹	2.9 ⁻¹	2.83 ⁻¹ (27)
15	2.5 ⁻¹	2.5 ⁻¹	2.4 ⁻¹	2.4 ⁻¹	2.35 ⁻¹ (25)
20	2.2 ⁻¹	2.2 ⁻¹	2.2 ⁻¹	2.1 ⁻¹	2.08 ⁻¹ (23)
30	2.1 ⁻¹	2.1 ⁻¹	2.0 ⁻¹	2.1 ⁻¹	1.81 ⁻¹ (21)
40	1.78 ⁻¹	1.93 ⁻¹	1.78 ⁻¹	1.88 ⁻¹	1.22 ⁻¹ (18)
60	6.4 ⁻²	8.3 ⁻²	7.6 ⁻²	8.3 ⁻²	8.20 ⁻² (26)
70	2.7 ⁻²	6.0 ⁻²	5.3 ⁻²	6.3 ⁻²	2.70 ⁻² (21)
90	5.9 ⁻²	1.02 ⁻¹	8.6 ⁻²	9.9 ⁻²	-5.10 ⁻² (29)
100	7.6 ⁻²	1.22 ⁻¹	1.04 ⁻¹	1.18 ⁻¹	-1.13 ⁻¹ (33)
120	1.00 ⁻¹	1.47 ⁻¹	1.26 ⁻¹	1.30 ⁻¹	-6.50 ⁻² (31)
140	8.4 ⁻²	1.19 ⁻¹	1.07 ⁻¹	9.8 ⁻²	-5.60 ⁻² (35)

The omission of RR transitions in SOO seems to be more serious for the 2p differential cross section (Table 3). Cross sections are underestimated by about 10% in most cases. This time the omission of the two monopole transitions in STA makes

less difference, and the compensating effect observed for 2s is not evident. The PR polarization potentials in CCO3 are not such a good approximation to STA, differences being again of order 10%. The data of Williams (1981) agree with CCO5 within experimental error, but STA is in error by more than a standard deviation in a few cases.

The angular correlation parameters λ and R (Table 4) for the 2p excitations are defined by

$$\sigma\lambda = \sigma_0, \quad \sigma R = \text{Re}\langle T_0 T_1 \rangle, \quad (14a, b)$$

where the subscripts refer to the projection quantum number m , and σ and T stand for differential cross sections and scattering amplitudes respectively. The approximations make very little difference to λ (Table 4a). At angles greater than 70° there is substantial disagreement with experiment. There is no evidence in Table 4a of any instability in the calculation that would cause such large changes when the approximations are improved by explicit inclusion of more channels in P space.

For the parameter R (Table 4b) however, there are large differences between CCO5 and the other approximations, occurring in just the large-angle region where theory and experiment disagree severely. This agrees with the generally accepted view that R is the most sensitive test of a scattering theory and that this is where experimental and theoretical work should concentrate to resolve the remaining discrepancies.

5. Conclusions

The coupled-channels optical approximation for electron-hydrogen scattering is remarkably close to experiment except for angular correlation parameters at large angles. One can omit matrix elements for transitions within the space R of channels adjacent to the experimentally observed ones of P , and even omit other small (non-dipole) matrix elements, and still get cross sections generally within the present experimental errors (of order 10% or more). This suggests approximations for including a larger R space explicitly in the coupled-channels calculation.

The calculation within P space using diagonal polarization potentials representing PR transitions is accurate for inelastic scattering within about 10% at 54.42 eV. The second-order method that is used for such calculations improves at higher energies. This is true also for the second-order optical approximation to the $P+R$ space calculation.

The overall agreement with experiment is strong evidence for the validity of the continuum contributions to the polarization potential, which cannot be tested in comparison with a more accurate theory as we have done here for discrete transitions.

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