Low-lying Negative-parity Levels of $^{17}$N and $^{18}$N

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Abstract

On the basis of a weak-coupling model, adjustments are made to the interactions used in the full shell model calculations of Millener in order to fit the experimental energies of the low-lying negative-parity levels of $^{16}$N and of the low-lying positive-parity levels of $^{18}$O and $^{19}$O. The predicted energies of the low-lying negative-parity levels of $^{17}$N then agree better with experiment, while those for $^{18}$N lead to suggested spin assignments for the observed levels.

1. Introduction

In a recent study of $^{18}$N with the reaction $^{18}$O($^7$Li, $^7$Be)$^{18}$N, Puttez et al. (1983) found strongly populated levels of $^{18}$N at excitation energies of 121 and 747 keV, while the ground state and a level at 580 keV were weakly populated. Only the ground state has a definite spin-parity assignment, which is $J^\pi = 1^-$ (Olness et al. 1982).

Full $0h0o$ and $1h0o$ shell model calculations by Millener (Olness et al. 1982; D. J. Millener, personal communication) indicated that the low-lying negative-parity states of both $^{17}$N and $^{18}$N are well described by the weak coupling of a $p_{1/2}$ proton hole to the low-lying $^{18}$O and $^{19}$O states. This is illustrated in Fig. 1, which also shows the similar situation for the low-lying states of $^{15}$N and $^{16}$N (Millener and Kurath 1975). In the $^{18}$N case, where the $2^-$ states can have mixed $^{19}$O parentage, the lower $2^-$ state contains only a 12.4% admixture of the $\frac{1}{2}^- \otimes \frac{3}{2}^+$ configuration (Olness et al. 1982). The experimental energies of the low-lying $^{15-18}$N and $^{16-19}$O levels are given in Fig. 2.

Millener's calculations used the Millener-Kurath (MK) (1975) interaction between nucleons from the p and sd shells. This was derived to give a good account of the non-normal parity states of a number of nuclei from $^{11}$Be to $^{16}$O. It does not, however, give the experimental ordering of the low-lying $^{16}$N levels or of the doublet members in $^{17}$N. This suggests that the predicted energies of the low-lying $^{18}$N levels might be significantly in error also.

We here adjust the MK interaction to fit the observed $^{16}$N energies and use the adjusted interaction to calculate the $^{17}$N and $^{18}$N energies. This is done by calculating the adjustments on the basis of a weak-coupling structure of the $^{16-18}$N levels and simple descriptions of the $^{17-19}$O levels, and assuming that the same relationship between the adjustments would hold in the full shell model calculation. Since the
Fig. 1. Spectra of low-lying negative-parity states of $^{15-18}N$ and of low-lying positive-parity states of $^{16-19}O$ calculated by Millener. Excitation energies are in keV. The weak-coupling parentage of the $^{15-18}N$ states is indicated.

Fig. 2. Experimental spectra of low-lying negative-parity states of $^{15-18}N$ and of low-lying positive-parity states of $^{16-19}O$ (Ajzenberg-Selove 1982, 1983; Olness et al. 1982; Putt et al. 1983).
Levels of $^{17}$N and $^{18}$N

sd-shell interaction of Chung and Wildenthal (Chung 1976) used by Millener does not reproduce exactly the observed $^{18}$O and $^{19}$O level energies, we also adjust this interaction to fit the observed energies, as was previously suggested for $^{19}$O by Olness et al. (1982).

2. Calculation

For the purpose of the weak-coupling calculation, the assumed $^{17-19}$O state descriptions, relative to a closed-shell $1s^24p^{12}$ $^{16}$O ground state core, are

$^{17}$O, $T = \frac{1}{2}$: \[ \psi(\frac{1}{2}^+) = |d_{3/2}, \frac{1}{2}\rangle, \quad \psi(\frac{3}{2}^+) = |s_{1/2}, \frac{1}{2}\rangle, \]

$^{18}$O, $T = 1$: \[ \psi(0^+) = |d_{3/2}, 0\rangle, \quad \psi(2^+) = |d_{5/2}, 2\rangle, \quad \psi(4^+) = |d_{5/2}, 4\rangle, \]

$^{19}$O, $T = \frac{3}{2}$: \[ \psi(\frac{1}{2}^+) = |d_{3/2}, \frac{1}{2}\rangle, \quad \psi(\frac{3}{2}^+) = |d_{5/2}, \frac{3}{2}\rangle, \quad \psi(\frac{5}{2}^+) = |d_{5/2}, 0\rangle_{s_{1/2}, \frac{1}{2}}. \]

The energy of the state $\psi(J^+)$ is denoted by $E(J)$. The energy $E_i(J)$ of the $^{16-18}$N state of spin $I^-$ obtained by weak coupling of a $p_{1/2}$ proton hole to the state $\psi(J^+)$ is then given by

$^{16}$N: \[ E_i(J) = E(J) + M\{\langle l_j p_{1/2} \rangle \}, \]

$^{17}$N:  

\[
\begin{align*}
E_i(J) &= E(J) + 2 \sum_{J' \neq J} U_1^2(\frac{5}{2}I_1\frac{1}{2}J_1J) M \{\langle d_{5/2} p_{1/2} \rangle \}, \\
E_i(J) &= E(J) + 3 \sum_{J' \neq J} \langle d_{5/2}^2 J \rangle \{\langle d_{5/2}^2 J \rangle \}^2 U_2(\frac{5}{2}I_1\frac{1}{2}J_1J) M \{(d_{5/2} p_{1/2} \rangle \},
\end{align*}
\]

$^{18}$N:  

\[
\begin{align*}
E_i(J) &= E(J) + \frac{1}{6} \sum_{J' \neq J} (2J + 1) M \{\langle d_{5/2} p_{1/2} \rangle \} + M \{(s_{1/2} p_{1/2} \rangle \},
\end{align*}
\]

Here $M \{\langle l_j p_{1/2} \rangle \}$ is the isospin 1 diagonal particle–hole matrix element ($l_j = d_{s/2}$ or $s_{1/2}$), and fractional parentage coefficients and Jahn $U$ coefficients are involved. It is convenient to consider the mean energy of a doublet

$E(J) = \frac{1}{2(2J+1)} \sum_{J' = J} (2J+1) E_i(J),$

and the doublet splitting

$D(J) = E_{J + \frac{1}{2}}(J) - E_{J - \frac{1}{2}}(J).$

If one writes

$S(J,J') = \bar{E}(J) - \bar{E}(J') - \{E(J) - E(J')\},$

for the separation of the mean doublet energies in $^{16-18}$N relative to the separation of the corresponding parent states in $^{17-19}$O, then one obtains the weak-coupling formulae

$^{16}$N: \[
S(\frac{1}{2}, \frac{3}{2}) = X, \quad D(\frac{5}{2}) = A_{32}, \quad D(\frac{1}{2}) = A_{10},
\]
where

$$ \Delta J' = M \{(l_j \frac{1}{2} J_j^1)J_1\} - M \{(l_j' \frac{1}{2} J_j'^1)J_1'\}, $$

$$ X = \frac{3}{4} \Delta_{10} - \frac{1}{2} \Delta_{32} - \Delta_{20}. $$

### Table 1. Values of energy separations and splittings in $^{16-18}\text{N}$

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Quantity</th>
<th>Value (keV)</th>
<th>Adjusted</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{16}\text{N}$</td>
<td>$S(\frac{3}{2}, \frac{5}{2})$</td>
<td>-716</td>
<td>-710</td>
</tr>
<tr>
<td></td>
<td>$D(\frac{5}{2})$</td>
<td>297</td>
<td>-229</td>
</tr>
<tr>
<td></td>
<td>$D(\frac{3}{2})$</td>
<td>278</td>
<td>212</td>
</tr>
<tr>
<td>$^{17}\text{N}$</td>
<td>$S(2, 0)$</td>
<td>-288</td>
<td>96</td>
</tr>
<tr>
<td></td>
<td>$S(4, 0)$</td>
<td>-148</td>
<td>465</td>
</tr>
<tr>
<td></td>
<td>$D(2)$</td>
<td>533</td>
<td>-42</td>
</tr>
<tr>
<td></td>
<td>$D(4)$</td>
<td>500</td>
<td>-272</td>
</tr>
<tr>
<td>$^{18}\text{N}$</td>
<td>$S(\frac{3}{2}, \frac{5}{2})$</td>
<td>-117</td>
<td>-117</td>
</tr>
<tr>
<td></td>
<td>$S(\frac{1}{2}, \frac{1}{2})$</td>
<td>-730</td>
<td>-736</td>
</tr>
<tr>
<td></td>
<td>$D(\frac{5}{2})$</td>
<td>34</td>
<td>560</td>
</tr>
<tr>
<td></td>
<td>$D(\frac{3}{2})$</td>
<td>21</td>
<td>372</td>
</tr>
<tr>
<td></td>
<td>$D(\frac{1}{2})$</td>
<td>200</td>
<td>266</td>
</tr>
</tbody>
</table>

* $^b$ Olness et al. (1982); D. J. Millener, personal communication.

Values of $S(J, J')$ and $D(J)$ obtained from experimental energies (Ajzenberg-Selove 1982, 1983) and from the full shell model calculations of Millener are given in the third and fourth columns of Table 1. The values for $^{18}\text{N}$ make use of energies corresponding to unmixed $2^-$ states of pure parentage, $E(2^-; \frac{1}{2}^- \otimes \frac{3}{2}^-) = 19 \text{ keV}$ and $E(2^-; \frac{1}{2}^- \otimes \frac{5}{2}^+) = 136 \text{ keV}$, which with a mixing matrix element of $\pm 51 \text{ keV}$ give eigenstates with the energies and $12.4\%$ mixing calculated by Millener.

We assume that adjustments to the MK interaction will give adjustments to the values of $S(J, J')$ and $D(J)$ for $^{16-18}\text{N}$ that are related in the same way as in the weak-coupling formulae (1). Thus, adjustment of the MK interaction to make the calculated values of $S(J, J')$ and $D(J)$ for $^{16}\text{N}$ agree with the experimental values implies changes in the values of $X$, $\Delta_{32}$ and $\Delta_{10}$ of $-6, 526$ and $66 \text{ keV}$ respectively, and these imply definite changes in the calculated values for $^{17}\text{N}$ and $^{18}\text{N}$. These adjusted values of $S(J, J')$ and $D(J)$ are given in the last column of Table 1. Adjusted values of the $^{16-18}\text{N}$ level energies are then obtained by using these adjusted values from Table 1, together with experimental values of $^{17-19}\text{O}$ energies. Since the experimental $^{17}\text{O}$ energies were fitted in the calculation of Millener and Kurath (1975), the adjusted $^{16}\text{N}$ energies agree with the experimental values. By using experimental values for
the $^{18,19}$O energies, we are effectively adjusting the sd-shell interaction used in the shell model calculations. The resultant adjusted $^{17,18}$N level energies are shown in Fig. 3. The 2$^-$ levels shown are the result of mixing the states of pure parentage with a mixing matrix element of $\pm 51$ keV, giving a lower 2$^-$ level containing only 1.3% of the $\frac{1}{2}^- \otimes \frac{3}{2}^+$ configuration.

Fig. 3. Spectra of low-lying negative-parity states of $^{17}$N and $^{18}$N calculated using adjusted shell model interactions.

<table>
<thead>
<tr>
<th></th>
<th>$^{17}$N</th>
<th>$^{18}$N</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1/2$^-$</td>
<td>2$^-$</td>
</tr>
<tr>
<td>4250</td>
<td>9/2$^-$</td>
<td></td>
</tr>
<tr>
<td>3733</td>
<td>7/2$^-$</td>
<td></td>
</tr>
<tr>
<td>1135</td>
<td>1$^-$</td>
<td></td>
</tr>
<tr>
<td>869</td>
<td>0$^-$</td>
<td></td>
</tr>
<tr>
<td>2236</td>
<td>5/2$^-$</td>
<td></td>
</tr>
<tr>
<td>1840</td>
<td>3/2$^-$</td>
<td></td>
</tr>
<tr>
<td>568</td>
<td>3$^-$</td>
<td></td>
</tr>
<tr>
<td>458</td>
<td>2$^-$</td>
<td></td>
</tr>
<tr>
<td>89</td>
<td>1$^-$</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>2$^-$</td>
<td></td>
</tr>
</tbody>
</table>

3. Discussion

It is seen from Table 1 and Fig. 3 that the adjusted interactions give better agreement with the experimental level energies of $^{17}$N than did the original Millener calculation, at least as far as the ordering and separations of the doublet members are concerned. This suggests that the predicted $^{18}$N energies for the adjusted interactions should be more accurate than those given by Olness et al. (1982).

There is still the difficulty, however, that the predicted ground state of $^{18}$N is 2$^-$, whereas the observed ground state is 1$^-$ (Olness et al. 1982). An argument for expecting a 1$^-$ ground state has been given by Sheline (1983) on the basis of the collective model. Putt et al. (1983) found the ground state and 580 keV level of $^{18}$N to be weakly populated in the reaction $^{18}$O($^7$Li,$^7$Be)$^{18}$N, while the 121 and 747 keV levels were strongly populated. Since the $\frac{3}{2}^+$ level of $^{19}$O is weakly populated relative to the $\frac{5}{2}^+$ ground state in $^{18}$O(d,p)$^{19}$O (Wiza and Middleton 1966), it is reasonable to suppose that $^{18}$N states of $\frac{1}{2}^- \otimes \frac{3}{2}^+$ structure would be populated weakly compared with those of $\frac{1}{2}^- \otimes \frac{5}{2}^+$ structure in $^{18}$O($^7$Li,$^7$Be)$^{18}$N. The requirements of minimal changes to the $^{18}$N spectrum of Fig. 3, of a 1$^-$ ground state, of weak population of the $\frac{1}{2}^- \otimes \frac{3}{2}^+$ states, and of small mixing of the $\frac{1}{2}^- \otimes \frac{3}{2}^+$ and $\frac{1}{2}^- \otimes \frac{5}{2}^+$ 2$^-$ states lead to suggested spin assignments of 2$^-$, 2$^-$ and 3$^-$ for the observed 121, 580 and 747 keV levels of $^{18}$N respectively. The main change to the spectrum of Fig. 3 is a reduction of the energy of the lower 1$^-$ state by about 200 keV. The non-observation of the $\frac{1}{2}^- \otimes \frac{3}{2}^+$ 0$^-$
and $1^-$ levels in $^{18}$O($^7$Li,$^7$Be)$^{18}$N is not surprising, since the similar reaction $^{16}$O($^7$Li,$^7$Be)$^{16}$N populates the low-lying $0^-$ and $1^-$ levels, of $\frac{1}{2}^- \otimes \frac{3}{2}^+$ structure, very weakly relative to the low-lying $2^-$ and $3^-$ levels, of $\frac{1}{2}^- \otimes \frac{5}{2}^+$ structure (L. K. Fifield, personal communication).

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**References**


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