# Ratio of the ${ }^{4} \mathrm{He}(\gamma, p)^{3} \mathbf{H}$ and ${ }^{4} \mathrm{He}(\gamma, \mathrm{n})^{3} \mathrm{He}$ Cross Sections 

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#### Abstract

Recent measurements of the ${ }^{4} \mathrm{He}(\gamma, \mathrm{p})^{3} \mathrm{H}$ and ${ }^{4} \mathrm{He}(\gamma, \mathrm{n})^{3} \mathrm{He}$ cross sections suggest that their ratio in the giant dipole resonance region is approximately $1 \cdot 6$. This is in marked disagreement with detailed model calculations, which have given a ratio close to unity. We here explain why the calculations yielded this result, by using a simple argument based on independent experimental data and some apparently reasonable assumptions. In order to remove the discrepancy with experiment, it appears to be necessary to relax one of these assumptions and to include contributions from states of higher configurations. A satisfactory fit to the cross sections is then possible, but the required isospinmixing matrix element is much larger than that derived from the Coulomb interaction plus a phenomenological charge-symmetry breaking interaction.


## 1. Introduction

Recent measurements of the total cross sections for ${ }^{4} \mathrm{He}(\gamma, \mathrm{p}){ }^{3} \mathrm{H}$ (McBroom et al. 1982; Calarco et al. 1983b) and ${ }^{4} \mathrm{He}(\gamma, \mathrm{n})^{3} \mathrm{He}$ (Berman et al. 1980; Ward et al. 1981) confirm earlier suggestions that they are appreciably different in the giant dipole resonance (GDR) region, with $R \equiv \sigma(\gamma, \mathrm{p}) / \sigma(\gamma, \mathrm{n}) \approx 1 \cdot 6$ for $E_{\gamma}=25-30 \mathrm{MeV}$ (Calarco et al. 1983a). This departure of $R$ from unity has been attributed by Berman et al. $(1971,1972,1980)$ to isospin mixing, the amount of mixing being determined by a simple formula that was originally derived (Barker and Mann 1957) for the ratio of the ${ }^{12} \mathrm{C}(\gamma, \mathrm{p}){ }^{11} \mathrm{~B}$ and ${ }^{12} \mathrm{C}(\gamma, \mathrm{n}){ }^{11} \mathrm{C}$ cross sections in the GDR region. The isospin mixing deduced in this way is larger than would be expected from Coulomb effects alone, and has been taken to imply a significant charge-symmetry breaking (CSB) component of the nuclear force (Berman et al. 1980; Calarco et al. 1983a).

Detailed calculations of $\sigma(\gamma, \mathrm{p})$ and $\sigma(\gamma, \mathrm{n})$ that included the Coulomb interaction but not a CSB nuclear interaction, for example a coupled-channel continuum shell model calculation (Londergan and Shakin 1972), a channel-corrected $R$-matrix calculation (Bevelacqua 1980), and a recoil-corrected continuum shell model (RCCSM) calculation (Halderson and Philpott 1981), have obtained $R \approx 1$. Moreover, a recent RCCSM calculation that included a reasonable CSB interaction produced values of $R$ only slightly greater than unity (Halderson and Philpott 1983). Here we give an explanation of these results by using a simple argument, based on independent experimental data and some apparently reasonable assumptions, which suggests that $R \approx 1$ irrespective of the amount of isospin mixing and therefore of the size of any

CSB interaction. This argument also indicates an essential difference between the ${ }^{4} \mathrm{He}$ and ${ }^{12} \mathrm{C}$ cases, which makes the use of the Barker and Mann formula inappropriate for calculating $R$ in the ${ }^{4} \mathrm{He}$ case. References to this argument have already been made by Langacker and Sparrow (1982), Calarco et al. (1983a, 1983b) and Barker (1983).

The difference between the calculated value $R \approx 1$ and the experimental value $R \approx 1.6$ suggests that at least one of the apparently reasonable assumptions should be relaxed. The requirements for removal of the discrepancy are discussed on the basis of a fit to the experimental $(\gamma, \mathrm{p})$ and $(\gamma, \mathrm{n})$ total cross sections with a two-level $R$-matrix approximation.

## 2. Simple Argument

In general, the total cross section for a photonucleon reaction may be written as (Lane and Thomas 1958)

$$
\begin{equation*}
\sigma(\gamma, \mathrm{N})=\frac{\pi}{2(2 I+1) k_{\gamma}^{2}} \sum_{J L p s l}(2 J+1)\left|U_{\gamma L p, \mathrm{~N} s l}^{J}\right|^{2}, \tag{1}
\end{equation*}
$$

where $\mathrm{N}(=\mathrm{p}, \mathrm{n})$ labels the nucleon emitted, $I$ is the target spin, $J$ is the total angular momentum of the system after $\gamma$ absorption, $L$ is the multipolarity of the $\gamma$ transition and $p$ is its mode ( 1 for electric, 0 for magnetic), $s$ is the channel spin, $l$ is the relative orbital angular momentum of the emitted nucleon, and $U$ is the collision matrix. For the particular case of ${ }^{4} \mathrm{He}(\gamma, \mathrm{N})$, we have $I=0, J=L$ and $s=0$ or 1. Contributions from other than E1, M1 and E2 transitions are not expected to be significant near the GDR energy. Measurements of the angular distribution and analysing power for the inverse proton capture reaction ${ }^{3} \mathrm{H}\left(\overrightarrow{\mathrm{p}}, \gamma_{0}\right){ }^{4} \mathrm{He}$ are consistent with negligible M1 contributions (as expected since M1 transitions are forbidden to the extent that the ${ }^{4} \mathrm{He}$ ground state belongs to a closed shell configuration) and show that E2 transitions contribute at most $5 \%$ to $\sigma(\gamma, \mathrm{p})$ throughout the GDR region (Weller and Roberson 1980). Similar ${ }^{3} \mathrm{He}\left(\overrightarrow{\mathrm{n}}, \gamma_{0}\right)^{4} \mathrm{He}$ measurements at the single energy $E_{\gamma}=27 \cdot 3 \mathrm{MeV}$ suggest even smaller E2 contributions to $\sigma(\gamma, \mathrm{n})$ (Weller et al. 1982). For E1 radiation, the nucleons are necessarily p wave. Thus, with perhaps $5 \%$ uncertainty, we can take

$$
\begin{equation*}
\sigma(\gamma, \mathrm{N})=\left(3 \pi / 2 k_{\gamma}^{2}\right) \sum_{s}\left|U_{\gamma 11, \mathrm{Ns} 1}^{1}\right|^{2}=\left(3 \pi / 2 k_{\gamma}^{2}\right) \sum_{s}\left|U_{\mathrm{Ns}}\right|^{2}, \tag{2}
\end{equation*}
$$

so that

$$
\begin{equation*}
R=\sum_{s}\left|U_{\mathrm{p} s}\right|^{2} / \sum_{s}\left|U_{\mathrm{n} s}\right|^{2}=\left|\frac{U_{\mathrm{p} 0}}{U_{\mathrm{n} 0}}\right|^{2} \frac{1+\left|U_{\mathrm{p} 1} / U_{\mathrm{p} 0}\right|^{2}}{1+\left|U_{\mathrm{n} 1} / U_{\mathrm{n} 0}\right|^{2}} \tag{3}
\end{equation*}
$$

The nucleon capture measurements (Weller and Roberson 1980; Weller et al. 1982) also give

$$
\begin{equation*}
\left.\left|U_{\mathrm{n} 1} / U_{\mathrm{n} 0}\right|^{2} \approx\left|U_{\mathrm{p} 1}\right| U_{\mathrm{p} 0}\right|^{2} \lesssim 2 \%, \tag{4}
\end{equation*}
$$

so that equation (3) becomes, still with about $5 \%$ uncertainty,

$$
\begin{equation*}
R=\left|U_{\mathrm{p} 0} / U_{\mathrm{n} 0}\right|^{2} . \tag{5}
\end{equation*}
$$

In order to show how the collision matrix element $U_{\mathrm{Ns}}$ depends on N , we use its expression in the $R$-matrix theory (Lane and Thomas 1958)

$$
\begin{equation*}
U_{\mathrm{N} s}=\exp \left\{\mathrm{i}\left(\omega_{\mathrm{N}}-\phi_{\mathrm{N}}\right)\right\} \mathrm{i}\left(2 P_{\mathrm{N}}\right)^{\frac{1}{2}} \sum_{\lambda \mu} \gamma_{\lambda, \mathrm{Ns}} \Gamma_{\mu, \gamma}^{\frac{1}{2}} A_{\lambda \mu} \tag{6}
\end{equation*}
$$

where $A_{\lambda \mu}$ are elements of a matrix in level space:

$$
\begin{equation*}
\left(A^{-1}\right)_{\lambda \mu}=\left(E_{\lambda}-E\right) \delta_{\lambda \mu}-\sum_{c}\left(S_{c}-B_{c}+\mathrm{i} P_{c}\right) \gamma_{\lambda c} \gamma_{\mu c} \tag{7}
\end{equation*}
$$

( $S_{c}$ is the shift factor, $B_{c}$ the boundary condition parameter and $P_{c}$ the penetration factor for the channel $c$ ). The dependence on N is shown explicitly in equation (6): $\omega_{\mathrm{N}},-\phi_{\mathrm{N}}, P_{\mathrm{N}}$ and $\gamma_{\lambda, \mathrm{N} s}$ are the Coulomb phase shift, hard-sphere phase shift, penetration factor and reduced width amplitude of the level $\lambda$, all for the p -wave nucleon channel, while $\Gamma_{\mu, \gamma}$, the E1 $\gamma$-ray width of the level $\mu$, and $A_{\lambda \mu}$ do not depend on N. We write

$$
\begin{equation*}
\gamma_{\lambda, \mathrm{N} s}=\sum_{T} \gamma_{\lambda, T s}\left(\left.\frac{1}{2} \frac{1}{2} M_{T}-M_{T} \right\rvert\, T 0\right) \tag{8}
\end{equation*}
$$

where $M_{T}=+\frac{1}{2}$ or $-\frac{1}{2}$ for $\mathrm{N}=\mathrm{p}$ or n , and $T(=0,1)$ labels the isospin component of the state $\lambda$, and we define reaction amplitudes for the pure-isospin states as

$$
\begin{equation*}
a_{T s}=\sum_{\lambda \mu} \gamma_{\lambda, T s} \Gamma_{\mu, \gamma}^{\frac{1}{2}} A_{\lambda \mu} \tag{9}
\end{equation*}
$$

Then equation (5) may be written as

$$
\begin{equation*}
R=\frac{P_{\mathrm{p}}}{P_{\mathrm{n}}}\left|\sum_{T} a_{T 0}\left(\left.\frac{11}{2} \frac{1}{2}-\frac{1}{2} \right\rvert\, T 0\right)\right|^{2} /\left|\sum_{T} a_{T 0}\left(\left.\frac{11}{2} \frac{11}{2} \frac{1}{2} \right\rvert\, T 0\right)\right|^{2}=\frac{P_{\mathrm{p}}}{P_{\mathrm{n}}} \frac{\left|a_{10}+a_{00}\right|^{2}}{\left|a_{10}-a_{00}\right|^{2}} \tag{10}
\end{equation*}
$$

We now make the apparently reasonable assumptions that
(i) the ${ }^{4} \mathrm{He}$ ground state belongs to the closed shell configuration $1 \mathrm{~s}^{4}$;
(ii) the ${ }^{3} \mathrm{H}$ and ${ }^{3} \mathrm{He}$ ground states belong to the configuration $1 \mathrm{~s}^{3}$;
(iii) only $1^{-}$states of ${ }^{4} \mathrm{He}$ belonging to the configuration $1 \mathrm{~s}^{3} 1 \mathrm{p}$ need to be considered.
Then the wavefunction of the $1^{-}$states at any energy may be written, in the $L S$ coupling representation as

$$
\begin{equation*}
\Psi\left(1^{-}\right)=\sum_{T S} c_{T S} \Psi\left(1 \mathrm{~s}^{3} 1 \mathrm{p}, T, S, L=1, J=1\right)=\sum_{T S} c_{T S} \Psi_{T S} \tag{11}
\end{equation*}
$$

The intrinsic spin $S$ is identical with the channel spin $s$, because the ${ }^{3} \mathrm{H}$ and ${ }^{3} \mathrm{He}$ ground states have $L=0$. The state $\Psi_{00}$ is spurious (Elliott and Skyrme 1955) since it is obtained by operating on the ${ }^{4} \mathrm{He}$ ground state with the c.m. position operator, and therefore any admissible state $\Psi\left(1^{-}\right)$must have $c_{00}=0$ at all energies. It then follows that all $\gamma_{\lambda, 00}$ must vanish, and therefore $a_{00}=0$. Thus equation (10) becomes

$$
\begin{equation*}
R=P_{\mathrm{p}} / P_{\mathrm{n}} \tag{12}
\end{equation*}
$$

We note that $P_{\mathrm{p}}$ and $P_{\mathrm{n}}$ are to be evaluated at nucleon energies corresponding to the same value of $E_{\gamma}$. For $E_{\gamma}=25-30 \mathrm{MeV}$ and channel radii in the range $2-7 \mathrm{fm}$, calculated values of $P_{\mathrm{p}} / P_{\mathrm{n}}$ lie between $1 \cdot 02$ and $1 \cdot 08$. We therefore get

$$
\begin{equation*}
R \approx 1 \cdot 05 \tag{13}
\end{equation*}
$$

with about $6 \%$ uncertainty. This is consistent with the results of the detailed calculations mentioned in Section 1, but is very different from the experimental value $R \approx 1 \cdot 6$.

## 3. Discussion of Previous Calculations

Essential parts of the present argument are that the radiation is predominantly E1, that different channel spins contribute incoherently, that the amount of channel spin 1 contribution is limited by the angular distribution and analysing power measurements, and that the $T=0$ part of the channel spin 0 contribution is spurious and vanishes. The amount of isospin mixing does not enter directly into the argument. This is because the contribution to $R$ from isospin mixing (due to $a_{01}$ ) is incoherent with the main contribution (from $a_{10}$ ) since it has different channel spin. In the ${ }^{12} \mathrm{C}$ case (Barker and Mann 1957), the contributions were coherent because they were assumed to have the same channel spin, which was possible because the $T=0, S=0$ states are not entirely spurious; furthermore the states $\Psi_{12}\left(C_{1} T\right)(T=0,1)$, which are analogous to the states $\Psi_{T 0}$ here, had the same structure apart from their $T$ value and were assumed to be degenerate, so that the reaction amplitudes $a_{T 0}$ in equation (10) could be replaced by the wavefunction expansion coefficients $c_{T 0}$ (the $\alpha_{T}$ of Barker and Mann).

Values of $R$ in agreement with experiment have been achieved in two calculations that included only the Coulomb interaction. Gibson (1972) obtained $R \approx 1 \cdot 8$, but incorrectly assumed that different channel spins contribute coherently to $\sigma(\gamma, \mathrm{N})$. Bevelacqua (1983) obtained $R=1.67$ in a channel-corrected $R$-matrix calculation by the ad hoc introduction of a $1^{-}, T=0$ state at 29 MeV excitation energy in ${ }^{4} \mathrm{He}$; it is not clear that this can be done without violating, for example, the experimental upper limit on the channel spin 1 contribution, given by equation (4) [the $T=0$, $S=1,1^{-}$state of the $1 \mathrm{~s}^{3} 1 \mathrm{p}$ configuration is presumably that observed at $24 \cdot 1 \mathrm{MeV}$ by Grüebler et al. (1981)].

The CSB interaction used by Halderson and Philpott (1983) in their RCCSM calculation, which gave $R \approx 1 \cdot 1$, was a phenomenological potential found by Shlomo (1978) to give a reasonable fit to certain Coulomb displacement energies. The interaction has a large matrix element coupling the two $1^{-}$states of ${ }^{4} \mathrm{He}$ with $S=1$, i.e. $\Psi_{01}$ and $\Psi_{11}$; since, however, the total amount of $S=1$ contribution is limited by experiment, this isospin mixing has little effect on the value of $R$. Likewise, Langacker and Sparrow (1982) derived a CSB potential based on quark mass differences, and found a large ( $\sim 175 \mathrm{keV}$ ) matrix element coupling the states $\Psi_{10}$ and $\Psi_{01}$. These states contribute to $\sigma(\gamma, \mathrm{N})$ with different channel spins, so that even large mixing affects $R$ only slightly. On the other hand, Delsanto et al. (1983) found that they could obtain $R \approx 1.6$ by reducing the pp residual interaction to about one half the nn residual interaction; they did not, however, eliminate spurious states [the earlier conclusion by Delsanto et al. (1977) that elimination of spurious states had little effect
on the value of $R$ was based on calculations in which the only CSB interaction was the Coulomb potential].

## 4. $\boldsymbol{R}$-matrix Fit to Data

The disagreement between our calculated value (13) and the experimental value of $R$ suggests that one or more of the assumptions (i)-(iii) must be invalid, and that other configurations are significant. Certain classes of higher configurations were included in the calculations of Bevelacqua (1980) and Halderson and Philpott (1981, 1983), without leading to values of $R$ very different from unity. One might expect that the assumption most likely to be at fault is (iii), and this for the $1^{-}$states with $T=0$ rather than with $T=1$. In order to obtain a value of $R$ appreciably greater than unity, it is seen from equation (10) that a nonzero value of $a_{00}$ is required. The probable source of this would be a $T=0, S=0,1^{-}$state belonging to configurations higher than the $1 s^{3} 1 \mathrm{p}$ assumed in (iii). Such a state would be expected at a considerably higher energy than the main $T=1,1^{-}$state. If we allow the isospin mixing of these two states, then $R>1$ in the region of the lower level implies $R<1$ near the upper level. This is just the behaviour observed (Calarco et al. 1983a), since the experimental values of $R$ decrease from about 1.6 in the GDR region to less than unity for $E_{\gamma}$ near 40 MeV .

We investigate to what extent a quantitative fit to the experimental values of $\sigma(\gamma, \mathrm{p})$ and $\sigma(\gamma, \mathrm{n})$ is possible with this simple picture of two $1^{-}$levels with $S=0$, the lower mainly $T=1$ and the higher mainly $T=0$, by using a two-level $R$-matrix approximation (Lane and Thomas 1958) with two-state isospin mixing (Barker 1966). The calculated cross sections are given by equations (2), (6) and (7), with $s=0$ only and with $\lambda=\mathrm{a}, \mathrm{b}$. We put $\Gamma_{\bar{\lambda}, \gamma}^{\frac{1}{2}}=E_{\gamma}^{3 / 2} \gamma_{\lambda \gamma}$. The sum in equation (7) is taken over the $s=0$ proton and neutron channels only, labelled $c=\mathrm{p}, \mathrm{n}$ (the $\mathrm{d}+\mathrm{d}$ channel cannot contribute to $J^{\pi}=1^{-}$with $s=0$ ). We put

$$
\begin{equation*}
\Psi_{\mathrm{a}}=\alpha \Psi_{0}+\beta \Psi_{1}, \quad \Psi_{\mathrm{b}}=\beta \Psi_{0}-\alpha \Psi_{1}, \tag{14a,b}
\end{equation*}
$$

where $\alpha^{2}+\beta^{2}=1$, so that

$$
\begin{array}{ll}
\gamma_{\mathrm{ap}}=2^{-\frac{1}{2}}\left(\alpha \gamma_{0}+\beta \gamma_{1}\right), & \gamma_{\mathrm{bp}}=2^{-\frac{1}{2}}\left(\beta \gamma_{0}-\alpha \gamma_{1}\right), \\
\gamma_{\mathrm{an}}=2^{-\frac{1}{2}}\left(-\alpha \gamma_{0}+\beta \gamma_{1}\right), & \gamma_{\mathrm{bn}}=2^{-\frac{1}{2}}\left(-\beta \gamma_{0}-\alpha \gamma_{1}\right), \\
\gamma_{\mathrm{a} \gamma}=\beta \gamma_{1 \gamma}, & \gamma_{\mathrm{b} \gamma}=-\alpha \gamma_{1 \gamma} . \tag{15e,f}
\end{array}
$$

The adjustable parameters are $E_{\mathrm{a}}, E_{\mathrm{b}}, \gamma_{0}, \gamma_{1}, \gamma_{1 \gamma}^{2}$ and $\alpha$. The values of $S_{c}$ and $P_{c}$ in equations (6) and (7) depend on the choice of the nucleon channel radius $a_{c}$. Previous $R$-matrix type calculations for $A=4$ systems have used $a_{c}=4 \mathrm{fm}$ (Werntz and Meyerhof 1968) and $a_{c}=7 \cdot 2 \mathrm{fm}$ (Halderson and Philpott 1979, 1981, 1983). The values of $B_{c}$ may be chosen arbitrarily (Barker 1972). Since our simple model cannot reproduce the experimental increase in $R$ to values greater than unity at high energies (Calarco et al. 1983a), we fit the data over the energy range from threshold to 45 MeV only.

With values of $a_{c}$ and $B_{c}$ from Werntz and Meyerhof (1968) ( $a_{c}=4 \mathrm{fm}, B_{\mathrm{p}}$ $=-0 \cdot 58, B_{\mathrm{n}}=-0 \cdot 68$ ), we obtain a best fit with $E_{\mathrm{a}}=30 \cdot 3 \mathrm{MeV}, E_{\mathrm{b}}=42 \cdot 5 \mathrm{MeV}$,
$\gamma_{0}=2.47 \mathrm{MeV}^{\frac{1}{2}}, \gamma_{1}=2.49 \mathrm{MeV}^{\frac{1}{2}}, \gamma_{1 \gamma}^{2}=0.313 \times 10^{-6} \mathrm{MeV}^{-2}$ and $\alpha=0.244$. The corresponding energies $E_{T}$ of the pure-isospin states are $E_{1}=31.0 \mathrm{MeV}$ and $E_{0}=41 \cdot 8 \mathrm{MeV}$. Werntz and Meyerhof fitted ${ }^{3} \mathrm{H}(\mathrm{p}, \mathrm{n})^{3} \mathrm{He}$ data by including three $1^{-}$states of the $1 \mathrm{~s}^{3} 1 \mathrm{p}$ configuration with $T S=01,10$ and 11 and by allowing mixing of the two $T=1$ states (but no isospin mixing). They obtained two solutions, one with the upper $T=1$ state mainly $S=0$ (WMI), the other with it mainly $S=1$ (WMII). In each solution, the upper eigenstate is at 30.5 MeV , and therefore corresponds to our $T=1$ state. Since we have included only $S=0$ states, this correspondence therefore favours WMI [other evidence also supports WMI-see Fiarman and Meyerhof (1973)]. Werntz and Meyerhof used $\gamma^{2} \equiv \gamma_{\mathrm{p}}^{2}+\gamma_{\mathrm{n}}^{2}=5 \cdot 5 \mathrm{MeV}$ for this level, which agrees well with our value of $\gamma_{1}^{2}=6 \cdot 2 \mathrm{MeV}$. The parameter values depend on the choice of $B_{c}$, and those with most physical significance are obtained when each $B_{c}$ is set equal to a mean value of the shift factor $S_{c}$ in the energy range of interest (Barker 1972). In the present case, this does not significantly change the values of $\gamma_{0}, \gamma_{1}$ or $\gamma_{1 \gamma}^{2}$, but gives $E_{\mathrm{a}}=27.4 \mathrm{MeV}, E_{\mathrm{b}}=39.6 \mathrm{MeV}$ and $\alpha=0 \cdot 218$. The corresponding value $E_{1}=28 \cdot 0 \mathrm{MeV}$ agrees with the value $E_{\mathrm{R}}=28.0 \mathrm{MeV}$ given by Werntz and Meyerhof.


Fig. 1. Values of $(a)$ the ${ }^{4} \mathrm{He}(\gamma, \mathrm{p})^{3} \mathrm{H}$ total cross section, (b) the ${ }^{4} \mathrm{He}(\gamma, \mathrm{n})^{3} \mathrm{He}$ total cross section, and (c) their ratio, as functions of the $\gamma$-ray energy. Hatched areas indicate experimental values (Calarco et al. 1983a). Curves are fits to the cross sections over the energy range from threshold to 45 MeV with a two-level $R$-matrix approximation. Solid curves are the best fit for a channel radius of 7 fm . Dashed curves are a fit in which the values of two parameters are restricted by model considerations.

Better fits to the $\sigma(\gamma, \mathrm{p})$ and $\sigma(\gamma, \mathrm{n})$ values are obtained for larger values of $a_{c}$. The solid curves in Fig. 1 show the best fit for $a_{c}=7 \mathrm{fm}$, the corresponding parameter
values being (for the mean $B_{c}$ values) $E_{\mathrm{a}}=27.9 \mathrm{MeV}, E_{\mathrm{b}}=38.9 \mathrm{MeV}$, $\gamma_{0}=1.83$ $\mathrm{MeV}^{\frac{1}{2}}, \gamma_{1}=1.63 \mathrm{MeV}^{\frac{1}{2}}, \gamma_{1 \gamma}^{2}=0.273 \times 10^{-6} \mathrm{MeV}^{-2}$ and $\alpha=0.230$. The fit to the data seems satisfactory, but we should also consider whether the parameter values are reasonable in view of the structures we have assumed for the two states involved.

The energy and reduced width of the $T=1$ state agree with the Werntz and Meyerhof (1968) values obtained from fitting independent data. This $T=1$ state is assumed to be the $S=0$ state of the $1 \mathrm{~s}^{3} 1 \mathrm{p}$ configuration, which has an excitation of $1 \hbar \omega$ above the lowest configuration $1 \mathrm{~s}^{4}$. The $T=0$ state belongs to higher configurations, of which the most important are expected to be those with $3 \hbar \omega$ excitation.* The energy difference of 11 MeV obtained in the fit is not unreasonable for states belonging to configurations differing by $2 \hbar \omega$, considering the highly unbound nature of these states. The $3 \hbar \omega$ configurations $1 \mathrm{~s}^{3} 2 \mathrm{p}, 1 \mathrm{~s}^{2} 1 \mathrm{p} 1 \mathrm{~d}, 1 \mathrm{~s}^{2} 1 \mathrm{p} 2 \mathrm{~s}$ and $1 \mathrm{~s} 1 \mathrm{p}^{3}$ contain seven $1^{-}$basis states with $T=0$ and $S=0$, five combinations of them being spurious and only two non-spurious. The large value of $\gamma_{0}$ obtained in the fit suggests a large $1 \mathrm{~s}^{3} 2 \mathrm{p}$ component in the $T=0$ state; the maximum fraction of $1 \mathrm{~s}^{3} 2 \mathrm{p}$ in a nonspurious state is $3 / 8$, for the state of maximum orbital symmetry. One would therefore expect a value of $\gamma_{0}$ considerably smaller than that of $\gamma_{1}$ (although some enhancement of $\gamma_{0} / \gamma_{1}$ as compared with the ratio of spectroscopic amplitudes could be attributed to the additional node in the 2 p radial wavefunction). The fit to the data does not determine $\gamma_{0}$ very precisely, and a fit with $\gamma_{0} / \gamma_{1}$ constrained to equal $(3 / 8)^{\frac{1}{2}}$ has an r.m.s. deviation only $39 \%$ larger than the best fit.

The isospin-mixing matrix element is given by

$$
\begin{equation*}
V_{01}=-\alpha \beta\left(E_{\mathrm{b}}-E_{\mathrm{a}}\right) \tag{16}
\end{equation*}
$$

and has the value -2.46 MeV for the best-fit parameter values. This is much larger than any known Coulomb mixing matrix element (Barker 1978). Again the value of $V_{01}$ is not fixed very precisely in the fit; for example, for $\gamma_{0} / \gamma_{1}=(3 / 8)^{\frac{1}{2}}$ and $\alpha=0 \cdot 15$, the r.m.s. deviation is $41 \%$ above the best-fit value and $V_{01}$ is -1.32 MeV . This fit is shown by the dashed curves in Fig. 1; it suggests that magnitudes of $V_{01}$ much smaller than this would not be consistent with these data. There is a tendency for calculated Coulomb mixing matrix elements to be larger between states of different configurations than between states of the same configuration (Barker 1957). In the present case, the matrix elements between the $T=1, S=0$ state of the $1 \mathrm{~s}^{3} 1 \mathrm{p}$ configuration and the $T=0, S=0$ basis states of the $3 \hbar \omega$ configurations are moderately large, of the same order as that between the $T=0$ and $T=1, S=1,1^{-}$states of the $1 \mathrm{~s}^{3} 1 \mathrm{p}$ configuration. This refers to the internal contributions; the surface contributions are small because the levels are far above the nucleon-channel thresholds and the barriers are small, leading to approximately equal proton and neutron shift factors (Barker 1978). The matrix element coupling the $T=1, S=0$ state to any of the five spurious combinations of the $T=0$ basis states must vanish (because the Coulomb interaction is a function of relative coordinates only), so that all the strength resides in the two non-spurious states. The matrix element for the non-spurious state of maximum orbital symmetry is $(5 / 7)^{\frac{1}{2}}$ of the maximum possible value. For harmonic oscillator radial wavefunctions with length parameter $b$, this matrix element is $-(5 / 54 \pi)^{\frac{1}{2}} e^{2} / b$, which has the value -148 keV for $b=1.67 \mathrm{fm}$ (Halderson and

[^0]Philpott 1979). This magnitude is large compared with the internal contributions to Coulomb matrix elements calculated for many cases of isospin mixing in ${ }^{8} \mathrm{Be},{ }^{12} \mathrm{C}$ and ${ }^{16} \mathrm{O}$ (Barker 1966, 1978), but is much smaller than the values required to fit the present data.

We therefore consider the contribution from an additional CSB interaction of the phenomenological form suggested by Shlomo (1978):

$$
\begin{equation*}
V^{\mathrm{CSB}}\left(r_{12}\right)=\left(V_{0}^{\sigma} m_{\sigma}^{3} / 4 \pi\right) Y\left(m_{\sigma} r_{12}\right)+\sigma_{1} \cdot \sigma_{2}\left(V_{0}^{\pi} m_{\pi}^{3} / 4 \pi\right) Y\left(m_{\pi} r_{12}\right), \tag{17}
\end{equation*}
$$

where $Y(x)=\exp (-x) / x$. With $m_{\sigma}=2.79 \mathrm{fm}^{-1}$ and $m_{\pi}=0.684 \mathrm{fm}^{-1}$, Shlomo found a reasonable fit to the differences between the pp and nn values of the ${ }^{1} \mathrm{~S}_{0}$ scattering length and effective range and to the Coulomb displacement energies for $A=3,15,17,18,39,41$ and 42 , with $V_{0}^{\sigma}=65 \mathrm{MeV} \mathrm{fm}^{3}$ and $V_{0}^{\pi}=20 \mathrm{MeV} \mathrm{fm}^{3}$. With harmonic oscillator radial wavefunctions, the matrix elements of $V^{\text {CSB }}$ between the $T=1, S=0$ state of $1 \mathrm{~s}^{3} 1 \mathrm{p}$ and the $T=0, S=0,3 \hbar \omega$ basis states are all proportional to the relative matrix element $\left\langle 1 \mathrm{~s}\left\|V^{\mathrm{CSB}}(r)\right\| 2 \mathrm{~s}\right\rangle$, with $\sigma_{1} \cdot \sigma_{2}=-3$, the coefficients being the same as those that occur in the Coulomb matrix elements as coefficients of $\left\langle 1 \mathrm{~s}\left\|e^{2} / r\right\| 2 \mathrm{~s}\right\rangle$. Thus the additional CSB interaction leads to an enhancement of the Coulomb matrix element by a factor independent of the particular combination of $3 \hbar \omega$ basis states, this factor being $3 \cdot 25$ for Shlomo's parameter values. Thus we obtain a mixing matrix element of -480 keV , which is still too small to fit the data.

## 5. Summary

There is disagreement between the experimental value of the ratio of the ${ }^{4} \mathrm{He}(\gamma, \mathrm{p})^{3} \mathrm{H}$ and ${ }^{4} \mathrm{He}(\gamma, \mathrm{n})^{3} \mathrm{He}$ total cross sections in the GDR region, $R \approx 1 \cdot 6$, and the results of detailed calculations, $R \approx 1$. The reason why these calculations could not obtain larger values of $R$ is suggested here, the explanation making use of independent experimental data and some apparently reasonable assumptions. In order to remove the discrepancy with experiment, it appears to be necessary to relax one of the assumptions, that only $1^{-}$states of ${ }^{4} \mathrm{He}$ of the configuration $1 \mathrm{~s}^{3} 1 \mathrm{p}$ need be considered, and to include $T=0,1^{-}$states of higher configurations. A two-level fit gives qualitative agreement with the observed cross sections, but the isospin-mixing matrix element required for the fit is very large, and cannot be explained by the Coulomb interaction alone. A charge-symmetry breaking interaction, chosen to remove the Okamoto-Nolen-Schiffer anomaly and to fit other data in light nuclei, produces a large matrix element, due to the concentration of the strength from many basis states into a few non-spurious combinations of them; this is still too small to fit the experimental value of $R$. If full use is made of the uncertainties in the data (Calarco et al. 1983a) and in the CSB interaction (Shlomo 1978), it is conceivable that the cross sections may be fitted over an extended energy range with a two-level $R$-matrix approximation with reasonable parameter values, corresponding to widely spaced $T=1$ and $T=0,1^{-}$states of different configurations. A more sophisticated calculation would be required, however, to show if this is the case.

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[^0]:    * Similar considerations have been reported recently by van Hees (1984).

