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The Possibility of a Photospheric Dynamo*

P. R. Wilson

Department of Applied Mathematics,
University of Sydney, Sydney, N.S.W. 2006.

Abstract

Several recent observations are discussed which suggest that the current model for the emergence and decay of photospheric flux does not provide a complete account of all the processes involved. An elementary two-dimensional dynamo is discussed and it is shown how this may be adapted to photospheric conditions in order to provide a plausible kinematic account of these observations.

1. Introduction

For some time now we (see e.g. Wilson and Simon 1983) have been greatly puzzled by a feature of our observations on the evolution of small-scale magnetic field knots; i.e. that flux of a given sign appears to grow, both in intensity and in area (i.e. in total flux), without the appearance of an equivalent quantity of negative flux in the vicinity as part of the expected pattern of emerging flux loops. The possibility that this result may be due, at least in part, to instrumental polarization, to atmospheric seeing, to variations in equivalent width, or to other effects has been considered in some detail (Simon and Wilson 1985), but none of these possible explanations were considered satisfactory.

All the details of these observations will not be repeated here. They have been discussed with many people, some of whom simply reject them while others, e.g. K. L. Harvey, H. P. Jones and J. K. Lawrence, admit to having seen similar phenomena in their own data, but which they consigned to the 'too hard basket'. As a result of a long discussion with A. M. Title and his group in 1983, some of their magnetograph data from Sacramento Peak were re-examined, and Topka and Tarbel (1983) found both flux increases and decreases in small unipolar magnetic regions with no obvious changes in any nearby opposite-polarity features.

On a larger scale, Wallenhorst and Howard (1981) have found that the flux associated with the decay of a spot group actually decreases *in situ*, and that this is due to a real change in the field rather than to dispersion by random walk or other processes. More recently, van Ballegooijen *et al.* (1985) discussed the formation and decay of active regions and, although they regard the formation process as relatively

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well understood, they concluded that some process, other than random walk and flux cancellation, is necessary to explain decay in which flux seems to disappear without any apparent motion towards neutral lines. The same mode of flux decay has also been observed in small-scale fields in our own data and also that of Topka and Tarbel (1983). However, here it appears to be simply the reverse of the growth process; a small knot of flux of one polarity appears to grow and then decay *in situ* without any lateral motion or apparent involvement with flux of opposite polarity. Thus, the question is raised of whether not only the decay but also the formation processes are completely and satisfactorily described by the current flux rope model.

Again this model requires that the large-scale (i.e. unipolar) field arises from the decay and random walk of active region fields. However, McIntosh (1981) reported that the first major large-scale field patterns of cycle 20 appeared in the southern hemisphere before the formation of any major active region. Indeed, the first major spot group did not appear until *four rotations* after the establishment of the large-scale field and, while the group appeared at the appropriate neutral line of the large-scale field (in order to conform to Hale's law), it did not significantly alter the development of the large-scale field which proved to be one of the major features of that cycle. (Thus, according to existing concepts, McIntosh's result tells us that the cart appeared before the horse.)

All of these observations therefore suggest that even if our current picture of the emergence of flux in the form of inert loops and their subsequent decay by random walk and cancellation is not entirely wrong, then there must at least be a serious omission. Altschuler (1973) has suggested that non-potential magnetic fields (i.e. electric currents) might be generated in the photospheric layers and that activity phenomena are the manifestations of local changes in these photospheric currents. More recently, Akasofu (1984) has suggested that, in place of the rising flux rope hypothesis, a photospheric dynamo associated with shear and vortex motions in the photosphere can supply the power needed for the formation of sunspot loops from the observed background (weak) field. Although these ideas have not yet been worked out in any detail and, in particular, although the rising flux rope hypothesis for the formation of the active regions cannot be rejected out of hand, the value of Akasofu's suggestion is in pointing out that the idea of the rising flux rope is simply a hypothesis and not an established fact. Further, in view of these recent observations, it is important to re-examine the basis of all our ideas concerning the evolution of magnetic fields, i.e. the hydromagnetic equation and, in particular, to see whether the growth and decay of photospheric fields *in situ* may be influenced by some form of local dynamo amplification process.

2. The Hydromagnetic Equation

Based on the complete form of the hydromagnetic equation as given by Pert (1977), for example, it is straightforward to show (Wilson 1984) that the rate of change of flux Φ associated with a given area of the photosphere S, such as that associated with a well-guided magnetograph, is given by

$$\frac{\mathrm{d}\Phi}{\mathrm{d}t} = \int_S \operatorname{curl}\left(\boldsymbol{V}_\mathrm{e} \times \boldsymbol{B} - \eta \operatorname{curl} \boldsymbol{B} + \frac{c}{e n_\mathrm{e}} \nabla p_\mathrm{e} + \frac{c}{e} \boldsymbol{\beta} . \nabla (k T_\mathrm{e}) \right) \mathrm{d}S . \qquad (1)$$

Here $\boldsymbol{V}_\mathrm{e}$ is the velocity of the electron gas, $\boldsymbol{B}$ the magnetic induction vector, η the magnetic diffusivity, and n_e, p_e and T_e the electron density, pressure and temperature, while β is the thermoelectric tensor (Braginskii 1965). The third and fourth terms in this integral, i.e. the pressure gradient term and the thermoelectric term, have been discussed elsewhere (Wilson 1984). For density and temperature gradients which appear to be reasonable under solar conditions, these terms contribute a growth rate of only a few G per year to a typical flux knot and thus cannot account for the observed flux changes. This is consistent with earlier estimates by Kopecky and Kuklin (1971). Thus, the change in the flux crossing S will be determined essentially by the first two terms. In the first we have $\boldsymbol{V}_e = \boldsymbol{u} + \boldsymbol{U}_e$, where $\boldsymbol{u}$ is the plasma velocity and

$$\boldsymbol{U}_e = -\boldsymbol{J}/e n_e,$$

$\boldsymbol{J}$ being the current density [$=(1/4\pi)\mathrm{curl}\,\boldsymbol{B}$]. This term thus describes the induction of new flux due to the motion of electrons relative to the existing field, while the second describes changes due to ohmic decay. Although there are many different definitions of dynamo action (see e.g. Moffatt 1978; Hide 1981), it is essentially determined by the competitive interaction between these two terms.

3. An Infinite Uniform Field

In order to understand the model presented here, we consider first an infinite uniform field $B_0\,\boldsymbol{k}$ which is perturbed by a velocity field defined by

$$\boldsymbol{u} = u_1(z/z_0)(\cos\omega t)\boldsymbol{i} + u_2(x/x_0)(\sin\omega t)\boldsymbol{k}. \tag{2}$$

Here $\boldsymbol{i}$ and $\boldsymbol{k}$ are orthogonal unit vectors, u_1 and u_2 are the velocity amplitudes and x_0 and z_0 are suitable cartesian scaling factors.

We seek a solution to the hydromagnetic equation

$$\partial\boldsymbol{B}/\partial t = \mathrm{curl}\{(\boldsymbol{u}\times\boldsymbol{B}) - \eta\,\mathrm{curl}\,\boldsymbol{B}\} \tag{3}$$

of the form

$$\boldsymbol{B} = b_1(t)\,\boldsymbol{i} + \{B_0 + b_2(t)\}\,\boldsymbol{k}, \tag{4}$$

with $b_1(0) = 0$ and $b_2(0) = 0$.

Substituting equations (2) and (4) into (3) gives the exact equations

$$\mathrm{d}b_1/\mathrm{d}t = \{B_0 + b_2(t)\}(u_1/z_0)\cos\omega t, \tag{5}$$

$$\mathrm{d}b_2/\mathrm{d}t = b_1(t)(u_2/z_0)\sin\omega t, \tag{6}$$

and these lead to second order differential equations of the Mathieu-type for b_1 and b_2. Although they cannot be solved analytically, they yield well-behaved solutions for a wide range of parameters u_1, u_2, z_0 and x_0, an example of which is shown in Fig. 1. If $b_2(t)/B_0 \ll 1$, then we have the approximate analytic solutions

$$b_1(t) = (B_0\,u_1/\omega z_0)\sin\omega t, \tag{7}$$

$$b_2(t) = (B_0\,u_1\,u_2/2\omega^2 z_0^2)(\omega t - \tfrac{1}{2}\sin 2\omega t), \tag{8}$$

and it is clear that the exact numerical solutions have a similar behaviour, except that the amplitude of $b_1(t)$ increases slowly with t (dashed curves in Fig. 1), while $b_2(t)$ fluctuates about an approximately exponential growth (solid curves). It should be emphasized, however, that these solutions are accurate only within the kinematic approximation, i.e. until the growth of the perturbed field requires a consideration of the interaction of $\boldsymbol{B}$ on $\boldsymbol{u}$ through the momentum equation, which we do not consider here.

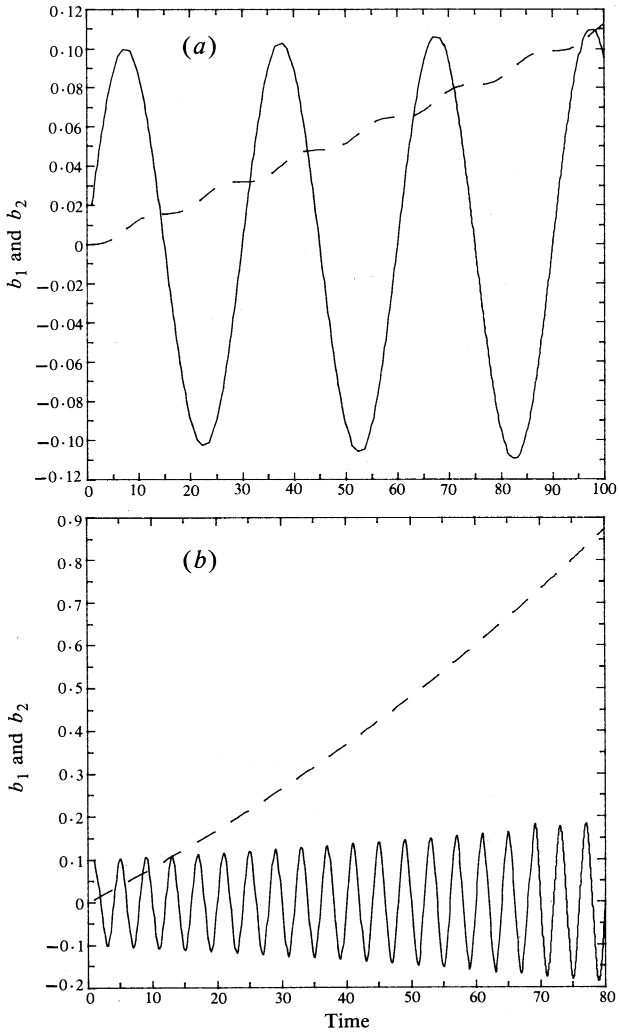

Fig. 1. Solutions to equations (5) and (6) for $u_1 = 0{\cdot}01$, $u_2 = 0{\cdot}01$, $\omega = 0{\cdot}1$ and $B_0 = 1$, where one unit on the time scale corresponds to (a) 0·21 s and (b) 1·68 s. The solid curves are for b_1 and the dashed curves for b_2.

4. A Cylindrical Model

Of course, the result in Section 3 is not very controversial, but neither is the cartesian model with linearly increasing amplitudes particularly appropriate for solar velocity fields. However, let us consider now a uniform (weak) magnetic field $B_0\,\boldsymbol{k}$ within the region $r < r_2$ and zero outside and assume that this is perturbed by the velocity field defined in polar coordinates by

$$\boldsymbol{u} = u_1 \sin(z/z_0)\, \boldsymbol{n}(r,\phi) \cos \omega t + u_2 f(r)(\sin\phi \sin \omega t)\boldsymbol{k}\,, \tag{9}$$

where

$$\boldsymbol{n}(r,\phi) = g(r)(\sin\phi)\,\hat{\boldsymbol{r}} + \{r\,g(r)\}'(\cos\phi)\hat{\boldsymbol{\phi}}\,.$$

The functions $f(r)$ and $g(r)$ need not be specified exactly but should be zero at the origin, increase monotonically to unity at some value $r = r_0$ and thereafter decrease to zero at $r = r_1$ where, for mathematical convenience, we take $r_1 < r_2$. We note that in the region defined by $z/z_0 \ll 1$, $f(r) \approx r/r_0 \approx 1$, $g(r) \approx 1$ and $\phi = \frac{1}{2}\pi$, the velocity corresponds to that defined by equation (2). However, this velocity field is confined within the region $r \leqslant r_1$ and may be taken to be uniform outside the region $-z_0/\pi < z < z_0/\pi$. Thus it describes an oscillatory transverse shear in the direction $\boldsymbol{n}(r,\phi)$, which is approximately the $\phi = \frac{1}{2}\pi$ direction, together with a non-axisymmetric vertical oscillation which is 90° out of phase, as shown in Fig. 2.

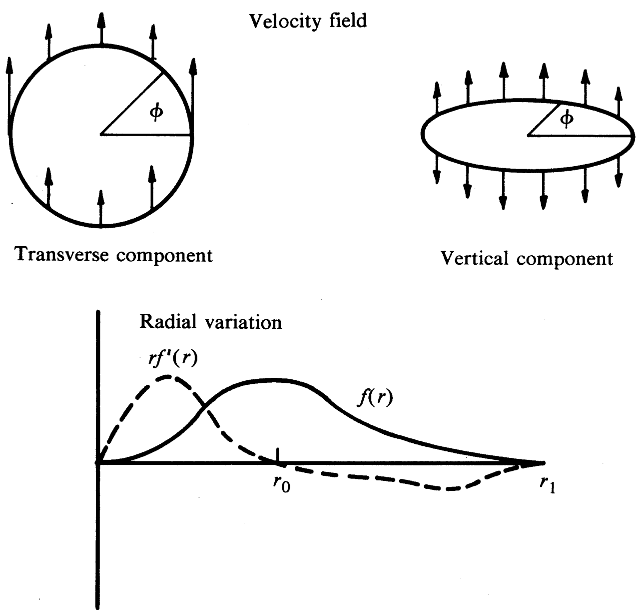

Fig. 2. Relative vectors for the transverse and vertical components of the velocity as a function of ϕ. Possible forms for $f(r)$ and $r f'(r)$ are also illustrated.

Although not typical of photospheric fields, these components may well be present from time to time in the Fourier components of a convective eddy when in the presence of a large-scale shear.

We now seek an appropriate solution of equation (3) of the form

$$B = \{ b_1(t) \cos(z/z_0) + b_2(t) f(r) \sin\phi \sin(z/z_0) \} n(r,\phi)$$
$$+ \{ B_0 + b_2(t)\, h(r,\phi) \cos(z/z_0) \} k\,, \tag{10}$$

where

$$h(r,\phi) \doteq z_0[f(r) \{ r\, g(r) \}' \cos^2\phi + r f'(r)\, g(r) \sin^2\phi]/r\,. \tag{11}$$

If $b_1(t)$ and $b_2(t)$ are as given by equations (7) and (8), the condition $b_1(t) \ll B_0$ for all t implies that $u_1/\omega z_0 \ll 1$. According to equation (8), $b_2(t)$ increases at first as $(\omega t)^3$ and subsequently as ωt. However, provided $\omega t \not\gtrless 1$, the condition $u_2/\omega z_0 \ll 1$ implies that $b_2(t) \ll b_1(t)$.

Thus, if equation (10) is substituted into equation (3) under these conditions, the left side is of order $b_2(t)\,\omega$ and this is equal to the right side provided that terms of order $b_2(t)\, u_2/z_0$ and the diffusion term are negligible compared with $b_2(t)\,\omega$. The first term is automatically satisfied by the assumed conditions while the diffusion term, which for $g(r) = 1$ becomes

$$\eta\{ b_1(t)/z_0^2 \} \cos(z/z_0)\{ (\sin\phi)\, \hat{r} + (\cos\phi)\hat{\phi} \}\,,$$

is negligible compared with $b_2(t)\,\omega$ provided that

$$\eta \ll u_2\, z_0\,.$$

Thus, to the accuracy of the approximation, the induced field consists of a transverse oscillatory component proportional to $b_1(t)$ and a monotonically increasing component proportional to $b_2(t)$, the form of which may be inferred by studying the r, ϕ independence of $h(r, \phi)$ (equation 11) and noting that, of course, div $B = 0$ for both oscillatory and increasing components.

Within the region $r \leqslant r_0$, $f'(r)$ and $\{ r\, g(r) \}'$ are positive and the induced field is thus positive and in the k direction when $z/z_0 \ll 1$. However, for $r_0 < r < r_1$, $f'(r)$ is negative while $\{ r\, g(r) \}'$ may be either positive or negative so that the induced field will be negative for most, but not all, values of ϕ. Reference to the transverse component at $z/z_0 \approx 1$ and the zero divergence condition shows that the increasing component of the induced field takes the form of a system of field loops, most having the positive footpoints within $r \leqslant r_0$ and negative in appropriate ranges of ϕ in $r_0 < r < r_1$, the loops being entirely contained within the volume $r < r_1$; $-\frac{1}{2}\pi z_0 < z < \frac{1}{2}\pi z_0$. This is illustrated in Fig. 3.

Of course, the resultant field at, for example, $t = \pi/\omega$ will be given by the vector sum of $B_0\, k$ and the induced field and, since the solution is valid only for $b_2 \ll B_0$, will correspond only to a deformation of the initial uniform field, the field being increased within r_0 and non-uniformly decreased in $r_0 < r < r_1$. Only if the perturbed field continues to increase and the effects of finite resistivity permit reconnection across an X-type neutral point in the annular region between r_0 and r_1 can dynamo action be seen to have occurred.

5. Comparison with Observation

We now consider whether the flux changes obtained above permit an explanation of the observations by Wilson and Simon (1983). Consider first the small knot of

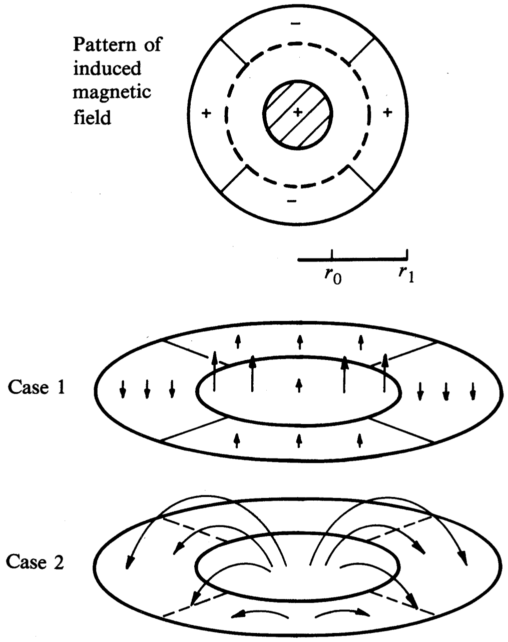

Fig. 3. Polarity pattern of the increasing component of the induced field illustrated as a function of ϕ in plan (top), in vector form at $z = 0$ (case 1) and at $z \approx \frac{1}{2} z_0 \pi$ (case 2).

magnetic flux enclosed within the box in Figs 2 and 3 of their paper and the subsequent development of this knot as shown in their Figs 4, 5 and 7. Initially (i.e. at 15:22:00) one intense and several weaker knots are observed in the field. The strongest parts of these knots have an intensity of $\sim$500 G against a background 'noise' of order $\pm$100 G (1 G $\equiv 10^{-4}$ T). Although some areas of negative flux are observed in the region outside the box, they rarely exceed this noise level. During the next 90 min the original bright (i.e. intense) knot and several of the weaker knots have greatly increased in both intensity and area, a particularly good example being the knot in the upper right centre which can just be seen at 15:22:00 and by 16:56:30 is the dominant feature of the pattern. During this time the rate of growth of field intensity within this region is of order 100 G hr^{-1}. Neglecting the other bright features, we postulate a velocity field, such as that given by equation (9), centred on this knot and extending over a region of transverse dimension r_0 corresponding to the transverse dimensions of the knot, i.e. $\sim$1000 km, and of vertical extent z_0, where z_0 is assumed to be comparable with r_0. Thus the increasing positive flux predicted by the model for $r < r_0$ would account for the brightening (i.e. intensification) of this feature. Substituting equation (10) into (1) yields for the rate of growth of flux within S, taken to be a circle of radius r_0,

$$\mathrm{d}\Phi/\mathrm{d}t = \tfrac{1}{2}\pi B_0(r_0\, u_1\, u_2/\omega z_0) f(r_0) \sin^2 \omega t . \tag{12}$$

Taking $B_0 \sim 10^2$ G, $u_1 = u_2 = 1{\cdot}0\ \mathrm{km\,s^{-1}}$, $r_0 = 1000$ km, $z_0 = 1000$ km and $\omega = 10^{-2}\ \mathrm{s^{-1}}$ gives a growth rate of $8\times10^{13}\ \mathrm{Mx\,s^{-1}}$ (1 Mx $\equiv 10^{-8}$ Wb) or, averaging over the region $r \leqslant r_0$, 20 G hr^{-1} which is comparable with the observed rate.

Although the dynamical problem of the interaction between the velocity and magnetic fields cannot be discussed here, it is worth noting that the kinematic energy density of the postulated velocity fields at unit optical depth is $\sim 10^3$ erg cm^{-3} (1 erg $\equiv 10^{-7}$ J), which is comparable with the magnetic energy density of the initial field of $\sim 4\times10^2$ erg cm^{-3}.

It is important to note, however, that while the model may, in part, explain some of the recent observations, it does not yet constitute dynamo action since the net result is an increase in flux concentration in one region with corresponding decreases in others. Within the limits of the approximation, no new field lines can be identified within the region. Only if the induced field continues to increase and reconnections across an X-type neutral point take place can new flux be generated within the region.

However, for simplicity, only a uniform background field has been considered for the model and, as is well known, solar magnetic fields tend to occur in discrete knots or flux tubes. In a more recent study the effects of oscillatory velocity fields on spatially varying initial fields have been investigated and these will be reported in a later paper.

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References

Akasofu, S.-I. (1984). *Planet. Space Sci.* **32**, 1469.
Altschuler, M. D. (1973). Proc. IAFE Flare Conf., Buenos Aires, Argentina.
Braginskii, S. I. (1965). *In* 'Reviews of Plasma Physics', Vol. 1 (Ed. M. A. Leontovish), p. 205 (Consultants Bureau: New York).
Hide, R. (1981). *J. Geophys. Res.* **86**, 11; 681.
Kopecky, M., and Kuklin, G. V. (1971). *In* 'Solar Magnetic Fields' (Ed. R. Howard), p. 534 (Reidel: Dordrecht).
McIntosh, P. S. (1981). *In* 'The Physics of Sunspots' (Eds L. E. Cram and J. H. Thomas), p. 7 (Sacramento Peak Observatory: Sunspot, NM).
Moffatt, K. H. (1978). 'Magnetic Field Generation by Fluid Motion' (Cambridge Univ. Press).
Pert, G. J. (1977). *J. Plasma Phys.* **18**, 227.
Simon, G. W., and Wilson, P. R. (1985). *Astrophys. J.* **295**, 241.
Topka, K., and Tarbel, T. (1983). *In* 'Small Scale Dynamical Processes in Quiet Stellar Atmospheres' (Ed. S. Keil), p. 278 (National Solar Observatory: Sunspot, NM).
van Ballegooijen, A. A., Title, A. M., and Spruit, H. C. (1985). Formation and decay of solar active regions. *Astrophys. J.* (submitted).
Wallenhorst, S. G., and Howard, R. (1981). *In* 'The Physics of Sunspots' (Eds L. E. Cram and J. H. Thomas), p. 55 (Sacramento Peak Observatory: Sunspot, NM).
Wilson, P. R. (1984). *Proc. Astron. Soc. Aust.* **5**, 500.
Wilson, P. R., and Simon, G. W. (1983). *Astrophys. J.* **273**, 805.

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