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Magnetic Moments of Baryons in a Relativistic Constituent Quark Model

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Abstract

We utilise the light-front dynamical formalism to compute the magnetic moments of baryons in a relativistic constituent quark model, investigating the dependence of the results on the mass of the constituent quarks and on the size of the hadron.

1. Introduction

One of the early successes of the nonrelativistic constituent quark model was the calculation of the ratio of the neutron and proton magnetic moments (de Rujula *et al.* 1975), providing a microscopic justification of the SU(6) result (Beg *et al.* 1964) and obtaining a value within 20% of the experimental value. Taking the quark model seriously, one can then determine the quark masses from the magnetic moments, and go on to predict the magnetic moments of the octet baryons. De Rujula *et al.* (1975) obtained magnetic moments which agreed, within the rather large experimental uncertainties, with the experimental values of the time.

However, that same paper contains estimates of the size of the hadron wavefunction through the hyperfine splitting calculation of the N–Δ mass difference which depends on the wavefunction when two quarks are close together. If we assume a gaussian wavefunction (see e.g. Igsur and Karl 1977) the N–Δ mass difference corresponds to an r.m.s. nucleon radius of 0·5 fm. Other estimates of the r.m.s. radius vary from 0·8 fm (from the charge radius) to 0·4 fm (from hyperon decays); these have been reviewed by Thomas and McKellar (1984). Through the uncertainty principle these values of the r.m.s. radius imply quark momenta in the range 250–500 MeV/c, which are to be compared with the constituent quark mass of 330 MeV/c^2. It is clear that, while the nonrelativistic quark model has a great deal of heuristic value, an adequate dynamical model of the structure of the hadrons will require a relativistic approach, as recently emphasised by Scadron *et al.* (1988).

Once one admits the necessity for relativistic dynamics, the success of the de Rujula *et al.* calculation of the hadron magnetic moments becomes somewhat mysterious. Corrections to the nonrelativistic magnetic moments

$$\mu = \sum_i \frac{Q_i}{2m_i} \tag{1}$$

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(in units with $\hbar = c = 1$) are of order $\langle p^2\rangle/m^2$ and can be expected to be large. However, in the extreme relativistic limit one expects on dimensional grounds that

$$\mu = \sum_i \frac{Q_i}{(\langle p_i^2\rangle)^{\frac{1}{2}}}. \tag{2}$$

As $(\langle p_i^2\rangle)^{\frac{1}{2}}$ is not too different from m_i, at least on the above estimates, and as the result must interpolate from (1) to (2), it is not surprising that the nonrelativistic quark model gives results which are approximately correct. Moreover, the most recent experimental values (Particle Data Group 1986; Wilkinson *et al.* 1987; Zapalac *et al.* 1986) for the moments of the octet differ from the static constituent quark model values by about 20% in some cases, so there is some indication that an improved calculation is necessary.

Table 1. Our results for the baryon magnetic moments

Magnetic moment (n.m.)	Experiment[A]	Static[B]	Present work[C]	Present work[D]
$\mu(\mathrm{p})$	2·793	2·79	2·53	2·94
$\mu(\mathrm{n})$	−1·913	−1·86	−1·50	−1·92
$\mu(\Lambda)$	−0·613±0·005	−0·60	−0·63	−0·71
$\mu(\Sigma^+)$	2·429±0·020	2·67	2·97	3·52
$\mu(\Sigma^-)$	−1·166±0·017	−1·05	−1·25	−1·40
$\mu(\Xi^0)$	−1·250±0·014	−1·39	−1·71	−2·03
$\mu(\Xi^-)$	−0·69±0·04	−0·46	−0·84	−0·72

[A] Particle Data Group (1986); Wilkinson *et al.* (1987); and Zapalac *et al.* (1986).
[B] de Rujula *et al.* (1975).
[C] Our results for $m_{\mathrm{u/d}} = 363$ MeV, $m_{\mathrm{s}} = 538$ MeV and $a = 250$ MeV (see Fig. 3).
[D] Our results for $m_{\mathrm{u/d}} = 300$ MeV, $m_{\mathrm{s}} = 500$ MeV and $a = 150$ MeV (see Fig. 2).

Table 1 shows the values of the magnetic moments of the octet baryons measured experimentally, together with those given by the nonrelativistic quark model and by the present calculation.

One way of performing a relativistic dynamical calculation is to use the light-front dynamics introduced by Dirac (1949). Light-front coordinates $x^{\pm} = x^0 \pm x^3$ and $\boldsymbol{x} = (x^1, x^2)$ are used, x^+ playing the role of time and $p_+ = (m^2 + \boldsymbol{p}^2)/2p_-$ that of the Hamiltonian. The advantage of this formalism is that, as both p_+ and p_- (which is conserved) are positive, the creation of virtual particle–antiparticle pairs from the vacuum is forbidden and the number of particles in the system is conserved.

Berestetskii and Terentev (1977) showed how to use this scheme, as it had been developed by Terentev and Berestetskii (1976) and Terentev (1976) to include the Melosh (1974) transformation to 'untangle' orbital angular momentum and spin operators, to provide a formalism for the calculation of magnetic moments and electromagnetic form factors of the baryons. Aznauryan and Ter-Isaakyan (1980) have used this method to estimate magnetic moments of the octet baryons, and Aznauryan *et al.* (1982*a*, 1982*b*) have introduced quark anomalous magnetic moments to improve the fit.

In one sense it may be suggested that these authors have solved the problem of a relativistic determination of the magnetic moments of the baryons. However, they give results only for a particular choice of the parameters (quark masses, mean momentum spread etc.) and give no indication of the sensitivity of the results to the parameters.

Dziembowski and Mankiewicz (1985, 1987) used a similar (but not identical) prescription to obtain magnetic moments from light plane wavefunctions which are claimed to be 'parameter free', but are again quoted only for one particular choice of parameters.

It is our purpose in this paper to investigate the sensitivity of the results obtained by light cone methods for the magnetic moments to the choice of parameters, thus showing how it recovers the static quark model results and the ultra-relativistic ('bag' type) results in the appropriate limits and how it interpolates between them.

2. Formalism

The baryon matrix element of the electromagnetic current j_μ defines the electromagnetic vertex function Γ_μ by

$$\langle B(P_{\text{out}})|j_\mu(k)|B(P_{\text{in}})\rangle = (2\pi)^6\delta^3(p_{\text{out}}+k-p_{\text{in}})\Gamma_\mu\,, \tag{3}$$

and as is well known current conservation and Lorentz invariance restrict Γ_μ to have the form

$$\Gamma_\mu = \bar{u}_{p_{\text{out}}}\{F_1(k^2)\gamma_\mu + F_2(k^2)\,(2m)^{-1}\sigma_{\mu\nu}\,k^\nu\}\,u_{p_{\text{in}}}\,. \tag{4}$$

With the following representation for the γ matrices

$$\gamma_3 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad \gamma_0 = -\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

$$\gamma_5 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}, \qquad \gamma_j = \mathrm{i}\epsilon_{jn}\begin{pmatrix} \sigma_n & 0 \\ 0 & -\sigma_n \end{pmatrix}, \tag{5}$$

($j = 1$ or 2) the free particle solution to the Dirac equation is

$$u(p) = 2^{-\frac{1}{4}}(2p_-)^{\frac{1}{2}}\begin{pmatrix} \phi_\lambda \\ (m - \mathrm{i}\,p\,.\,\epsilon\,.\,\sigma)/(2p_-)^{\frac{1}{2}}\phi^\lambda \end{pmatrix}; \qquad \phi^\gamma(\alpha) = \delta^\gamma_\alpha\,, \tag{6}$$

and using the quark charge operator C_ϵ ($\epsilon = a, b, c$) the quark current is

$$w_-(k) = \sum_{\epsilon=a,b,c} C_\epsilon \exp(-\mathrm{i}\,k_\mu\,x^\mu_\epsilon)\,. \tag{7}$$

So (3) becomes

$$\{F^i_1\,\delta_{s's} + (2M_{\mathrm{B}})^{-1}(k\,.\,\epsilon\,.\,\sigma_{s's})F^i_2\}\delta_{ii'} = 3\int \mathrm{d}\Gamma\,\psi^*_{i's'}\,C_c\exp(-\mathrm{i}\,k_j\,X^j_c)\psi_{is} \tag{8}$$

in the infinite momentum limit. In (8) the symmetry of the wavefunction has been utilised in recasting the matrix element in (3) into a form which only involves the position X^j_c and charge C_c operators for one of the quarks. The integration measure $\mathrm{d}\Gamma$ for the intrinsic degrees of freedom and details of the derivation of (8) can be found in Berestetskii and Terentev (1977). The expression is essentially identical with that by Brodsky and Drell (1980).

In the limit $k \to 0$

$$F_1^i \delta_{i'i} \delta_{s's} = 3\langle \chi'_{i's'} | C_c | \chi_{is} \rangle = C_i \delta_{i'i} \delta_{s's}, \tag{9}$$

so that

$$\lim_{k \to 0} F_1^i = C_i,$$

where

$$C_i = \sum_{\epsilon = a,b,c} C_\epsilon$$

is the nuclear charge, and also

$$(\mathrm{i}/2M_\mathrm{B})(k.\epsilon.\sigma_{s's})F_2^i(0) = -3 \int \psi^*_{i's'} C_c \eta k_j \frac{\partial}{\partial Q_j} \psi_{is} \,\mathrm{d}\Gamma, \tag{10}$$

where the internal variables η, ξ, Q, q are identical with that used by Berestetskii and Terentev (1977). Explicitly, the momenta of the three quarks a, b, c are given in terms of the internal variables, the total transverse momentum $\boldsymbol{P}$ and total 'minus' momenta P_-:

$$\boldsymbol{p}_a = \boldsymbol{q} + \xi \boldsymbol{Q} + \xi \eta \boldsymbol{P},$$

$$\boldsymbol{p}_b = -\boldsymbol{q} + (1-\xi)\boldsymbol{Q} + (1-\xi)\eta \boldsymbol{P},$$

$$\boldsymbol{p}_c = -\boldsymbol{Q} + (1-\eta)\boldsymbol{P};$$

$$p_{a-} = \xi\eta P_-, \qquad p_{b-} = (1-\xi)\eta P_-, \qquad p_{c-} = (1-\eta)P_-.$$

Since $F_2^i(0) = \kappa$, the anomalous moment, by operating with the Melosh transformation on the operator in (10) we then obtain

$$(k.\epsilon.\sigma_{s's})\kappa^i/2M_\mathrm{B} = 3k.\epsilon_j \int \psi^*_{i's'}(C_c/2M_0) \sum_{\epsilon = a,b,c} \sigma_j^\epsilon \gamma^\epsilon \psi_{is} \,\mathrm{d}\Gamma, \tag{11}$$

with

$$\gamma^a = -\frac{2\xi\eta}{1-\eta} \frac{(1-\eta)\xi\eta M_0^2 + (1-\eta)m_a M_0 - \frac{1}{2}\boldsymbol{Q}\cdot(\boldsymbol{q}+\xi\boldsymbol{Q})}{(\boldsymbol{q}+\xi\boldsymbol{Q})^2 + (m_a + \xi\eta M_0)^2},$$

$$\gamma^b = -\frac{2(1-\xi)\eta}{1-\eta} \frac{(1-\eta)(1-\xi)M_0^2 + (1-\eta)m_b M_0 - \frac{1}{2}\boldsymbol{Q}\cdot\{\boldsymbol{q}+(1-\xi)\boldsymbol{Q}\}}{\{\boldsymbol{q}+(1-\xi)\boldsymbol{Q}\}^2 + \{m_b + (1-\xi)\eta M_0\}^2},$$

$$\gamma^c = 2\frac{\eta(1-\eta)M_0^2 + \eta m_c M_0 - \frac{1}{2}Q^2}{Q^2 + \{m_c + (1-\eta)M_0\}^2},$$

$$M_0^2 = \frac{Q^2}{\eta(1-\eta)} + \frac{q^2}{\eta(1-\xi)\xi} + \frac{m_a^2}{\eta\xi} + \frac{m_b^2}{\eta(1-\xi)} + \frac{m_c^2}{1-\eta}. \tag{12}$$

If the quark masses are identical, as for the nucleons, there is a high degree of symmetry in this expression and it can be shown that

$$\gamma^a = \gamma^b = -\left(\frac{1-\eta}{\eta}\right)\gamma^c \tag{13}$$

under the integral in (11). Using this result we obtain

$$(k.\epsilon.\sigma_{s's})\kappa^T/2M_N = -3k.\epsilon_j \int \psi^*_{TS'}(C_c/2M_0)\gamma^c\left\{\frac{\sigma^c_j}{\eta} - \left(\frac{1-\eta}{\eta}\right)\sigma_j\right\}\psi_{TS}\,\mathrm{d}\Gamma \tag{14}$$

for the nucleon case.

We use the usual SU(6) spin–isospin wavefunctions listed by Thirring (1966), and so obtain

$$\kappa^T/2M_N = \int \mathrm{d}\Gamma\,(|\phi|^2/2M_0)\gamma^c\{4\tau_3+(\tfrac{2}{3}-\eta)(2\tau_3-\tfrac{1}{2})/\eta\}, \tag{15}$$

with γ^c given in (12), and $\phi(\eta, \xi, Q, q)$ is the momentum wavefunction. Equation (15) was derived by Berestetskii and Terentev (1977) and was utilised by Aznauryan *et al.* (1982*a*, 1982*b*) in their derivation of nucleon moments. It is a six-fold integral over all internal momenta, and for evaluation requires a choice of wavefunction and quark masses. Similar expressions for hyperon moments may be obtained in the same way, and have been listed by Aznauryan and Ter-Isaakyan (1980). It is important to note that in the nonrelativistic limit we reproduce the SU(6) relation $\mu_n/\mu_p = -\frac{2}{3}$ of Beg *et al.* (1964), but cannot reconcile this theory with that of the nonrelativistic quark model result by de Rujula *et al.* (1975), as in this limit we obtain

$$\mu_p = -1+\tfrac{2}{3}M_N/m, \qquad \mu_n = -\tfrac{2}{3}M_N/m,$$

against their result $\mu_p = M_N/m$ and $\mu_n = -\frac{2}{3}M_N/m$. However, in this limit we are constrained to $m = M_N/3$, whereas they are not. When this constraint is imposed the results agree. Aznauryan *et al.* have chosen a spherically symmetric wavefunction in momentum space dependent on the total momentum of the quarks in a gaussian fashion:

$$\Phi(k^2) = N\exp(-M_0^2/\alpha^2), \tag{16}$$

with M_0 defined in equation (12). This constant α and the up/down quark mass (denoted hereafter by m) are free parameters not determined by the theory.

In order to investigate the light-front approach fully we have chosen to treat quarks as Dirac point particles, excluding the quark anomalous moments introduced by Aznauryan *et al.* This reduces the number of 'free' parameters to a minimum.

3. Results and Discussion

Aznauryan *et al.* (1982*a*, 1982*b*) have used the integral (15) with wavefunction (16) to calculate nucleon magnetic moments for a chosen set of values for m and α. In addition, they calculated correction terms using the unknown quark anomalous magnetic moments, and obtained a fit to these parameters to reproduce experimental results accurately. In quoting suitable m and α for the fit they included large uncertainties (of order 15%). Aznauryan and Ter-Isaakyan (1980) in the same way calculated a number of baryon octet moments, using a very different choice of the available parameters, but obtaining results in good agreement with experimental values of the time. Again, quoted uncertainties in parameters were quite large. Unfortunately, the number of undetermined parameters in the theory is high and so its applicability is unclear.

To clarify this situation we have calculated baryon magnetic moments for nucleons and hyperons in the above formalism, excluding the anomalous moment corrections. We have reduced the six-fold integral (15) to a one parameter three-fold integral, which

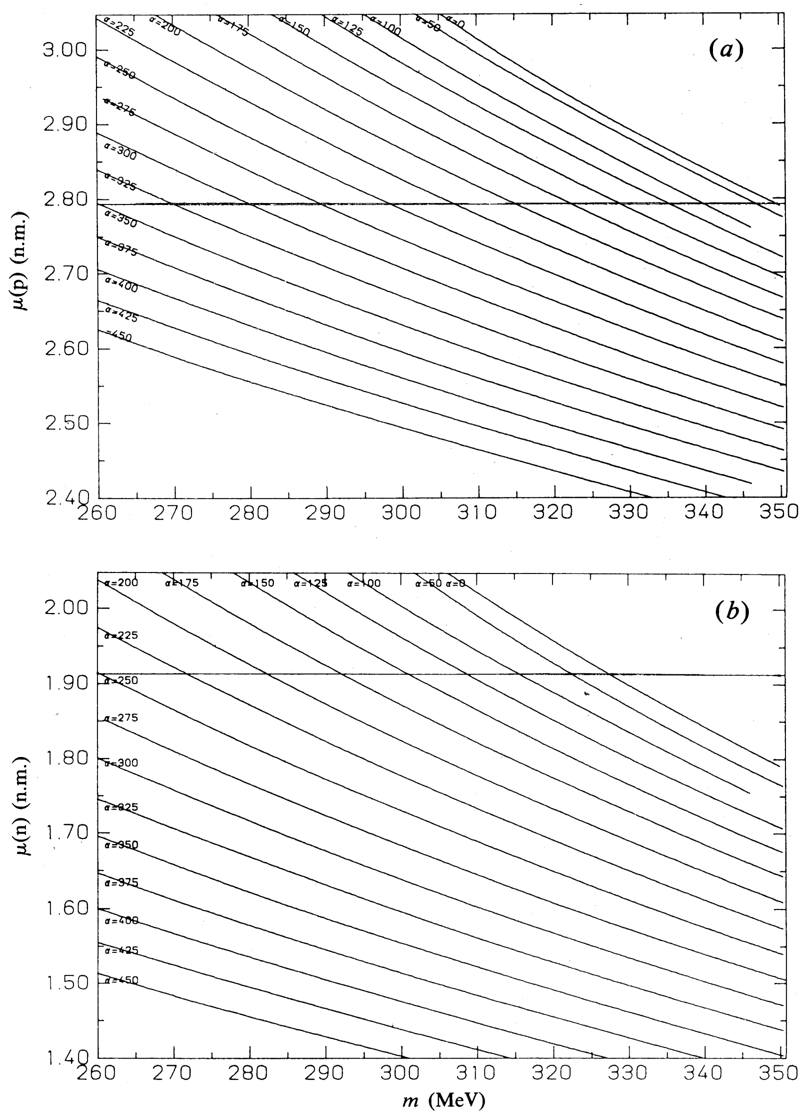

Fig. 1. Calculated (a) proton and (b) neutron magnetic moments as functions of the up/down quark mass m and α.

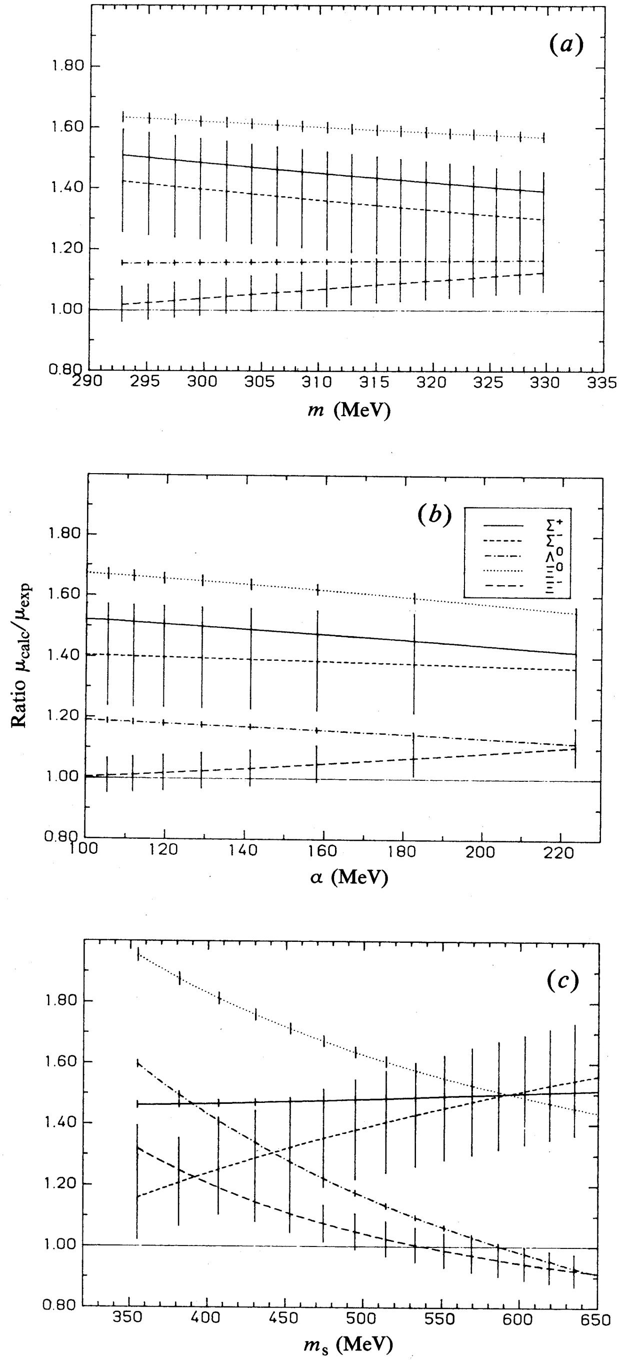

Fig. 2. Calculated hyperon magnetic moments as functions of (*a*) up/down mass m (α = 150 MeV and m_s = 500 MeV), (*b*) α (m = 300 MeV and m_s = 500 MeV) and (*c*) strange mass m_s (α = 150 MeV and m = 300 MeV).

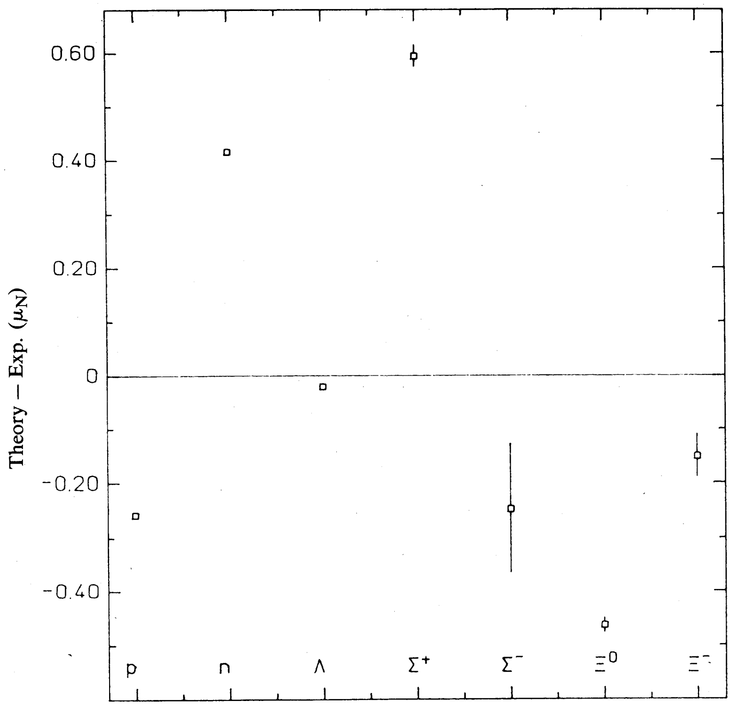

Fig. 3. Plot of the difference between theoretical and experimental values of baryon magnetic moments with $m = 363$ MeV, $\alpha = 250$ MeV and $m_s = 538$ MeV.

we have evaluated numerically using a Chebychev polynomial technique. Results were found for values of α ranging between 100 and 220 MeV, m ranging between 290 and 335 MeV, and m_s (the strange quark mass) between 320 and 650 MeV. These values were chosen as most similar theories acknowledge reasonable quark masses and momenta to lie in this region. We show our results in Figs 1–3.

Several observations may be made from these figures. Some features of Fig. 1 are:

The general inverse dependence of the nucleon anomalous moment on α. As α increases, the curves tend to zero. As α decreases, the curves tend to the nonrelativistic form $|\kappa^T| = 2M_N/3m$. This is a model independent feature of the magnetic moment operator.

The nucleon graphs have a very similar form, due to the almost identical nature of the integral expressions. This illustrates the isospin symmetry of the constituent quarks.

There is no choice for α and m for which both the empirical proton and neutron magnetic moments are obtained. There is a wide range of parameters for which theory and experiment agree moderately well, though due to their interdependence a choice in one restricts the choice of the other.

Similar features may be observed in Fig. 2. These graphs show the same general dependence on α and m, but are complicated by the large strange mass. This mass is a new parameter on which results depend to the same degree as α and m. Again, it is impossible to reconcile the calculated results with the experimental values for any single set of α, m and m_s values. Discrepancies, particularly in the case of the Ξ^0

particle, range up to 20%. Again, inclusion of quark anomalous moments is able to correct these discrepancies, at the expense of predictive power.

Dziembowski and Mankiewicz (1985) used similar techniques to those described in this paper to calculate SU(6) baryon moments, with a different choice of wavefunction and a particular choice of values for the relevant parameters. They were able to achieve a remarkably close fit of hyperon moments to observed values, although their nucleon results are somewhat worse. Fig. 3 shows results obtained from our calculations using their choice of parameters (cf. graph 1 of their paper). The results are comparable for the nucleon moments, but differ markedly for most hyperon values. This is perhaps not surprising given the sensitivity of the expressions for hyperon moments, in particular to the choice of wavefunction.

Taking the limit $\alpha \rightarrow \infty$ in expression (16) we would expect this formalism to be comparable with results obtained from bag model calculations (Thomas 1985). In fact, as with the nonrelativistic case the proton charge contribution to the moment is independent of this limit, in contrast to its dependence on bag radius in bag model calculations. The bag model result is that the total moment μ is proportional to R, whereas we find the anomalous moment κ to be proportional to R. From the Drell–Hearn (1966) sum rule we expect that $\kappa \propto R$, so we regard this discrepancy between the light cone results and the bag results as a point in favour of the light cone approach.

Nevertheless, for a comparison with the bag results we compare neutron moments to avoid this problem. Our results show that

$$\mu_n = -1\cdot 12 M_{\mathrm{N}}/\alpha\,(\mathrm{n.m.}) \quad (\alpha \rightarrow \infty),$$

whereas the bag results show that

$$\mu_n = -0\cdot 53 M_{\mathrm{N}}/\alpha\,(\mathrm{n.m.}) \quad (\alpha \rightarrow \infty).$$

For a bag we use the value of the mean squared momentum to define an equivalent value of α

$$\tfrac{9}{8}\alpha^2 = \langle q^2 \rangle = (2\cdot 04/R)^2$$

in the ultra-relativistic limit.

The two limits are certainly identical in form and similar in magnitude. However, this could well be a manifestation of dimensional analysis considerations in that there is only one momentum scale in this limit, so the two results must scale in similar ways.

4. Conclusions

Our results show how a consistent description of relativistic hadron dynamics is sensitive to the choice of parameters. In particular, they highlight the need for care in the choice of these parameters in hadron calculations. As noted previously, it is difficult to obtain a simultaneous close fit to all baryon moments using valence quarks alone. This suggests that corrections are needed to account for contributions from sea quarks. Two different approaches to this problem have been used. Aznauryan *et al.* (1982*a*, 1982*b*) invoked the concept of effective 'quark anomalous moments' which they included in their calculations, while Cohen and Weber (1985) included pionic corrections in their moment calculations. Of the two approaches, the latter is perhaps more desirable, as all couplings are determined by SU(3), while the quark moments of Aznauryan *et al.* enter as free parameters at this stage of our understanding.

Parenthetically, we remark that these results differ appreciably from those which would be obtained as $(\alpha/m)^2$ corrections to the nonrelativistic results (Aznauryan *et al.* 1982); this reflects the relativistic nature of the problem in that α/m is not a small parameter.

When considering the similarity between results from this and the nonrelativistic theory one might have expected that the present approach would agree at zeroth order with the findings of de Rujula *et al.* (1975) and introduce higher order corrections to that result. However, we approach the de Rujula *et al.* results only in the very strict nonrelativistic limit that we choose $m = M_N/3$ in both cases. This is due to the form of our equation (9)—for this component of current there is no contribution from F_1 linear in k, so we evaluate the integral for $F_2^i(0) = \kappa$ only. This is an important feature of the light-front formalism and infinite momentum limit employed here. It shows that the present formalism correctly boosts the total charge to obtain the normal contribution to the moment.

Furthermore, the light cone analysis is known to reproduce the familiar QED values for the electron moment (Brodsky and Drell 1980). We therefore believe that the light cone formalism offers the best available way of approaching the problem of computing the magnetic moments of the baryons.

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