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Nonlinear Shift of Wave Parameters of Whistlers in the Ionosphere in the Presence of Negative Ions

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Abstract

In this paper the nonlinear shift of the wave number of ion–cyclotron whistlers propagating through the ionosphere in the presence of negative ions is estimated. The results are discussed both numerically and graphically for the proton whistler. It is seen that under some physical situations the nonlinear shift of the wave number of the proton whistler is very significant. Furthermore, the nonlinear shift of the wave number depends significantly on the variation of negative ion concentration when the wave frequency is very close to an ion–cyclotron frequency.

1. Introduction

Linear propagation (Gurnett *et al.* 1965; Gurnett and Brice 1966; Singh *et al.* 1976; Das and Sur 1986; Das *et al.* 1987) as well as nonlinear propagation (Matsumoto *et al.* 1980; Brinca 1981; Murtaza and Shukla 1984; Nunn 1984; Serra 1984) of whistlers has already been studied both theoretically and experimentally and used as a diagnostic tool of the ionosphere. Nonlinear propagation of whistlers in the ionosphere gives interesting results which aids the investigation of different kinds of instabilities as well as other properties of the propagating waves (Karpman *et al.* 1974*a*, 1974*b*; Matsumoto *et al.* 1980; Brinca 1981; Murtaza and Shukla 1984; Nunn 1984; and others). Recently Chakraborty *et al.* (1986) developed a theory of propagation of whistlers in a nonlinear medium and showed that due to nonlinear interactions the group travel time of the proton whistler may be changed considerably under suitable conditions. Paul *et al.* (1987) and Chakraborty *et al.* (1987) then considered the dispersion relations of Chakraborty *et al.* (1986) and studied the wave number shift and instability of the ion–cyclotron whistlers respectively. Actually, the nonlinearities of the medium create other important phenomena, e.g. shifts of frequency and wave number of the whistler which lead to modulation instabilities relevant for understanding various physical processes in solar and terrestrial atmospheres (Ter Haar and Tsytovich 1981).

However, these investigators have not considered the effects of negative ions on whistler propagation. Smith (1965) was the first to consider negative ion effects on whistler propagation in the ionosphere. Subsequently other workers (Branscomb 1964; Whitten *et al.* 1965; Das and Uberoi 1972; Das 1975; Das *et al.* 1987) have shown the effective role of negative ions on the low frequency waves in the ionosphere as well as in laboratory plasmas. However, all of these investigations only discussed

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the propagation of whistlers in a linearised theory. In the present paper, we consider nonlinear propagation of whistlers in the ionosphere in the presence of negative ions. We derive here the nonlinear dispersion relation of the whistler and calculate the wave number shift of the ion–cyclotron whistler. We also show both numerically and graphically the variation of the wave number shift with the power of the electric field and with different concentrations of negative ions.

2. Basic Equations and Dispersion Relations

The ionospheric plasma assumed here is cold, collision-less, homogeneous and consisting of one type of positive ion, one type of negative ion and electrons. The basic system of equations governing the plasma dynamics is

$$\frac{\delta \boldsymbol{v}_\alpha}{\delta t}+(\boldsymbol{v}_\alpha.\nabla)\boldsymbol{v}_\alpha=\frac{q_\alpha}{m_\alpha}\left(\boldsymbol{E}+\frac{\boldsymbol{v}_\alpha\times\boldsymbol{H}}{c}\right), \tag{1}$$

$$\frac{\delta n_\alpha}{\delta t}+\nabla.(n_\alpha\,\boldsymbol{v}_\alpha)=0, \tag{2}$$

$$\nabla\times\boldsymbol{E}=-\frac{1}{c}\frac{\delta\boldsymbol{H}}{\delta t}, \tag{3}$$

$$\nabla\times\boldsymbol{H}=\frac{1}{c}\frac{\delta\boldsymbol{E}}{\delta t}+\frac{4\pi}{c}\sum n_\alpha\,q_\alpha\,\boldsymbol{v}_\alpha, \tag{4}$$

$$\nabla.\boldsymbol{E}=4\pi\sum n_\alpha\,q_\alpha, \tag{5}$$

$$\nabla.\boldsymbol{H}=0, \tag{6}$$

where $\alpha = \mathrm{i}, \mathrm{j}, \mathrm{e}$ for positive ions, negative ions and electrons respectively, and q_α represent the charges of the αth species with proper sign. All other symbols have their usual meanings.

Now, we consider the field parameters to be perturbed:

$$\boldsymbol{E}=0+\epsilon^1\boldsymbol{E}^{(1)}+\epsilon^2\boldsymbol{E}^{(2)}+\epsilon^3\boldsymbol{E}^{(3)}+\ldots, \tag{7a}$$

$$\boldsymbol{H}=\boldsymbol{H}^{(0)}+\epsilon^1\boldsymbol{H}^{(1)}+\epsilon^2\boldsymbol{H}^{(2)}+\epsilon^3\boldsymbol{H}^{(3)}+\ldots, \tag{7b}$$

$$n_\alpha=n_\alpha^{(0)}+\epsilon^1 n_\alpha^{(1)}+\epsilon^2 n_\alpha^{(2)}+\epsilon^3 n_\alpha^{(3)}+\ldots, \tag{7c}$$

$$\boldsymbol{v}_\alpha=\epsilon^1\boldsymbol{v}_\alpha^{(1)}+\epsilon^2\boldsymbol{v}_\alpha^{(2)}+\epsilon^3\boldsymbol{v}_\alpha^{(3)}+\ldots, \tag{7d}$$

where the terms on the right-hand side with superscripts $0, 1, 2$ etc. represent zeroth, first and second order etc. values of their respective parameters.

Moreover, we assume that the first order electric field of the whistler is purely transverse and circularly polarised, and given by

$$E_\pm^{(1)}=a(\mathrm{e}^{\mathrm{i}\theta_\pm}+\mathrm{e}^{-\mathrm{i}\theta_\mp}), \tag{8}$$

where $\theta_\pm = K_\pm z - \omega t$ and a is the amplitude of the wave. The upper and lower signs in the subscripts for E, θ, K etc. represent the left circularly polarised (LCP) and right circularly polarised (RCP) waves respectively, while ω and K are the frequency and wave number of the whistler wave.

By using (7) and (8) in (1)–(6), we obtain the values of the first order quantities as

$$H_{\pm}^{(1)} = \pm \frac{\mathrm{i}\, c a}{\omega}(K_{\pm}\, \mathrm{e}^{\mathrm{i}\theta_{\pm}} + K_{\mp}\, \mathrm{e}^{-\mathrm{i}\theta_{\mp}}), \tag{9a}$$

$$v_{\alpha\pm}^{(1)} = \frac{\mathrm{i}\, q_\alpha\, a}{m_\alpha}\left(\frac{\mathrm{e}^{\mathrm{i}\theta_{\pm}}}{\omega \mp \phi_\alpha\, \Omega_\alpha} - \frac{\mathrm{e}^{-\mathrm{i}\theta_{\mp}}}{\omega \pm \phi_\alpha\, \Omega_\alpha}\right), \tag{9b}$$

$$E_z^{(1)} = 0, \qquad H_z^{(1)} = 0, \qquad v_{\alpha z}^{(1)} = 0, \qquad n_\alpha^{(1)} = 0, \tag{9c}$$

where

$$H^{(0)} = (0, 0, H^{(0)}) = \text{constant},$$

$$\Omega_\alpha = |\, q_\alpha\, H^{(0)} / m_\alpha\, c\,|,$$

$$\phi_\alpha = +1, \qquad \text{for positive ions}$$

$$= -1, \qquad \text{for electrons and negative ions.}$$

So, the first order dispersion relation of the whistler becomes

$$n_{\pm}^2 = 1 - \sum_\alpha \frac{X_\alpha}{1 \mp \phi_\alpha\, Y_\alpha}, \tag{10}$$

where $X_\alpha = \omega_{\mathrm{p}\alpha}^2/\omega^2$ and $Y_\alpha = \Omega_\alpha/\omega$, and the plasma frequency of the αth species is

$$\omega_{\mathrm{p}\alpha} = \left(\frac{4\pi\, n_\alpha^{(0)}\, e^2}{m_\alpha}\right)^{\frac{1}{2}}.$$

Using (8) and (9) in the second order equations obtained from (1)–(7) the excited field variables are derived as

$$E_z^{(2)} = \sum_\alpha \frac{\omega_{\mathrm{p}\alpha}^2\, q_\alpha\, a^2}{2\mathrm{i}\, m_\alpha\, \omega}\left(\frac{K_+}{\omega + \phi_\alpha\, \Omega_\alpha} + \frac{K_-}{\omega - \phi_\alpha \Omega_\alpha}\right)\frac{\mathrm{e}^{\mathrm{i}(\theta_+ + \theta_-)} - \mathrm{e}^{-\mathrm{i}(\theta_+ + \theta_-)}}{\omega_{\mathrm{p}}^2},$$

$$v_{\alpha z}^{(2)} = \left\{\frac{q_\alpha^2\, a^2}{m_\alpha^2}\left(\frac{K_+}{\omega + \phi_\alpha\, \Omega_\alpha} + \frac{K_-}{\omega - \phi_\alpha\, \Omega_\alpha}\right) - \frac{q_\alpha^2\, a^2(\omega_{\mathrm{p}}^2 + 4\omega^2 - \omega_{\mathrm{p}\alpha}^2)}{4\, m_\alpha^2\, \omega^2}\right.$$

$$\times \left(\frac{K_+}{\omega + \phi_\alpha\, \Omega_\alpha} + \frac{K_-}{\omega - \phi_\alpha\, \Omega_\alpha}\right) + \sum_\beta \frac{\omega_{\mathrm{p}\beta}^2\, a^2\, q_\alpha\, q_\beta}{4\, m_\alpha\, m_\beta\, \omega^2}$$

$$\left.\times \left(\frac{K_+}{\omega + \psi_\beta\, \Omega_\beta} + \frac{K_-}{\omega - \psi_\beta\, \Omega_\beta}\right)\right\}\frac{\mathrm{e}^{\mathrm{i}(\theta_+ + \theta_-)} + \mathrm{e}^{-\mathrm{i}(\theta_+ + \theta_-)}}{\omega_{\mathrm{p}}^2},$$

$$\sum_{\alpha} n_{\alpha}^{(2)} \phi_{\alpha} = \sum_{\alpha} \frac{\omega_{p\alpha}^2 a^2 \phi_{\alpha}(K_+ + K_-)}{8\pi m_{\alpha} \omega} \left(\frac{K_+}{\omega + \phi_{\alpha} \Omega_{\alpha}} + \frac{K_-}{\omega - \phi_{\alpha} \Omega_{\alpha}} \right)$$

$$\times \frac{e^{i(\theta_+ + \theta_-)} + e^{-i(\theta_+ + \theta_-)}}{\omega_p^2},$$

$$E_{\pm}^{(2)} = 0, \qquad H_{\pm}^{(2)} = 0, \qquad H_z^{(2)} = 0, \qquad v_{\alpha\pm}^{(2)} = 0, \tag{11}$$

where $\psi_{\beta} = +1$ and -1 for positive ions and for electrons and negative ions respectively, and $\omega_p^2 = (\Sigma\, \omega_{p\alpha}^2) - 4\omega^2$. Also, $\beta = \mathrm{i}, \mathrm{j}, \mathrm{e}$, but $\beta \neq \alpha$ at the same time. Now, to obtain the dispersion relation for the third order field variables we derive the two equations

$$\left(\frac{\delta}{\delta t} \pm \mathrm{i}\, \phi_{\alpha} \Omega_{\alpha} \right) v_{\alpha\pm}^{(3)} + \left(\frac{\delta v_{\alpha\pm}^{(1)}}{\delta z} \mp \frac{\mathrm{i}\, q_{\alpha} H_{\pm}^{(1)}}{m_{\alpha}} \right) v_{\alpha z}^{(2)} - \frac{q_{\alpha}}{m_{\alpha}} E_{\pm}^{(3)} = 0, \tag{12}$$

$$\left(\frac{\delta^2}{\delta t^2} - c^2 \frac{\delta^2}{\delta z^2} \right) E_{\pm}^{(3)} + 4\pi \frac{\delta}{\delta t} \left(\sum_{\alpha} n_{\alpha}^{(0)} q_{\alpha} v_{\alpha\pm}^{(3)} + \sum_{\alpha} n_{\alpha}^{(2)} q_{\alpha} v_{\alpha\pm}^{(1)} \right) = 0. \tag{13}$$

Substituting (10) and (11) in (12) and (13) and then considering the first harmonic part of the third order electric field, the nonlinear dispersion relation for the LCP and RCP waves are obtained as

$$\frac{c^2 K_{\pm}^2}{\omega^2} = 1 - \sum \frac{\omega_{p\alpha}^2}{\omega(\omega \mp \phi_{\alpha} \Omega_{\alpha})} \pm \frac{1}{\omega_p^2} \sum_{\alpha,\beta} \phi_{\alpha} \left(\frac{(\omega_{p\alpha}^2 - \omega_p^2)\omega_{p\alpha}^2 q_{\alpha}^2}{4 m_{\alpha}^2 \omega^4} \right.$$

$$\times \left. \frac{a^2 \Omega_{\alpha} K_{\mp}}{\omega^2 - \Omega_{\alpha}^2} + \frac{\omega_{p\alpha}^2 \omega_{p\beta}^2 q_{\alpha}^2 a^2 \Omega_{\beta} K_{\mp}}{4 m_{\alpha} m_{\beta} \omega^4 (\omega^2 - \Omega_{\beta}^2)} \right) \left(\frac{K_+}{\omega + \phi_{\alpha} \Omega_{\alpha}} + \frac{K_-}{\omega - \phi_{\alpha} \Omega_{\alpha}} \right)$$

$$+ \frac{a^2 (K_+ + K_-)}{2\omega_p^2} \sum_{\alpha} \frac{q_{\alpha}^2 \omega_{p\alpha}^2 (\omega_{p\alpha}^2 - \omega_p^2)}{4 m_{\alpha}^2 \omega^4} \left(\frac{K_{\pm}}{(\omega \pm \phi_{\alpha} \Omega_{\alpha})^2} \right.$$

$$\left. + \frac{K_{\mp}}{\omega^2 - \Omega_{\alpha}^2} \right) + \frac{a^2 (K_+ + K_-)}{2\omega_p^2} \sum_{\alpha,\beta} \phi_{\alpha} \psi_{\alpha} \frac{\omega_{p\alpha}^2 \omega_{p\beta}^2 q_{\alpha}^2}{4 m_{\alpha} m_{\beta} \omega^4 (\omega \pm \phi_{\alpha} \Omega_{\alpha})}$$

$$\times \left(\frac{K_+}{\omega + \psi_{\beta} \Omega_{\beta}} + \frac{K_-}{\omega - \psi_{\beta} \Omega_{\beta}} \right), \tag{14}$$

where $K_{\pm}$ on the right-hand side represents the wave number of the LCP and RCP waves arising from the first order solution respectively (and again the upper and lower signs represent the LCP and RCP waves respectively).

For an ion–cyclotron whistler, the wave frequency asymptotically approaches the ion–cyclotron frequency. In the resonance case, i.e. when the wave frequency is very close to an ion–cyclotron frequency, the whistler is most prominent and the dispersion relation (14) simplifies to

$$\frac{c^2K_+^2}{\omega^2}=\frac{\omega_{\rm pi}^2}{\Omega(\Delta\omega)}-\frac{3\alpha^2c^2\omega_{\rm pi}^2K_-^2(\omega^2+3\Omega\omega+2\Omega^2)}{8\Omega^3(\Delta\omega)^2(\Omega+\omega)}$$

$$-\frac{\alpha^2c^2\omega_{\rm pi}^2K_+K_-(3\Omega+\omega)}{8\Omega^3(\Delta\omega)(\omega+\Omega)^2}+\frac{\chi\alpha^2c^2\omega_{\rm pi}^2K_+K_-(3\omega^2-2\Omega\omega+\Omega^2)}{8\Omega^3(\Delta\omega)^3(\omega+\Omega)}$$

$$+\frac{\chi\alpha^2c^2\omega_{\rm pi}^2K_-^2(\omega^2+5\omega\Omega+2\Omega^2)}{8\Omega^3(\Delta\omega)(\omega+\Omega)^2}+\frac{\chi\alpha^2c^2\omega_{\rm pi}^2K_+^2(2\Omega-\omega)}{8\Omega^3(\Delta\omega)(\Omega+\omega)},\tag{15}$$

$$\frac{c^2K_-^2}{\omega^2}=\frac{\chi\omega_{\rm pi}^2}{\Omega(\Delta\omega)}-\frac{\alpha^2c^2\omega_{\rm pi}^2}{8\Omega^3(\Delta\omega)(\Omega+\omega)^2}-\frac{\chi\alpha^2c^2\omega_{\rm pi}^2K_+^2}{8\Omega^2(\Delta\omega)^2}$$

$$+\frac{\chi\alpha^2c^2K_+K-\omega_{\rm pi}^2}{8\Omega^3(\omega+\Omega)}+\frac{\alpha^2c^2\omega_{\rm pi}^2K_+K_-}{8\Omega^3(\omega+\Omega)}$$

$$+\frac{\chi\alpha^2c^2\omega_{\rm pi}^2K_-^2}{8\Omega^2(\omega+\Omega)^2}-\frac{\alpha^2c^2\omega_{\rm pi}^2K_-^2(2\omega-\Omega)}{8\Omega^3(\Delta\omega)^2},\tag{16}$$

where $\alpha=ea/m_{\rm i}\,\omega c$ denotes the dimensionless amplitude of the wave, $\chi=n_{\rm j}^{(0)}/n_{\rm i}^{(0)}$, $\Delta\omega=\Omega-\omega$, and $\Omega_{\rm i}=\Omega_{\rm j}=\Omega$. Since K_+ and K_- on the right-hand sides of (15) and (16) arise from the first order solution, we substitute from the linearised approximation $K_+=\omega_{\rm pi}\,\omega^{1/2}/c(\Delta\omega)^{1/2}$ and $K_-=\chi^{1/2}\omega^{1/2}\omega_{\rm pi}/c(\Delta\omega)^{1/2}$ and obtain

$$\frac{c^2K_+^2}{\omega^2}=\frac{\omega_{\rm pi}^2}{\Omega(\Delta\omega)}-\frac{\alpha^2(\chi^{\frac{1}{2}}-2\chi^2)\omega_{\rm pi}^4}{2(\Delta\omega)^2(\Omega+\omega)^2}+\frac{\alpha^2(\chi^{\frac{3}{2}}-3\chi)\omega_{\rm pi}^4}{4(\Delta\omega)^3(\Omega+\omega)},\tag{17}$$

$$\frac{c^2K_-^2}{\omega^2}=\frac{\chi\omega_{\rm pi}^2}{\Omega(\Delta\omega)}-\frac{\alpha^2(1-\chi^2)\omega_{\rm pi}^4}{2(\Delta\omega)^2(\Omega+\omega)^2}+\frac{\alpha^2\chi^{\frac{1}{2}}(1+\chi)\omega_{\rm pi}^4}{8\Omega^2(\Delta\omega)(\Omega+\omega)}$$

$$-\frac{\alpha^2\chi\omega_{\rm pi}^4}{4\Omega(\Delta\omega)^3}.\tag{18}$$

3. Nonlinear Wave Number Shift

To obtain the wave number shift of the waves we substitute $K_+=K+\delta k_+$ and $K_-=K+\delta k_-$ in equations (17) and (18), where δk_+ and δk_- are the wave number shifts from their initial values K, with $|K|>|\delta k_\pm|$ (Chakraborty and Paul 1983; Paul *et al.* 1987). Neglecting the higher order terms in δk_+ and δk_- we obtain

$$\delta k_+=-\frac{\alpha^2\omega_{\rm pi}^3}{4c\omega^2(Y-1)^{\frac{3}{2}}(Y+1)}\left(\frac{3\chi-\chi^{\frac{3}{2}}}{2(Y-1)}+\frac{\chi^{\frac{1}{2}}-2\chi^2}{Y+1}\right),\tag{19}$$

$$\delta k_{-} = -\frac{\alpha^2\omega_{\mathrm{pi}}^3}{4c\chi^{\frac{1}{2}}(Y-1)^{\frac{1}{2}}}\left(\frac{1-\chi^2}{\omega^2(Y+1)^2(Y-1)} - \frac{\chi^{\frac{1}{2}}(1+\chi)}{4\Omega^2(Y+1)} + \frac{\chi}{2\Omega\omega(Y-1)^2}\right), \tag{20}$$

where $Y = \Omega/\omega$. The two quantities are obviously the mutually complementary nonlinear wave number shifts of the LCP and RCP waves. The average shift $\delta K = \frac{1}{2}(\delta k_{+} + \delta k_{-})$ is the average nonlinearly correct wave number shift of the whistler wave and is given by

$$\delta K = -\frac{\alpha^2\omega_{\mathrm{pi}}^3}{8c(Y-1)^{\frac{1}{2}}}\left\{\frac{1}{(Y-1)(Y+1)\omega^2}\left(\frac{3\chi-\chi^{\frac{3}{2}}}{2(Y-1)} + \frac{\chi^{\frac{1}{2}}-2\chi^2}{Y+1}\right) + \chi^{-\frac{1}{2}}\left(\frac{1-\chi^2}{(Y+1)^2(Y-1)\omega^2} - \frac{\chi^{\frac{1}{2}}(1+\chi)}{4\Omega^2(Y+1)} + \frac{\chi}{2\Omega\omega(Y-1)^2}\right)\right\}. \tag{21}$$

It can be seen from (21) that the wave number shift depends on the intensity of the electric field (i.e. on α), the negative ion concentration relative to positive ions, the ion–cyclotron frequency, and the plasma frequency.

4. Results and Discussion

For a numerical estimate of the dependence of $|\delta K|$ on α and χ, we consider the proton whistler record on 11 January 1963 in Injun-3 (Gurnett and Brice 1966), for which $\Omega = 410{\cdot}3$ Hz, $\Omega - \omega = 3{\cdot}7$ Hz and $n(\mathrm{H}^+) = 12{\cdot}5\times10^3\ \mathrm{cm}^{-3}$. Moreover, we have taken the value of α to be of the order of 10^{-4}, i.e. $a \sim 10^{-1}\ \mathrm{V\,m}^{-1}$ which is relevant for the ionospheric cold plasma where the density of the plasma is small and the magnetic field is weak (Paul *et al.* 1987). Since the negative ion concentration relative to the positive ion concentration in the ionosphere varies with height from 0 to $0{\cdot}5$ (Arnold and Krankowsky 1979), we consider low as well as high concentrations of negative ions. To show the variational effects of negative ions, as well as nonlinear effects on the estimate of wave number shifts, we plot $|\delta K|$ against χ in Fig. 1 for four values of α. For the particular value $\alpha = 1{\cdot}8\times10^{-4}$, $|\delta K|$ varies from $2{\cdot}41\times10^{-7}$ to $6{\cdot}67\times10^{-7}\ \mathrm{cm}^{-1}$, when χ varies from $0{\cdot}1$ to $0{\cdot}5$. It is to be noted that the initial values of $K = \frac{1}{2}(k_{+} + k_{-})$ are $5{\cdot}37\times10^{-6}$, $6{\cdot}31\times10^{-6}$ and $6{\cdot}96\times10^{-6}\ \mathrm{cm}^{-1}$ for $\chi = 0{\cdot}1$, $0{\cdot}3$ and $0{\cdot}5$ respectively. So, we see that the wave number shift due to nonlinear interaction of the wave and the plasma will be $4{\cdot}5\%$ to $9{\cdot}6\%$ when $\alpha = 1{\cdot}8\times10^{-4}$. We also plot $|\delta K|$ against α in Fig. 2 for different concentrations. When χ is constant, i.e. $\chi = 0{\cdot}3$ (say), $|\delta K|$ varies from $1{\cdot}48\times10^{-7}$ to $10{\cdot}03\times10^{-7}\ \mathrm{cm}^{-1}$ when α varies from 1×10^{-4} to $2{\cdot}6\times10^{-4}$. The above estimates show that the nonlinear wave number shifts of the whistler become $2{\cdot}35\%$ to 16%, which could be experimentally verified.

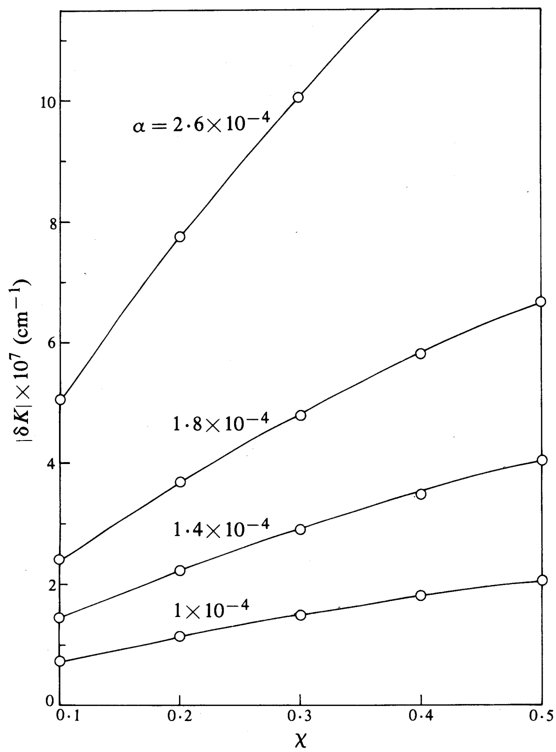

Fig. 1. Variation of $|\delta K|$ with χ.

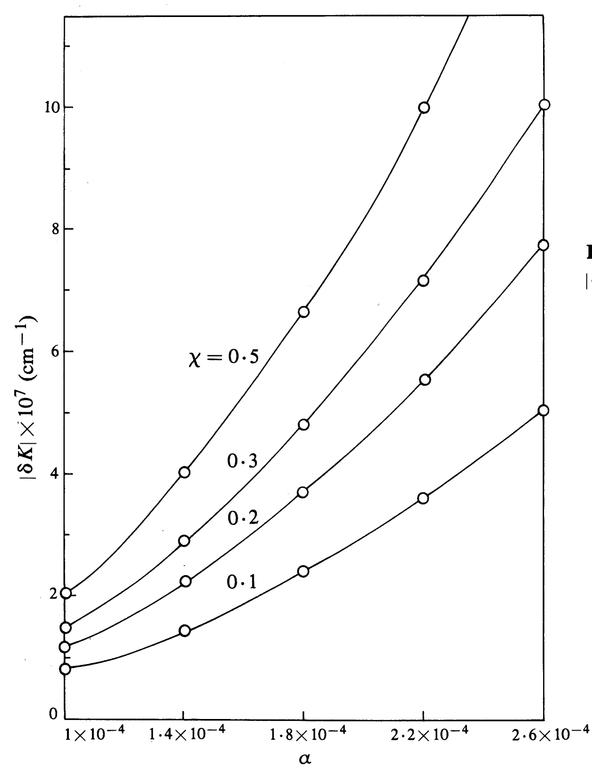

Fig. 2. Variation of $|\delta K|$ with α.

5. Concluding Remarks

In the present paper we have obtained the wave number shift of ion–cyclotron whistlers in the presence of negative ions as well as nonlinear effects. This theoretical work shows that negative ions effects have a measurable contribution in addition to the nonlinear effects. Thus, the inclusion of negative ion effects in the estimation of the wave number shift is essential to obtain a more accurate result. However, our present work has some limitations and could be extended by considering the following effects:

(1) Consideration of thermal effects on the propagation of whistlers (Singh and Tolpadi 1975; Singh *et al.* 1976; Das and Sur 1986) in nonlinear plasmas would give more interesting results.

(2) The stream velocities of the species of the plasma and the newly born ion effects should be taken into account (Das and Sur 1986) in a further study of whistler waves.

(3) The slow variation of the number density of the plasma species in the ionosphere will have a significant role in whistler propagation. This propagation problem could be studied by using the WKB method (Budden 1961; Wait 1962; Chakraborty 1970; Khan and Paul 1977).

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