Aust. J. Phys., 1983, **36**, 859–65

Townsend Discharges in Transverse Magnetic Fields

H. A. Blevin and M. J. Brennan

School of Physical Sciences, Flinders University of South Australia,
Bedford Park, S.A. 5042.

Abstract

Expressions are derived for the electron concentration in Townsend discharges in the presence of a transverse magnetic field for both steady state and pulsed conditions. These results indicate that the two components of the electron drift velocity and the four diffusion coefficients required to describe the concentration distribution can be determined by observation of photons emitted from the discharge.

1. Introduction

The influence of a transverse magnetic field on gas breakdown at low gas pressures can be quite spectacular. For example, the breakdown voltage between coaxial cylinders at low gas pressures can be reduced from some tens of kilovolts to a few hundred volts by the application of magnetic fields of the order 0·02 T (Harcombe *et al.* 1963). Nevertheless many aspects of electron swarm behaviour under these conditions remain unexplored. Most studies related to gas breakdown in transverse magnetic fields have concentrated on the Townsend primary and secondary ionization coefficients and this work is described in a review by Heylen (1980). Comparatively little experimentation has been devoted to electron transport parameters in crossed fields. Experiments such as those initiated by Townsend and Tizard (1913) or Huxley and Zaazou (1949) used weak magnetic fields so that $\omega/\nu \ll 1$, where ω is the electron cyclotron frequency and ν is the collision frequency for momentum transfer. These techniques yield values of W_M, the magnetic drift velocity, which in the weak field limit approximates to W, the drift velocity in the absence of a magnetic field. In the strong field limit, $\omega/\nu \gg 1$, Bernstein (1962) measured drift velocities and the diffusion of electrons along the magnetic field. As far as we are aware there have not been any measurements of the other components of the diffusion tensor.

In this paper we explore the possibility of measuring parameters such as the ionization rate ν_i, the components of the electron drift velocity W_x and W_z, and the four diffusion coefficients D_x, D_y, D_z, D_s in crossed field discharges (Huxley and Crompton 1974). We adopt a coordinate system with the electric field $\boldsymbol{E}$ in the $-Z$ direction and the magnetic field $\boldsymbol{B}$ along the Y axis. The diffusion coefficient D_s relates to the particle flux produced in a direction mutually perpendicular to the local density gradient and the magnetic field. The proposed experimental method

0004-9506/83/060859$02.00

is that presently used in these laboratories, whereby the spatial and temporal development of the swarm is monitored by observation of photons emitted from excited gas molecules (Blevin *et al.* 1976*a*). In principle this approach could also yield excitation rates to various levels and their ω/v dependence. This should reduce difficulties experienced in arriving at unambiguous collision cross sections when comparing measured transport and rate coefficients with theoretical values calculated from the Boltzmann equation.

In analysing data from these experiments it is necessary to determine the integrated electron concentration along a line of sight through the swarm. Analytical expressions for this quantity are derived in the following sections for both pulsed and steady state discharges. Numerical examples are used to illustrate the potentialities of this approach for the determination of transport coefficients.

2. Theory

(*a*) *Pulsed Electron Swarms*

The continuity equation describing the evolution of the electron concentration $n(X, Y, Z, t)$ is

$$\frac{\partial n}{\partial t}+W_x\frac{\partial n}{\partial X}+W_z\frac{\partial n}{\partial Z}-D_x\frac{\partial^2 n}{\partial X^2}-D_y\frac{\partial^2 n}{\partial Y^2}-D_z\frac{\partial^2 n}{\partial Z^2}-D_s\frac{\partial^2 n}{\partial X\partial Z}=n\nu_{\mathrm{i}}\,. \tag{1}$$

Only an outline of the solution of this equation is given here. A more detailed treatment is given by Huxley and Crompton (1974), although the term involving D_s is discarded in their analysis which was intended for application in the low field limit.

Substituting $n = n_1\exp(\nu_{\mathrm{i}}\,t)$, $X_1 = X-W_x t$ and $Z_1 = Z-W_z t$ into equation (1) gives

$$\frac{\mathrm{d}n_1}{\mathrm{d}t}-D_x\frac{\partial^2 n_1}{\partial X_1^2}-D_y\frac{\partial^2 n_1}{\partial Y^2}-D_z\frac{\partial^2 n_1}{\partial Z_1^2}-D_s\frac{\partial^2 n_1}{\partial X_1\,\partial Z_1}=0\,. \tag{2}$$

The mixed derivative can be removed by a rotation of X_1 and Z_1 about the Y axis to obtain the usual form of the diffusion equation in this coordinate system. The solution for a δ-function source at $(0,0,0,0)$ can then be written in terms of the original coordinates as

$$n(X,Y,Z,t)=\{n_0/(D_x D_y D_z K)^{\frac{1}{2}}\}(4\pi t)^{-3/2}\exp(\nu_{\mathrm{i}}\,t)\exp(-Y^2/4D_y t)$$
$$\times\exp\left\{\frac{-1}{4Kt}\left(\frac{(X-W_x t)^2}{D_x}+\frac{(Z-W_z t)^2}{D_z}-\frac{D_s}{D_x D_z}(X-W_x t)(Z-W_z t)\right)\right\},$$

where $K = 1-D_s^2/4D_x D_z$.

At any time, contours of equal concentration in the X–Z plane are ellipses centred at $X = W_x t$ and $Z = W_z t$ and with axes rotated through an angle θ with respect to the X axis, where $\tan 2\theta = D_s/(D_x-D_z)$. This is shown in Fig. 1 where we have chosen $W_x = W_z$ and $\theta = \frac{1}{8}\pi$ for illustrative purposes.

For a region bounded at the cathode ($Z = 0$) and the anode ($Z = d$), the boundary condition $n = 0$ can be included by using the method of images. Allowance must be made for the rotation of the electron distribution so that to satisfy the cathode boundary condition a pole source is introduced at $(D_s h/2D_z, 0, h)$ and the 'image' term at $(-D_s h/2D_z, 0, -h)$ with a relative strength of $-\exp(-hW_z/D_z)$. In the limit of $h \to 0$ the electron concentration for this dipole source at the cathode becomes

$$n(X, Y, Z, t) = (CZ/t^{5/2})\exp\{v_i t - (Y^2/4D_y t)\}\exp\{-(Z - W_z t)^2/4D_z t\}$$
$$\times \exp[-\{X - W_x t - (D_s/2D_z)(Z - W_z t)\}^2/4D_x Kt], \qquad (3)$$

where C is a constant.

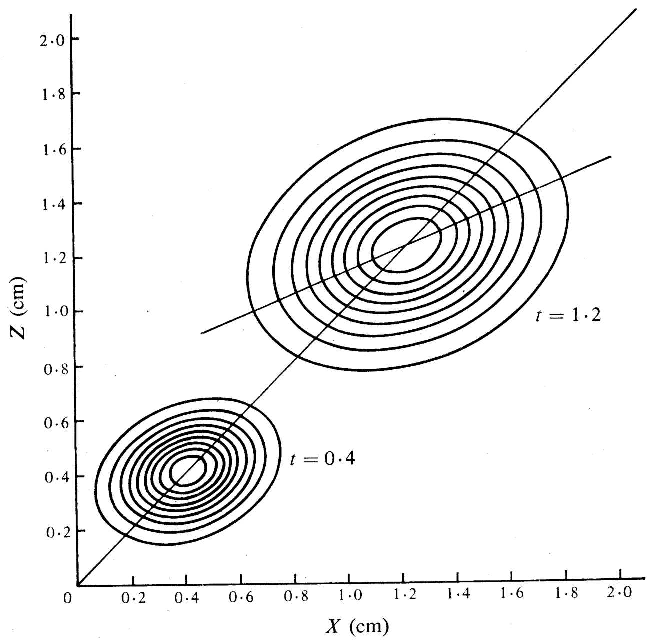

Fig. 1. Electron density distribution in the $Y = 0$ plane for an isolated swarm at two times, $t = 0 \cdot 4$ and $1 \cdot 2$, where $t = 1 \cdot 0$ is the transit time for 1 cm drift in the Z direction. The contours represent concentrations of $0 \cdot 1, 0 \cdot 2, \ldots, 0 \cdot 9$ of the peak concentrations in each case. We have chosen $W_x = W_z$, $D_x = 5 \times 10^5\ \mathrm{cm^2\,s^{-1}}$, $D_z = 3 \times 10^5\ \mathrm{cm^2\,s^{-1}}$ and $D_s = 2 \times 10^5\ \mathrm{cm^2\,s^{-1}}$.

The anode boundary conditions are satisfied by taking a dipole 'image' term at $(2dD_s/2D_z, 0, 2d)$ with a relative strength of $\exp(dW_z/D_z)$. The subsequent change in the electron concentration at the cathode is negligible for cases of practical interest so that no additional image terms are required. The final expression for the electron concentration is

$$\begin{aligned} n(X, Y, Z, t) = \; & (C/t^{5/2})\exp(v_i t)\exp(-Y^2/4D_y t) \\ & \times \exp[-\{X - W_x t - (D_s/2D_z)(Z - W_z t)\}^2/4D_x Kt] \\ & \times [Z\exp\{-(Z - W_z t)^2/4D_z t\} \\ & + (Z - 2d)\exp(dW_z/D_z)\exp\{-(Z - 2d - W_z t)^2/4D_z t\}]. \qquad (4) \end{aligned}$$

(b) Steady Stream

The electron concentration for a steady stream can be obtained by the superposition of the pulsed swarm solution given by equation (4) for an infinite sequence of sources (Blevin *et al.* 1976*a*, 1976*b*):

$$n(X,Y,Z) = C\exp\left(\frac{W_x X}{2D_x K}+\frac{W_z Z}{2D_z K}-D_s\frac{XW_z+ZW_x}{4D_x D_z K}\right)$$

$$\times\{ZS_0^{-3/2}\,\mathrm{K}_{3/2}(S_0)+(Z-2d)S_1^{-3/2}\,\mathrm{K}_{3/2}(S_1)\},$$

where

$$S_0^2 = \left(\frac{X^2}{D_x K}+\frac{Y^2}{D_y}+\frac{Z^2}{D_z K}-\frac{D_s XZ}{D_x D_z K}\right)\left(\frac{W_x^2}{4D_x K}+\frac{W_z^2}{4D_z K}-\frac{D_s W_x W_z}{4D_x D_z K}-\nu_\mathrm{i}\right),$$

$$S_1^2 = \left(\frac{X^2}{D_x K}+\frac{Y^2}{D_y}+\frac{Z^2}{D_z K}-\frac{D_s XZ}{D_x D_z K}+\frac{4d(d-Z)}{D_z}\right)\left(\frac{W_x^2}{4D_x K}+\frac{W_z^2}{4D_z K}-\frac{D_s W_x W_z}{4D_x D_z K}-\nu_\mathrm{i}\right),$$

and $\mathrm{K}_{3/2}(S_0)$ and $\mathrm{K}_{3/2}(S_1)$ are modified spherical Bessel functions.

The electron current density at the anode which results from the diffusion flux at $Z = d$ can be calculated from equation (4) for pulsed swarms. The total current is found by integration of this flux over all X and Y coordinates and the steady stream current obtained by integration over all source times as before. Although the details of these calculations are not relevant to the present study we note that the total current for a steady stream is described by the usual Townsend current growth equation

$$i = i_0\exp(\alpha_\mathrm{T} d),$$

where $\alpha_\mathrm{T} = \lambda_z\{1-(1-2\nu_\mathrm{i}/\lambda_z W_z)^{\frac{1}{2}}\}$ is the first Townsend ionization coefficient and $\lambda_z = W_z/2D_z$.

3. Application to Photon Detection Experiments

In these experiments the electron number density distribution in a swarm is determined by measurement of the emitted photon flux using a collimated detector. There is not a simple relationship between the measured photon flux and electron density since allowance must be made for variations in the local energy distribution function and lifetimes of excited states (Blevin *et al.* 1976*b*; Fletcher and Reid 1980). However, the starting point for the analysis of experimental data requires an expression for the integrated concentration along a line through the electron swarm and this can be obtained from equation (4).

(a) Pulsed Swarms

There are practical difficulties in measuring photon fluxes along a line of sight parallel to the electric field and the following work is restricted to line integrals of $n(X, Y, Z, t)$ along the Y and X axes which we define to be $N_y(X, Z, t)$ and $N_x(Y, Z, t)$

respectively. Provided that measurements are not taken close to the anode, these integrals can be evaluated using equation (3). Hence

$$N_y(X,Z,t) = (CZ/t^2)\exp[\nu_i t - \{(Z-W_z t)^2/4D_z t\}]$$
$$\times \exp[(-1/4D_x Kt)\{X - W_x t - (D_s/2D_z)(Z-W_z t)\}^2], \qquad (5)$$
$$N_x(Y,Z,t) = (CZ/t^2)\exp[\nu_i t - (Y^2/4D_y t) - \{(Z-W_z t)^2/4D_z t\}]. \qquad (6)$$

These results indicate that experiments using pulsed swarms can yield swarm parameters in the same manner as in the absence of a magnetic field (Blevin *et al.* 1976*a*). For example the Z-dependence of $N_x(0,Z,t)$ from equation (6) allows W_z and D_z to be found while observation of $N_x(0,Z,t)$ at $Z = W_z t$ determines ν_i. Similarly W_x and KD_x are determined from the X-dependence of $N_y(X,Z,t)$ at $Z = W_z t$. In principle, the diffusion coefficient D_s can be found from the shape of the contours of constant $N_y(X,Z,t)$ at any time. However, the accuracy of this measurement depends on the values of D_s and $D_x - D_z$ as discussed in Section 2*a*.

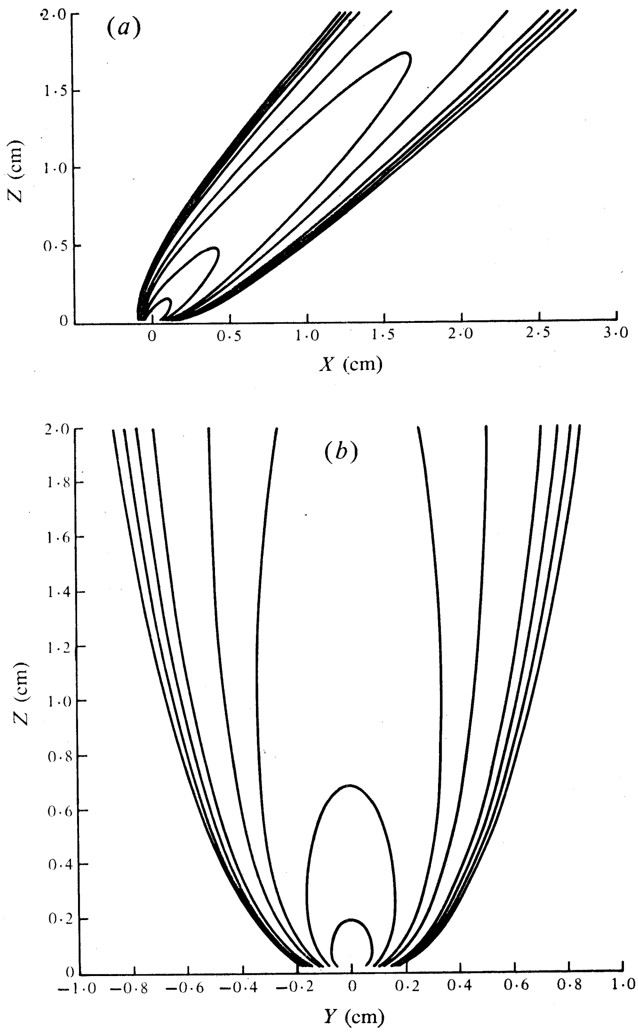

Fig. 2. Steady stream density distribution integrated along lines of sight (*a*) parallel to the magnetic field and (*b*) perpendicular to the electric and magnetic fields. The contours represent integrated concentrations which are 1/10, 1/20, ..., 1/80 of the peak value.

(b) Steady Stream

As before the steady stream results for $N(X,Z)$ and $N(Y,Z)$ can be obtained from equations (5) and (6) by integration over all source times to give

$$N_y(X,Z) = (CZ/S_1)\mathrm{K}_1(S_1)\exp\left(\frac{W_x X}{2D_x K} + \frac{W_z Z}{2D_z K} - \frac{D_s}{4D_x D_z K}(XW_z + ZW_x)\right), \quad (7)$$

where

$$S_1^2 = \frac{1}{K^2}\left(\frac{X^2}{D_x} + \frac{Z^2}{D_z} - \frac{D_s XZ}{D_x D_z}\right)\left(\frac{W_x^2}{4D_x} + \frac{W_z^2}{4D_z} - \frac{D_s W_x W_z}{4D_x D_z} - \nu_i K\right),$$

and

$$N_x(Y,Z) = (CZ/S_2)\mathrm{K}_1(S_2)\exp(ZW_z/2D_z K), \quad (8)$$

where

$$S_2^2 = \left(\frac{Y^2}{D_y} + \frac{Z^2}{D_z K}\right)\left(\frac{W_z^2}{4D_z K} - \nu_i\right).$$

Examples of the distributions described by equations (7) and (8) are shown in Fig. 2 where we have chosen $\nu_i = 0$, $W_x = W_z = 2\times10^7\ \mathrm{cm\,s^{-1}}$, $D_x = D_z = 6\cdot2\times10^5\ \mathrm{cm^2\,s^{-1}}$, $D_y = 1\cdot24\times10^6\ \mathrm{cm^2\,s^{-1}}$ and $D_s = 9\cdot3\times10^4\ \mathrm{cm^2\,s^{-1}}$. Equation (8) (Fig. 2*b*) is similar to the steady stream results obtained in the absence of a magnetic field (Blevin *et al.* 1976*a*) and it is possible to measure $\lambda_y = W_z/2D_y$ and the first Townsend ionization coefficient from this distribution. As evident from Fig. 2*a* it is also possible to measure W_x/W_z from $N_y(X,Z)$. However, it is not evident that other parameters such as $\lambda_x = W_x/2D_x$, $\lambda_z = W_z/2D_z$ and $W_z/2D_s$ are so readily obtained. Numerical calculations of the sensitivity of $N_y(X,Z)$ to changes in these parameters indicate that their evaluation is sensitive to the value of W_x/W_z (and hence ω/ν) selected.

4. Conclusions

The results of the above studies indicate that the optical method for measuring swarm evolution in a transverse magnetic field can be used to determine ionization rates, electron drift velocities and components of the diffusion tensor. Experiments using a steady stream are considerably easier to conduct, but yield a restricted range of information.

The transport and rate coefficients in crossed electric and magnetic fields can be calculated from the Boltzmann equation using published cross sections (Govinda and Gurumurthy 1978). Experimental verification of these calculations under conditions where the cyclotron frequency and the collision frequency are comparable are important, since the presence of a magnetic field gives a different weighting to these cross sections in the energy range of interest. Consequently, any errors in the cross sections used in the calculations would become evident.

Acknowledgments

This work was financed through the Australian Research Grants Scheme and in part by Flinders University research funds. The authors are indebted to Dr J. Fletcher for valuable discussion.

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Manuscript received 6 June, accepted 2 August 1983